

Department of Mathematics
 CSJM University Kanpur
 Abstract Algebra (MMA201)
 Semester II:2021-22(Even Semester)
 1st Mid Sem Examination

Time: 1.5 h

M.M:20

Section A (8*1=8)

Q.1. All questions are compulsory.

- (a) If G is abelian group then all subgroups of G are *normal*
- (b) Number of cyclic subgroups of order 2 in S_3 = $\frac{\# \text{ elements of order 2 in } S_3}{\phi(2)} = \frac{3}{1} = 3$
- (c) If G is a finite group then any two sylow-p subgroups of G are *conjugate*
- (d) $Z_3 \times Z_7$ is always *abelian* group.
- (e) Number of elements of order 2 in D_6 = $\begin{cases} n+1 & n \text{ even} \\ n & n \text{ odd} \end{cases}$
- (f) Class equation of Z_5 = is $1 + 1 + 1 + 1 + 1$
- (g) Define Normal series with example. $G_0 \supseteq G_1 \supseteq G_2 \supseteq \dots \supseteq G_m$ s.t. $G_i \triangleleft G_{i+1}$
- (h) Write one normal subgroup of Q_8 . $\{1, -1\}$

Section B (2*3=6)

Q.2. Attempt any two of the following:

- (a) Define Nilpotent group and Solvable group with suitable examples each.
- (b) If order of G is 20, then how many Sylow-5 subgroups of order 5 in G . $n_5 = 1$
- (c) If $G = Z_6 \times Z_{15}$ then find the number of subgroups of order 9 in G . $n_3 = 1$

Section C (1*6=6)

Q3. Attempt any one of the following:

- (a) (i) Show that any subgroup of order 2 in D_7 is not normal subgroup of D_7 . $\therefore n_2 = \frac{7}{2} = 7$
 \therefore not normal
- (ii) Show that any 3-SSG in S_3 are conjugate to each other.
- (b) Let G be a finite group then any two composition series of G are equivalent.