

UNIVERSITY INSTITUTE OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF ELECTRONICS & COMM. ENGG.

UNIT - 1
NUMBER SYSTEMS

- Introduction about digital system
- Philosophy of number systems
- Complement representation of negative numbers
- Binary arithmetic
- Binary codes
- Error detecting & error correcting codes
- Hamming codes

INTRODUCTION ABOUT DIGITAL SYSTEM

A Digital system is an interconnection of digital modules and it is a system that manipulates discrete elements of information that is represented internally in the binary form.

Now a day's digital systems are used in wide variety of industrial and consumer products such as automated industrial machinery, pocket calculators, microprocessors, digital computers, digital watches, TV games and signal processing and so on.

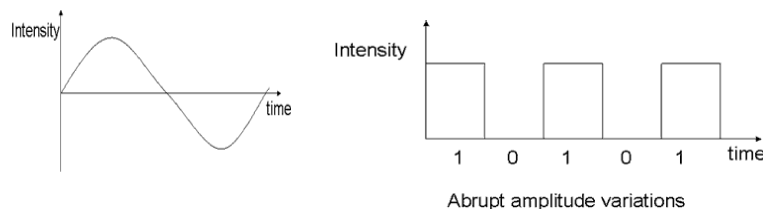
Characteristics of Digital systems

- Digital systems manipulate discrete elements of information.
- Discrete elements are nothing but the digits such as 10 decimal digits or 26 letters of alphabets and so on.
- Digital systems use physical quantities called signals to represent discrete elements.
- In digital systems, the signals have two discrete values and are therefore said to be binary.
- A signal in digital system represents one binary digit called a bit. The bit has a value either 0 or 1.

Analog systems vs Digital systems

Analog system process information that varies continuously i.e; they process time varying signals that can take on any values across a continuous range of voltage, current or any physical parameter.

Digital systems use digital circuits that can process digital signals which can take either 0 or 1 for binary system.



Advantages of Digital system over Analog system

1. Ease of programmability

The digital systems can be used for different applications by simply changing the program without additional changes in hardware.

2. Reduction in cost of hardware

The cost of hardware gets reduced by use of digital components and this has been possible due to advances in IC technology. With ICs the number of components that can be placed in a given area of Silicon are increased which helps in cost reduction.

3. High speed

Digital processing of data ensures high speed of operation which is possible due to advances in Digital Signal Processing.

4. High Reliability

Digital systems are highly reliable one of the reasons for that is use of error correction codes.

5. Design is easy

The design of digital systems which require use of Boolean algebra and other digital techniques is easier compared to analog designing.

6. Result can be reproduced easily

Since the output of digital systems unlike analog systems is independent of temperature, noise, humidity and other characteristics of components the reproducibility of results is higher in digital systems than in analog systems.

Disadvantages of Digital Systems

- Use more energy than analog circuits to accomplish the same tasks, thus producing more heat as well.
- Digital circuits are often fragile, in that if a single piece of digital data is lost or misinterpreted the meaning of large blocks of related data can completely change.
- Digital computer manipulates discrete elements of information by means of a binary code.
- Quantization error during analog signal sampling.

NUMBER SYSTEM

Number system is a basis for counting various items. Modern computers communicate and operate with binary numbers which use only the digits 0 & 1. Basic number system used by humans is Decimal number system.

For Ex: Let us consider decimal number 18. This number is represented in binary as 10010.

We observe that binary number system takes more digits to represent the decimal number. For large numbers we have to deal with very large binary strings. So this fact gave rise to three new number systems.

- i) Octal number systems
- ii) Hexa Decimal number system
- iii) Binary Coded Decimal number (BCD) system

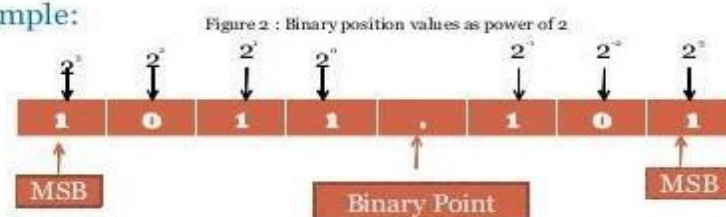
To define any number system we have to specify

- Base of the number system such as 2, 8, 10 or 16.
- The base decides the total number of digits available in that number system.
- First digit in the number system is always zero and last digit in the number system is always base-1.

Binary number system:

The binary number has a radix of 2. As $r = 2$, only two digits are needed, and these are 0 and 1. In binary system weight is expressed as power of 2.

• Example:



The left most bit, which has the greatest weight is called the Most Significant Bit (MSB). And the right most bit which has the least weight is called Least Significant Bit (LSB).

For Ex: $1001.01_2 = [(1) \times 2^3] + [(0) \times 2^2] + [(0) \times 2^1] + [(1) \times 2^0] + [(0) \times 2^{-1}] + [(1) \times 2^{-2}]$

$$1001.01_2 = [1 \times 8] + [0 \times 4] + [0 \times 2] + [1 \times 1] + [0 \times 0.5] + [1 \times 0.25]$$

$$1001.01_2 = 9.25_{10}$$

Decimal Number system

The decimal system has ten symbols: 0,1,2,3,4,5,6,7,8,9. In other words, it has a base of 10.

Octal Number System

Digital systems operate only on binary numbers. Since binary numbers are often very long, two shorthand notations, octal and hexadecimal, are used for representing large binary numbers. Octal systems use a base or radix of 8. It uses first eight digits of decimal number system. Thus it has digits from 0 to 7.

Hexa Decimal Number System

The hexadecimal numbering system has a base of 16. There are 16 symbols. The decimal digits 0 to 9 are used as the first ten digits as in the decimal system, followed by the letters A, B, C, D, E and F, which represent the values 10, 11,12,13,14 and 15 respectively.

Decima l	Binar y	Octal	Hexadeci mal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number Base conversions

The human beings use decimal number system while computer uses binary number system. Therefore it is necessary to convert decimal number system into its equivalent binary.

- i) Binary to octal number conversion
- ii) Binary to hexa decimal number conversion

The binary number: 001 010 011 000 100 101 110 111
 The octal number: 1 2 3 0 4 5 6 7

The binary number: 0001 0010 0100 1000 1001 1010 1101 1111
 The hexadecimal number: 1 2 5 8 9 A D F

- iii) Octal to binary Conversion

Each octal number converts to 3 binary digits

Code
0 - 000
1 - 001
2 - 010
3 - 011
4 - 100
5 - 101
6 - 110
7 - 111

To convert 653_8 to binary, just substitute code:

6 5 3
 ↓ ↓ ↓
 110 101 011

- iv) Hexa to binary conversion
- 0100 1111 1101 0111**

- v) Octal to Decimal conversion

Ex: convert 4057.06_8 to octal

$$=4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$$

$$=2048 + 0 + 40 + 7 + 0 + 0.0937$$

$$=2095.0937_{10}$$

vi) Decimal to Octal Conversion

Ex: convert 378.93_{10} to octal

378₁₀ to octal: Successive division:

$$\begin{array}{r} 8 \mid 378 \\ \hline 8 \mid 47 \text{ --- } 2 \\ \hline 8 \mid 5 \text{ --- } 7 \quad \uparrow \\ \hline 0 \text{ --- } 5 \end{array}$$

$$=572_8$$

0.93₁₀ to octal :

$$\begin{array}{r} 0.93 \times 8 = 7.44 \\ 0.44 \times 8 = 3.52 \quad \downarrow \\ 0.53 \times 8 = 4.16 \\ 0.16 \times 8 = 1.28 \\ =0.7341_8 \end{array}$$

$$378.93_{10} = 572.7341_8$$

vii) Hexadecimal to Decimal Conversion

Ex: $5C7_{16}$ to decimal

$$=(5 \times 16^2) + (C \times 16^1) + (7 \times 16^0)$$

$$=1280 + 192 + 7$$

$$=147_{10}$$

viii) Decimal to Hexadecimal Conversion

Ex: 2598.675_{10}

$$\begin{array}{r} 16 \overline{) 2598} \\ 16 \overline{) 2} \quad -6 \\ \hline 10 \quad -2 \end{array}$$

$$= A26_{(16)}$$

$$0.675_{10} = 0.675 \times 16 \rightarrow 10.8$$

$$= 0.800 \times 16 \rightarrow 12.8 \quad \downarrow$$

$$= 0.800 \times 16 \rightarrow 12.8$$

$$= 0.800 \times 16 \rightarrow 12.8$$

$$= 0.ACCC_{16}$$

$$2598.675_{10} = A26.ACCC_{16}$$

ix) Octal to hexadecimal conversion:

The simplest way is to first convert the given octal no. to binary & then the binary no. to hexadecimal.

Ex: 756.603_8

7	5	6	.	6	0	3
111	101	110	.	110	000	011
0001	1110	1110	.	1100	0001	1000
1	E	E	.	C	1	8

x) Hexadecimal to octal conversion:

First convert the given hexadecimal no. to binary & then the binary no. to octal.

Ex: $B9F.AE_{16}$

B	9	F	.	A	E		
1011	1001	1111	.	1010	1110		
101	110	011	111	.	101	011	100
5	6	3	7	.	5	3	4

$$= 5637.534$$

Complements:

In digital computers to simplify the subtraction operation & for logical manipulation complements are used. There are two types of complements used in each radix system.

- i) The radix complement or r's complement
- ii) The diminished radix complement or (r-1)'s complement