Subject Name: Optimization Techniques

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Subject Topic: Graphical Solution of LPP

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Graphical Solution of Linear Programming Problems

Some commonly used terms in linear programming problems are:

- **Objective function**
- **Constraints**

    Non-negative constraints: $x > 0$, $y > 0$ etc.

    General constraints: $x + y > 40$, $2x + 9y \geq 40$ etc.

- **Optimization problem:** A problem that seeks to maximization or minimization of variables of linear inequality problem is called optimization problems.
- **Feasible region:** A common region determined by all given issues including the non-negative ($x \geq 0$, $y \geq 0$) constrain is called the feasible region (or solution area) of the problem. The region other than the feasible region is known as the infeasible region.
- **Feasible Solutions:** These points within or on the boundary region represent feasible solutions of the problem. Any point outside the scenario is called an infeasible solution.
- **Optimal(most feasible) solution:** Any point in the emerging region that provides the right amount (maximum or minimum) of the objective function is called the optimal solution.

Theorem for graphical solution

- **Theorem 1:** Let R be the feasible region for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
- **Theorem 2:** Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.
- **Remark:** If R is unbounded, then a maximum or a minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of R.

Corner Point Method

- This method of solving linear programming problem is referred as **Corner Point Method.** The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = ax + by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. **(i)** When the feasible region is **bounded,** M and m are the maximum and minimum values of Z.

(ii) An **unbounded feasible region** can not be enclosed in a circle, no matter how big the circle is. If the coefficients on the objective function are all positive, then an **unbounded feasible region** will have a minimum but no maximum.

Example 1

Solve the given linear programming problems graphically:

Maximize: $Z = 8x + y$

and the constraints are :

$x + y \leq 40$,

$2x + y \leq 60$,

$x \geq 0, y \geq 0$

Solution:

- Convert inequalities constraints to equality constraints
- Solve linear equations and find coordinate values including nonnegative values of x and y.
- Draw a linear graph and identify the region of feasible solutions and the corner points of the of the feasible region.

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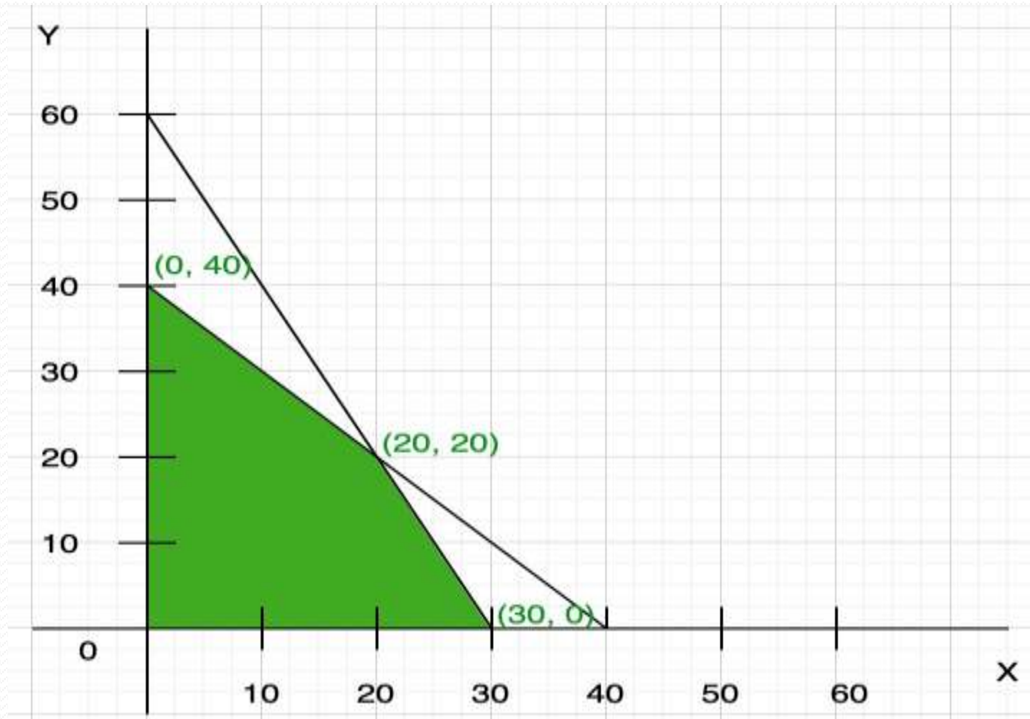
Points	Corresponding values of Z
(0,0)	0
(0,40)	40
(20,20)	180
(30,0)	240



- So the maximum value of Z = 240 at point x = 30, y = 0.

Example 2

Solve the following LPP by graphical method

Maximize: $P = 7x_1 + 5x_2$

Subject to:

$4x_1 + 3x_2 \leq 240$

$2x_1 + x_2 \leq 100$

$x_1 \geq 0$, $x_2 \geq 0$.

Solution:

- Convert inequalities constraints to equality constraints
- Solve linear equations and find coordinate values including nonnegative values of x_1 and x_2.
- Draw a linear graph and identify the region of feasible solutions and the corner points of the of the feasible region.

Continue…….

Corner Points	Corresponding values of Z
(0,0)	0
(50,0)	350
(30,40)	410
(0,80)	400

At the point (30,40) the value of Z is highest and value 410