Subject Name: Optimization Techniques

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Subject Topic: Introduction to Linear Programming

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Introduction

- Linear Programming is a special and versatile technique which can be applied to a variety of management problems viz. Advertising, Distribution, Investment, Production, Refinery Operations, and Transportation analysis. The linear programming is useful not only in industry and business but also in non-profit sectors such as Education, Government, Hospital, and Libraries. The linear programming method is applicable in problems characterized by the presence of decision variables. The objective function and the constraints can be expressed as linear functions of the decision variables. The decision variables represent quantities that are, in some sense, controllable inputs to the system being modeled. An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption.

- There is always some practical limitation on the availability of resources viz. man, material, machine, or time for the system. These constraints are expressed as linear equations involving the decision variables. Solving a linear programming problem means determining actual values of the decision variables that optimize the objective function subject to the limitation imposed by the constraints.

- The main important feature of linear programming model is the presence of linearity in the problem. The use of linear programming model arises in a wide variety of applications. Some model may not be strictly linear, but can be made linear by applying appropriate mathematical transformations. Still some applications are not at all linear, but can be effectively approximated by linear models. The ease with which linear programming models can usually be solved makes an attractive means of dealing with otherwise intractable nonlinear models.

Linear Programming Problem Formulation

- The linear programming problem formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products. A product has a certain margin of profit per unit, and uses a common pool of limited resources. In this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints.

Example 1:

- Suppose an industry is manufacturing tow types of products P1 and P2. The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P1 and P2. Formulate the problem in the form of linear programming model.

Profit/Kg	P1 Rs. 30	P2 Rs. 40	Total available Machine hours/day
M1	3	2	600
M2	3	5	800
M3	5	6	1100

Solution:

The procedure for linear programming problem formulation is as follows: Introduce the decision variable as follows:

Let x_1 = amount of P1

x_2 = amount of P2

In order to maximize profits, we establish the objective function as

$$30x_1 + 40x_2$$

Since one Kg of P1 requires 3 hours of processing time in machine 1 while the corresponding requirement of P2 is 2 hours. So, the first constraint can be expressed as

$$3x_1 + 2x_2 \leq 600$$

Solution:

Similarly, corresponding to machine 2 and 3 the constraints are

$$3x_1 + 5x_2 \leq 800$$
$$5x_1 + 6x_2 \leq 1100$$

In addition to the above there is no negative production, which may be represented algebraically as

$$x_1 \geq 0 \ ; \quad x_2 \geq 0$$

Thus, the product mix problem in the linear programming model is as follows:

Maximize $30x_1 + 40x_2$

Subject to:

$$3x_1 + 2x_2 \leq 600$$
$$3x_1 + 5x_2 \leq 800$$
$$5x_1 + 6x_2 \leq 1100$$

Non Negative Condition:

$$x_1 \geq 0, \ x_2 \geq 0$$

Example 2:

A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low quality flour. The company has entered a contract to supply flour to a firm every month with at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the company Rs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4 quintals of high, medium and low quality flour, Mill B produces 2, 4 and 12 quintals of high, medium and low quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically.

Solution:

Let us define x_1 and x_2 are the mills A and B. Here the objective is to minimize the cost of the machine runs and to satisfy the contract order. The linear programming problem is given by

Minimize $2000x_1 + 1500x_2$

Subject to:

$$6x_1 + 2x_2 \geq 8$$
$$2x_1 + 4x_2 \geq 12$$
$$4x_1 + 12x_2 \geq 24$$

Non Negative Condition:

$$x_1 \geq 0, x_2 \geq 0$$