Subject Name: Optimization Techniques

Subject Code: BCA-404 N

Subject Topic: Special Cases of Graphical Solution for LPP

Abhishek Dwivedi

Assistant Professor

Department of Computer Application

UIET, CSJM University, Kanpur

Graphical Method-Special cases

- **Unique optimal solution**
- **Multiple optimal solution**
- **Unbound optimal solution**
- **No feasible solution**

Solve the following linear programming problem graphically:

Max. $Z = 4x + y$ ... (1)

subject to the constraints:

$x + y \leq 50$ ... (2)

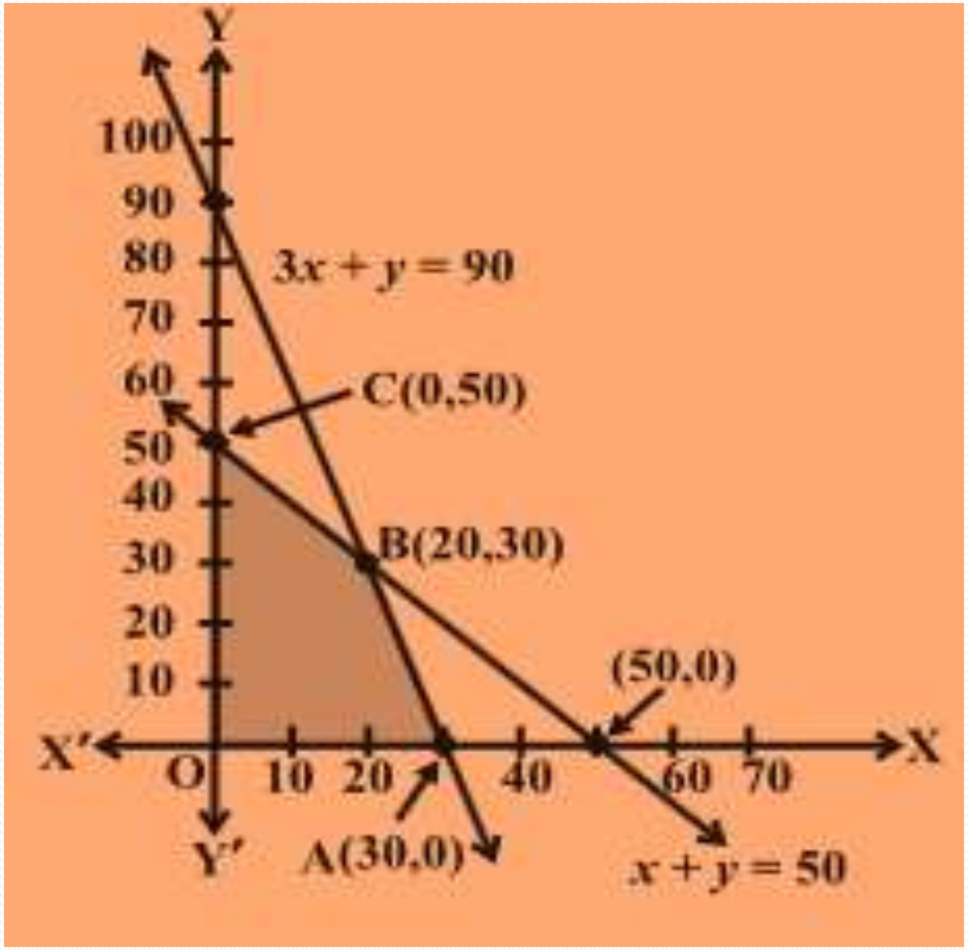
$3x + y \leq 90$ ... (3)

$x \geq 0, y \geq 0$ ... (4)

Solution:

- The shaded region given in the figure below is the feasible region determined by the system of constraints (2) to (4).
- We observe that the feasible region OABC is bounded. So, we now use Corner Point Method to determine the maximum value of Z.
- The coordinates of the corner points O, A, B and C are (0, 0), (30, 0), (20, 30) and (0, 50) respectively.

Continue……



Corner Point	Corresponding value of Z	
(0, 0)	0	
(30, 0)	**120** ←	Maximum
(20, 30)	110	
(0, 50)	50	

Hence, maximum value of Z is 120 at the point (30, 0).

Case 1:UNIQUE OPTIMAL SOLUTION

Solve the following linear programming problem graphically:

Min. $Z = 200x + 500y$ ... (1)

subject to the constraints:

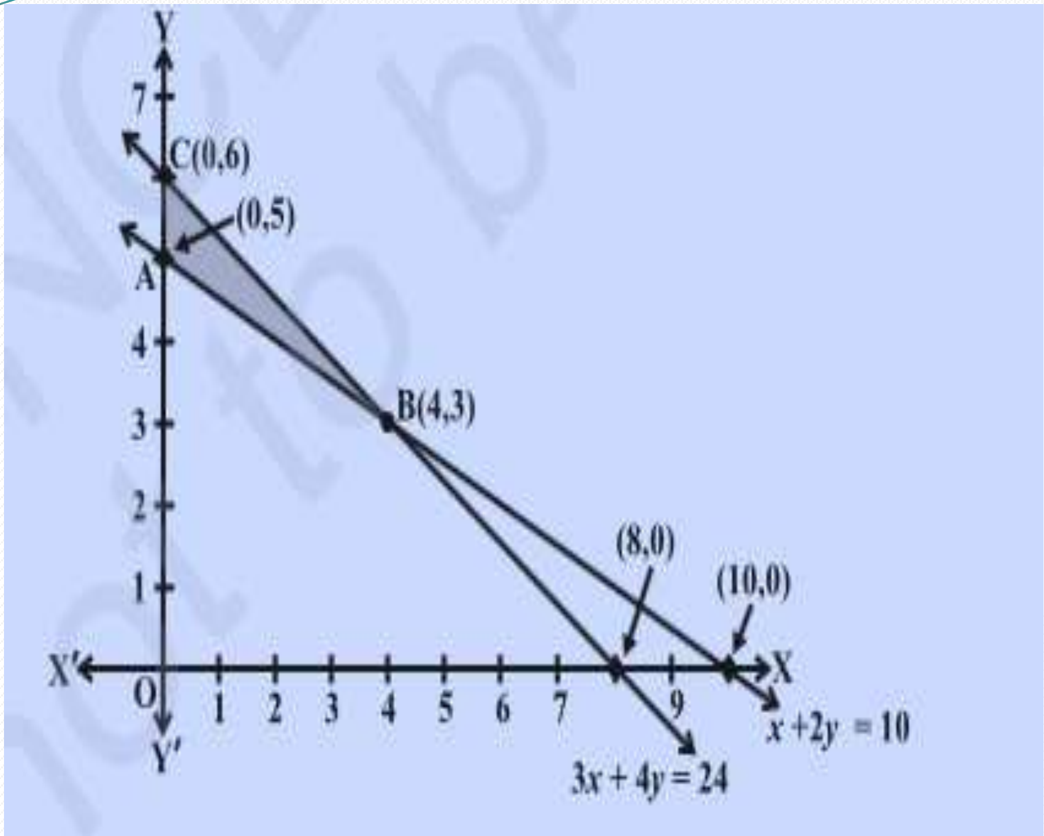
$x + 2y \geq 10$ ... (2)

$3x + 4y \leq 24$ ... (3)

and $x, y \geq 0$ ... (4)

- The shaded region in Fig B is the feasible region ABC determined by the system of constraints (2) to (4), which is bounded.
- The coordinates of corner points A, B and C are (0,5), (4,3) and (0,6) respectively.
- Now we evaluate $Z = 200x + 500y$ at these points.

Continue………



Corner Point	Corresponding value of Z
(0, 5)	2500
(4, 3)	**2300** ← Minimum
(0, 6)	3000

Hence, minimum value of Z is 2300 attained at the unique point (4, 3) .

Case2:MULTIPLE OPTIMAL SOLUTION

Solve by using graphical method

Max Z = 4x1 + 3x2

Subject to constraints

$4x1 + 3x2 \leq 24$

$x1 \leq 4.5$

$x2 \leq 6$

and $x1 , x2 \geq 0$

Solution:

The first constraint $4x1 + 3x2 \leq 24$, written in a form of equation

$4x1 + 3x2 = 24$

Put x1 =0, then x2 = 8

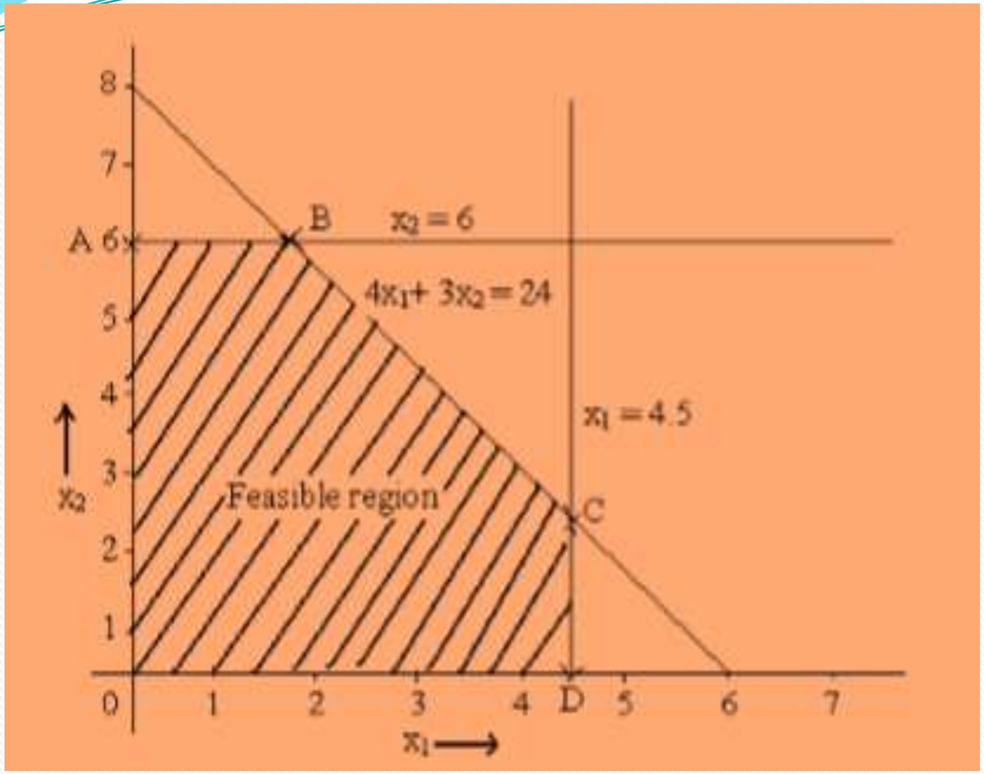
Put x2 =0, then x1 = 6

The coordinates are (0, 8) and (6, 0)

The second constraint $x1 \leq 4.5$, written in a form of equation x1 = 4.5

The third constraint $x2 \leq 6$, written in a form of equation x2 = 6

Continue………



Corner Point	Corresponding value of Z
A (0,6)	18
B (1.5,6)	24
C (4.5,2)	24
D (4.5,0)	18

- The corner points of feasible region are A, B, C and D.
- Max Z = 24, which is achieved at both B and C corner points. It can be achieved not only at B and C but every point between B and C.
- Hence the given problem has multiple optimal solutions.

Case 3:UNBOUNDED SOLUTION

Solve by graphical method

Max $Z = 3x1 + 5x2$

Subject to $2x1 + x2 \geq 7$

$x1 + x2 \geq 6$

$x1 + 3x2 \geq 9$

and $x1 , x2 \geq 0$

- **Solution:**

The first constraint $2x1 + x2 \geq 7$, written in a form

of equation $2x1 + x2 = 7$

Put $x1 = 0$, then $x2 = 7$

Put $x2 = 0$, then $x1 = 3.5$

The coordinates are (0, 7) and (3.5, 0)

Continue……

The second constraint $x1 + x2 \geq 6$, written in a form of equation $x1 + x2 = 6$

Put $x1 = 0$, then $x2 = 6$

Put $x2 = 0$, then $x1 = 6$

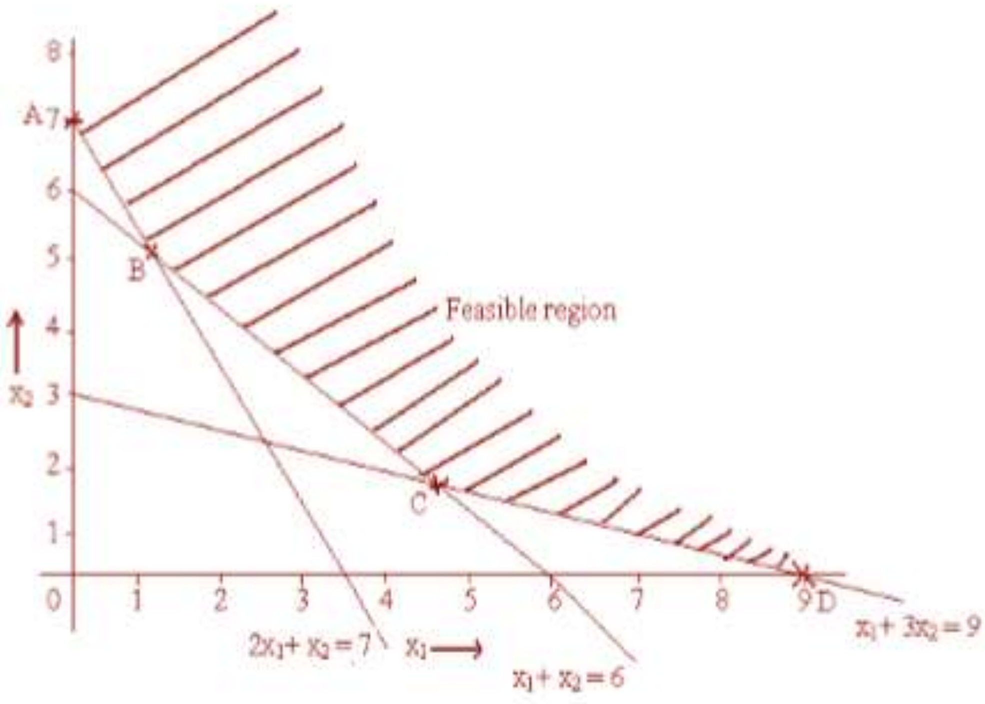
The coordinates are (0, 6) and (6, 0).

The third constraint $x1 + 3x2 \geq 9$, written in a form of equation $x1 + 3x2 = 9$

Put $x1 = 0$, then $x2 = 3$

Put $x2 = 0$, then $x1 = 9$ The coordinates are (0, 3) and (9, 0)

Continue……



Corner Points	Value of Z
A (0,7)	35
B (1, 5) (Solve the two equations 2x1+ x2 = 7 and x1+ x2 = 6 to get the coordinates)	28
C (4.5, 1.5) (Solve the two equations x1+ x2 = 6 and x1+ 3x2 = 9 to get the coordinates)	21
D (9, 0)	27

Continue……..

- The corner points of feasible region are A, B, C and D.
- The values of objective function at corner points are 35, 28, 21 and 27.
- But there exists infinite number of points in the feasible region which is unbounded.
- The value of objective function will be more than the value of these four corner points i.e. the maximum value of the objective function occurs at a point at ∞.
- Hence the given problem has unbounded solution.

Case 4:NO FEASIBLE SOLUTION

Solve graphically

Max Z = 3x1 + 2x2

Subject to

x1+ x2 ≤ 1

x1+ x2 ≥ 3

and x1 , x2 ≥ 0

Solution:

The first constraint x1+ x2 ≤ 1, written in a form of equation x1+ x2 = 1

Put x1 =0, then x2 = 1

Put x2 =0, then x1 = 1

The coordinates are (0, 1) and (1, 0)

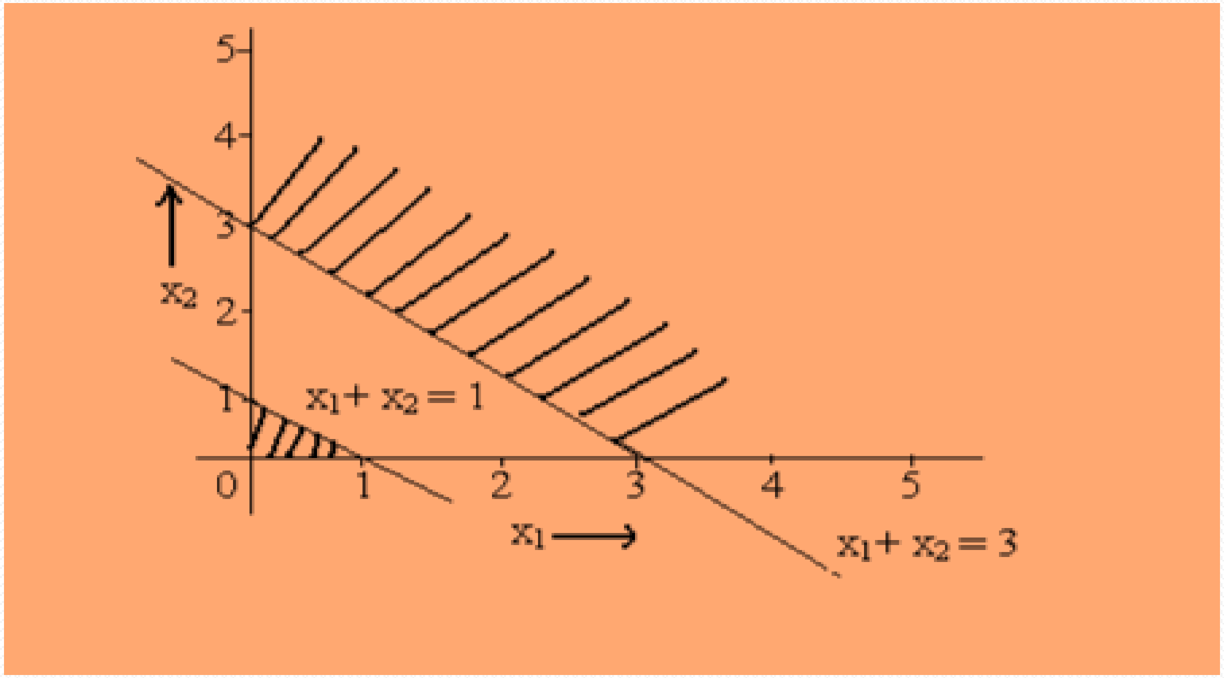
Continue……….

The first constraint $x1+ x2 \geq 3$, written in a form of equation $x1+ x2 = 3$

Put $x1 = 0$, then $x2 = 3$

Put $x2 = 0$, then $x1 = 3$

The coordinates are (0, 3) and (3, 0)



Continue………

- There is no common feasible region generated by two constraints together i.e. we cannot identify even a single point satisfying the constraints.

- Hence there is no feasible solution.