1. If $x + \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6$ then x is equal to

(a) 2, -3 (b) 2 only

(c) 1 (d) 3

Solution:

$\mathbf{x = \log_{10} 10^x}$

$x + \log_{10}(1 + 2^x) = x\log_{10} 5 + \log_{10} 6$

$\mathbf{\log_{10} 10^x} + \log_{10}(1 + 2^x)$

$= \log_{10} 5^x + \log_{10} 6$

$\log_{10} 10^x(1 + 2^x) = \log_{10} 5^x \times 6$

$10^x(1 + 2^x) = 5^x \times 6$

$5^x\{2^x(1 + 2^x) - 6\} = 0$

$2^x(1 + 2^x) - 6 = 0 \quad (5^x \neq 0)$

Let $2^x = y$

$y(1 + y) - 6 = 0$

$y^2 + y - 6 = 0$

$(y + 3)(y - 2) = 0$

$y - 2 = 0$

$2^x - 2 = 0$

$x = 1$

Answer: (c)

2 The remainder and the quotient of the binary division $(101110)_2 \div (110)_2$ are respectively

(a) $(111)_2 + (100)_2$

(b) $(100)_2 + (111)_2$

(c) $(101)_2 + (101)_2$

(d) $(100)_2 + (100)_2$

Solution:

$(101110)_2 = 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^5$

$(101110)_2 = 46$

$(110)_2 = 1 \times 2^1 + 1 \times 2^2 = 6$

$(101110)_2 \div (110)_2 = 46 \div 6$

Quotient = 7 = $(111)_2$

Remainder = 4 = $(100)_2$

Answer: (a)

3. The matrix A has x rows and x + 5 columns. The matrix B has y rows and 11 – y columns. Both AB and BA exist. What are the values of x and y respectively?

(a) 8 and 3 (b) 3 and 4

(c) 3 and 8 (d) 8 and 8

Solution:

Since AB and BA both exists.

(1) .Column of A = Rows of B.

$x + 5 = y$

(2) Column of B = Rows of A.

$11 - y = x$

Solve these two equations we get,

$x = 3 \text{ and } y = 8$

Answer: (c)

4 If $S_n = nP + \frac{n(n-1)Q}{2}$, where S_n denotes the sum of the first n terms of an AP, then the common difference is

(a) P + Q (b) 2P + 3Q

(c) 2Q (d) Q

Solution:

Sum of n^{th} term of an A.P. whose first term is a and common difference is d.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$nP + \frac{n(n-1)Q}{2} = \frac{n}{2}(2a + (n-1)d)$$

$$\frac{n}{2}(2P + (n-1)Q) = \frac{n}{2}(2a + (n-1)d)$$

$a = P, d = Q$

Answer: (d)

5. The roots of the equation $(q - r)x^2 + (r - p)x + (p - q) = 0$ are

(a) $(r - p)/(q - r), 1/2$

(b) $(p - q)/(q - r), 1$

(c) $(q - r)/(p - q), 1$

(d) $(r - p)/(p - q), 1/2$

Solution:

$$(q - r)x^2 + (r - p)x + (p - q) = 0$$

Put x = 1 $(q - r) + (r - p) + (p - q) = 0$

So x =1 is a root.

Product of roots

$$x_1x_2 = \frac{p-q}{q-r}$$

$$x_2 = \frac{p-q}{q-r}$$

Answer: (b)

6. The sum of all real roots of the equation $|x-3|^2 + |x-3| - 2 = 0$ is

(a) 2 (b) 3

(c) 4 (d) 6

Solution: $|x-3|^2 + |x-3| - 2 = 0$

Let $y = |x-3|$

$$y^2 + y - 2 = 0$$

$$y^2 + 2y - y - 2 = 0$$

$$(y+2)(y-1) = 0$$

$$y - 1 = 0$$

$$y = 1$$

$$|x-3| = 1$$

$$x = 2, 4$$

Sum of roots is equal to 6

Answer: (d)

7. If $y = x + x^2 + x^3 + \ldots$ up to infinite terms where x < 1, then which one of the following is correct?

(a) $x = \frac{y}{1+y}$ (b) $x = \frac{y}{1-y}$

(c) $x = \frac{1+y}{y}$ (d) $x = \frac{1-y}{y}$

Solution:

Series x, x^2, x^3 are in G.P.

Summation of series =

$$\frac{a}{1-r}$$

$$y = \frac{x}{1-x}$$

$$y(1-x) = x$$

$$y - yx = x$$

$$x = \frac{y}{1+y}$$

Answer: (a)

8. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is

(a) $3x^2 + 8x + 16 = 0$

(b) $3x^2 - 8x - 16 = 0$

(c) $3x^2 + 8x - 16 = 0$

(d) $x^2 + 8x + 16 = 0$

Solution: Equation of quadratic equation is $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$$x^2 - (\alpha + \beta^{-1} + \beta + \alpha^{-1})x + (\alpha + \beta^{-1}) \times (\beta + \alpha^{-1}) = 0$$

If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$ then $\alpha + \beta = -\frac{2}{3}$ and $\alpha\beta = \frac{1}{3}$

$$\alpha + \beta^{-1} + \beta + \alpha^{-1} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$$

$$= -\frac{2}{3} + \frac{-\frac{2}{3}}{\frac{1}{3}} = -\frac{2}{3} - 2$$

$$= -\frac{8}{3}$$

$$(\alpha + \beta^{-1}) \times (\beta + \alpha^{-1}) = \frac{(\alpha\beta + 1)^2}{\alpha\beta}$$

$$= \frac{\left(\frac{1}{3} + 1\right)^2}{\frac{1}{3}} = \frac{\frac{16}{9}}{\frac{1}{3}} = \frac{16}{3}$$

$$x^2 + \frac{8}{3}x + \frac{16}{3} = 0$$

$$3x^2 + 8x + 16 = 0$$

Answer: (a)

9. The value of $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots$ up to infinite terms is

(a) $\log_e 9$ (b) 0

(c) 1 (d) $\log_e 3$

Solution: $\frac{1}{\log_3 e}, \frac{1}{\log_3 e^2}, \frac{1}{\log_3 e^4} + \ldots$ it is G.P. series

First term a = $\frac{1}{\log_3 e}$

Common ratio $r = \frac{\textit{Second term}}{\textit{first term}} = \frac{\frac{1}{\log_3 e^2}}{\frac{1}{\log_3 e}} = \frac{1}{2}$

Sum = $\frac{a}{1-r} = \frac{\frac{1}{\log_3 e}}{1-\frac{1}{2}} = \frac{2}{\log_3 e}$

$$= 2\log_e 3 = \log_e 9$$

Answer: (a)

10. If ΔPQR, angle R $= \frac{\pi}{2}$. if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a = b + c$

(b) $b = c + a$

(c) $c = a + b$

(d) $b = c$

Solution:

If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$

$$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$$

$$\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right) = \frac{c}{a}$$

In ΔPQR, $\angle P + \angle Q + \angle R = \pi$

Given $\angle R = \frac{\pi}{2}$

$$\angle P + \angle Q = \frac{\pi}{2}$$

$$\frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$$

$$\tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \tan\frac{\pi}{4} = 1$$

$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = 1$$

$$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = 1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)$$

$$-\frac{b}{a} = 1 - \frac{c}{a}$$

$$-b = a - c$$

$$a + b = c$$

Answer: (c)

11. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to

(a) $1 + \sqrt{3}$ (b) $1 + \sqrt{5}$

(c) $1 - \sqrt{5}$ (d) $\sqrt{5} - 1$

Solution: Let $z = x + iy$

$$z - \frac{4}{z} = x + iy - \frac{4}{x + iy} \times \frac{x - iy}{x - iy}$$

$$= x + iy - \frac{4}{x^2 + y^2}(x - iy)$$

$$= x - \frac{4x}{x^2 + y^2} + i\left(y + \frac{4y}{x^2 + y^2}\right)$$

$$\left|z - \frac{4}{z}\right| = \sqrt{\left(x - \frac{4x}{x^2 + y^2}\right)^2 + \left(y + \frac{4y}{x^2 + y^2}\right)^2}$$

$$= \sqrt{x^2 + y^2 + \frac{16x^2}{(x^2 + y^2)^2} - \frac{8x}{x^2 + y^2} + \frac{8y^2}{x^2 + y^2} + \frac{16y^2}{(x^2 + y^2)^2}}$$

$$= \sqrt{x^2 + y^2 + \frac{16}{x^2 + y^2} + 8\frac{y^2 - x^2}{x^2 + y^2}} = 2$$

square both side we get

$$x^2 + y^2 + \frac{16}{x^2 + y^2} + 8\frac{y^2 - x^2}{x^2 + y^2} = 4$$

$$(x^2 + y^2)^2 + 8(y^2 - x^2) - 4(x^2 + y^2) + 16 = 0$$

12. The value of $\sqrt{3}\text{cosec } 20^0 - \sec 20^0$ is equal to

(a) 4 (b) 2

(c) 1 (d) -4

Solution:

$$\sqrt{3}\text{cosec } 20^0 - \sec 20^0$$

$$\frac{\sqrt{3}}{\sin 20^0} - \frac{1}{\cos 20^0}$$

$$\frac{\sqrt{3}\cos 20^0 - \text{si} \quad ^0}{\sin 20^0 \ \cos 20^0}$$

$$\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0\right)}{\frac{\sin 40^0}{2}}$$

$$\frac{4(\sin 60^0 \cos 20^0 - \cos 60^0 \sin 20^0)}{\sin 40^0}$$

$$\frac{4\sin 40^0}{\sin 40^0} = 4$$

Answer: (a)

13. $\sqrt{1 + \sin A} = -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$ is true if

(a) $\frac{3\pi}{2} < A < \frac{5\pi}{2}$ only

(b) $\frac{\pi}{2} < A < \frac{3\pi}{2}$ only

(c) $\frac{3\pi}{2} < A < \frac{7\pi}{2}$ only

(d) $0 < A < \frac{3\pi}{2}$ only

Solution:

$\sqrt{1+\sin A} = \sqrt{sin^2\frac{A}{2} + \cos^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}}$

$= \left|\sin\frac{A}{2} + \cos\frac{A}{2}\right|$

if $\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right) < 0$ then

$\left|\sin\frac{A}{2} + \cos\frac{A}{2}\right| = -\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$

$\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right) < 0$

$\sqrt{2}\sin\left(\frac{A}{2} + \frac{\pi}{4}\right) < 0$

$\pi < \left(\frac{A}{2} + \frac{\pi}{4}\right) < 2\pi$

$\frac{3\pi}{2} < A < \frac{7\pi}{2}$

Answer: (c)

14. The principal value of $\sin^{-1} x$ lies in the interval

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) $\left[0, \frac{\pi}{2}\right]$ (d) $[0, \pi]$

Solution:

$y = \sin^{-1} x$

Domain = $(-1, 1)$

Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: (b)

15. The points (a, b) , (0, 0), (-a, -b) and (ab, b^2) are

(a) the vertices of a parallelogram

(b) the vertices of a rectangle

(c) the vertices of a square

(d) collinear

Solution:

Slope of line joining (a, b) and (0, 0)

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{b-0}{a-0} = \frac{b}{a}$

Slope of line joining (o, o) and (-a, -b)

$= \frac{-b-0}{-a-0} = \frac{b}{a}$

Slope of line joining (0, 0) and (ab, b^2)

$= \frac{b^2}{ab} = \frac{b}{a}$

Answer: (d)

16 . The length of the normal form origin to the plane $x + 2y - 2z = 9$ is equal to

(a) 2 units (b) 3 units

(c) 4 units (d) 5 units

Solution:

Equation of plane P: $x + 2y - 2z = 9$

Direction ratio of normal to the plane is (1, 2, -2)

Equation of line passing through origin and perpendicular to plane P.

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-2} = k$$

$$x = k, y = 2k, z = -2k$$

Line passing through origin and perpendicular to plane P intersect at point Q. The coordinate of point Q satisfy the equation of the plane.

$$k + 4k + 4k = 9$$

$$k = 1$$

$x = 1, x = 2, z = -2$

OQ = $\sqrt{1 + 2^2 + (-2)^2} = 3$

Answer: (b)

17. If α, β and γ are the angles which the vector $\overrightarrow{OP}$ (O being the origin) makes with positive direction of the coordinate axes, then which of the following are correct?

1. $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$

2. $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$

3. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Solution: if l , m and n are direction cosine of line.

$$l^2 + m^2 + n^2 = 1$$

$$l = \cos\alpha$$

$$m = \cos\beta$$

$$n = \cos\gamma$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2\beta + 1 - \sin^2\gamma = 1$$
$$\cos^2\alpha + \cos^2\beta = \sin^2\gamma$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$
$$2 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$
$$1 + \cos^2\gamma = \sin^2\alpha + \sin^2\beta$$

18. The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is β. Which one of the following is correct?

(a) $\alpha = \beta$ (b) $\alpha > \beta$

(c) $\alpha < \beta$ (d) $\alpha = 2\beta$

Solution:

Slope of line $x + y - 3 = 0$ is $m_1 = -1$

Slope of line $x - y + 3 = 0$ is $m_2 = 1$

Angle between two lines is

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$m_1 m_2 = -1 \times 1 = -1$$

Lines are perpendicular to each other.

Slope of line $x - \sqrt{3}y + 2\sqrt{3} = 0$ is $m_1 = \frac{1}{\sqrt{3}}$

Slope of line $\sqrt{3}x - y + 1 = 0$ is $m_2 = \sqrt{3}$

Angle between two lines is

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \times \sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\theta = 30^0$$

19. Let $\vec{\alpha} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$ be three vectors. If $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to the vector $\vec{\delta}$ and $\vec{\delta}.\vec{\gamma} = 10$, then what is the magnitude of $\vec{\delta}$?

(a) $\sqrt{3}$ units (b) $2\sqrt{3}$ units

(c) $\frac{\sqrt{3}}{2}$ unit (d) $\frac{1}{\sqrt{3}}$ unit

Solution:

Let $\vec{\delta} = x\hat{i} + y\hat{j} + z\hat{k}$

Vector perpendicular to $\vec{\alpha}$ and $\vec{\beta}$

$$= \vec{\alpha} \times \vec{\beta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 5\hat{i} - 8\hat{j} - 5\hat{k}$$

Since $\vec{\delta}$ is perpendicular to $\vec{\alpha} \times \vec{\beta}$.

$$\vec{\delta}.(\vec{\alpha} \times \vec{\beta}) = 0$$

$$5x - 8y - 5z = 0$$

$$\vec{\delta} = k(\vec{\alpha} \times \vec{\beta}) = (5k)\hat{i} - (8k)\hat{j} - (5k)\hat{k}$$

$$\vec{\delta}.\vec{\gamma} = 10$$

$$[(5k)\hat{i} - (8k)\hat{j} - (5k)\hat{k}].[2\hat{i} + \hat{j} + 6\hat{k}] = 10$$

$$10k - 8k - 30k = 10$$

$$-28k = 10$$

$$|\vec{\delta}| = \sqrt{25k^2 + 64k^2 + 25k^2} = \sqrt{114}|k|$$

$$= \sqrt{114} \times \frac{10}{28}$$

20 For any vector $\vec{a}$

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is equal to

(a) $|\vec{a}|^2$ (b) $2|\vec{a}|^2$

(c) $3|\vec{a}|^2$ (d) $4|\vec{a}|^2$

Solution:

$$|\vec{a} \times \hat{i}| = |\vec{a}||\hat{i}|\sin\alpha = |\vec{a}|\ \sin\alpha$$
$$|\vec{a} \times \hat{j}| = |\vec{a}||\hat{j}|\sin\beta = |\vec{a}|\ \sin\beta$$
$$|\vec{a} \times \hat{k}| = |\vec{a}||\hat{k}|\sin\gamma = |\vec{a}|\ \sin\gamma$$

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = |\vec{a}|^2\{\sin^2\alpha + \sin^2\beta + \sin^2\gamma\}$$

$$= |\vec{a}|^2\{3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)\}$$

$$(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 1$$

$$= 2|\vec{a}|^2$$

Answer: (b)

21. The distance of the point (1, 3) from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$, is

(a) $\frac{5}{\sqrt{13}}$ units (b) $\frac{3}{\sqrt{13}}$ units

(c) $\sqrt{17}$ units (d) $\frac{\sqrt{17}}{2}$ units

Solution:

Equation of line parallel to the line $4x + y = 4$ and passing through point (1,3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -4(x - 1)$$

$$y = -4x + 7.$$

Intersection of line $y = -4x + 7$ and $2x + 3y = 6$ is (3/2 , 1).

Distance between (3/2 ,1) and (1, 3) is

$$\sqrt{\left(\frac{3}{2} - 1\right)^2 + (3 - 1)^2} = \frac{\sqrt{17}}{2}$$

Answer: (d)

22. The position of the point (1,2) relative to the ellipse $2x^2 + 7y^2 = 20$ is

(a) outside the ellipse

(b) inside the ellipse but not at the focus

(c) on the ellipse

(d) at the focus

Solution:

Equation of ellipse $2x^2 + 7y^2 = 20$.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 > 0$ then point lies outside the ellipse

$$2 \times 1^2 + 7 \times 2^2 - 20 = 10 > 0$$

So point (1, 2) lies outside the ellipse.

Answer: (a)

23. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle 120^0 with positive direction of x-axis is

(a) $y + \sqrt{3}x + 5 = 0$

(b) $y - \sqrt{3}x + 5 = 0$

(c) $y + \sqrt{3}x - 5 = 0$

(d) $y - \sqrt{3}x - 5 = 0$

Solution:

Slope intercept form of equation of line

$$y = mx + c$$

m = slope of line

$$m = \tan\theta = \tan 120^0 = -\cot 30^0 = -\sqrt{3}$$

$c = y - \text{intercept}$

$c = -5$

$y = -\sqrt{3}x - 5$

$y + \sqrt{3}x + 5 = 0$

Answer: (a)

24. Equation of line passing through lines $2x - 3y + 7 = 0$ and $7x + 4y + 2 = 0$ is

$$(2x - 3y + 7) + \lambda(7x + 4y + 2) = 0$$

Point (2, 3) passes through line of intersection. So it must satisfy the equation.

$$(2 \times 2 - 3 \times 3 + 7) + \lambda(7 \times 2 + 4 \times 3 + 2) = 0$$

$$2 + 28\lambda = 0$$

$$\lambda = -\frac{1}{14}$$

$$(2x - 3y + 7) - \frac{1}{14}(7x + 4y + 2) = 0$$

$$28x - 42y + 98 - 7x - 4y - 2 = 0$$

$$21x - 46y + 96 = 0$$

Answer: (b)

25 The equation of the circle which passes through the points (1, 0), (0, -6) and (3, 4) is

(a) $4x^2 + 4y^2 + 142x + 47y + 140 = 0$
(b) $4x^2 + 4y^2 - 142x - 47y + 138 = 0$
(c) $4x^2 + 4y^2 - 142x + 47y + 138 = 0$
(d) $4x^2 + 4y^2 + 150x - 49y + 138 = 0$

Solution: Centre of circle lies on point of intersection of line passing through perpendicular bisector of line joint AB and BC.

A(1, 0) , B(0, -6) and C(3, 4).

Coordinate of midpoint of A(1, 0) , B(0, -6)

$$\left(\frac{1+0}{2}, \frac{0-6}{2}\right) \equiv \left(\frac{1}{2}, -3\right)$$

Slope of line AB

$$\frac{-6-0}{0-1} = 6$$

Equation of line passing through $\left(\frac{1}{2}, -3\right)$ and perpendicular to AB.

$$y + 3 = -\frac{1}{6}\left(x - \frac{1}{2}\right)$$

$$2x + 12y + 35 = 0$$

Coordinate of mid point A and C.

$$\left(\frac{x_A + x_c}{2}, \frac{y_A + y_c}{2}\right) \equiv \left(\frac{1+3}{2}, \frac{0+4}{2}\right) \equiv (2,2)$$

Slope of AC =

$$\frac{y_c - y_A}{x_c - x_A} = \frac{4-0}{3-1} = 2$$

Equation of line passing through $(2, 2)$ and perpendicular to AC.

$$y - 2 = -\frac{1}{2}(x - 2)$$

$$x + 2y - 6 = 0$$

Intersection of line L_1 $2x + 12y + 35 = 0$ and L_2 : $x + 2y - 6 = 0$

$$y = -\frac{47}{8}, \quad x = \frac{71}{4}$$

Radius = $\sqrt{\left(\frac{71}{4} - 1\right)^2 + \left(-\frac{47}{8} - 0\right)^2}$

$$R = \sqrt{\left(\frac{67}{4}\right)^2 + \left(\frac{47}{8}\right)^2}$$

Equation of circle:

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$x^2 + y^2 - 2xx_0 - 2yy_0 + x_0^2 + y_0^2 - R^2 = 0$$

$$x^2 + y^2 - 2x \times \frac{71}{4} + 2y \times \frac{47}{8} + \left(\frac{71}{4}\right)^2 + \left(\frac{47}{8}\right)^2 - \left(\frac{67}{4}\right)^2 - \left(\frac{47}{8}\right)^2 = 0$$

$$4x^2 + 4y^2 - 142x + 47y + \frac{(71-67)(71+67)}{4} = 0$$

$$4x^2 + 4y^2 - 142x + 47y + 138 = 0$$

Answer: (c)

26 A function is defined as follows:

$$f(x) = \begin{cases} -\frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(a) f(x) is continuous at x = 0 but not differentiable at $x = 0$
(b) f(x) is continuous at x = 0 as well as differentiable at $x = 0$
(c) f(x) is discontinuous at x = 0
(d) None of the above

$$f(x) = \begin{cases} -\frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\sqrt{x^2} = |x|$$

$$f(x) = \begin{cases} -\frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$f(x) = \begin{cases} -\frac{x}{-x} = 1 & x < 0 \\ -\frac{x}{x} = -1 & x > 0 \\ 0, & x = 0 \end{cases}$$

So f(x) is discontinuous at x =0.

Answer: (c)

27. if $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 0 (b) 1

(c) $\frac{x-1}{x+1}$ (d) $\frac{x+1}{x-1}$

Solution: Let $\sec^{-1}\left(\frac{x+1}{x-1}\right) = \theta$

$$\sec\theta = \frac{x+1}{x-1}$$

$$\cos\theta = \frac{x-1}{x+1}$$

$$\theta = \cos^{-1}\left(\frac{x-1}{x+1}\right)$$

$$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right) = \frac{\pi}{2}$$

$$\frac{dy}{dx} = 0$$

28 Which one of the following is correct in respect of the derivative of the function, i.e. $f'(x)$?

(a) $f'(x) = 2x$ for $0 < x \le 1$

(b) $f'(x) = -2x$ for $0 < x \le 1$

(c) $f'(x) = -2x$ for $0 < x < 1$

(d) $f'(x) = 0$ for $0 < x < \infty$

Solution:

$$f'(x) = \begin{cases} -2x & \text{for } 0 < x < 1 \\ \frac{1}{x} & \text{for } 1 < x \le 2 \\ 0.5x & \text{for } 2 < x < \infty \end{cases}$$

At x = 1, Let hand derivative is not equal to right hand derivative. Therefore function f(x) is not differential at x = 1.

Answer: (c)

29. The maximum value of $\frac{\ln}{x}$ is

(a) e

(b) $\frac{1}{e}$

(c) $\frac{2}{e}$

(d) 1

Solution:

$$y = \frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{x\frac{d(\ln x)}{dx} - \ln x\frac{dx}{dx}}{x^2} = \frac{1 - \ln x}{x^2}$$

$$\frac{dy}{dx} = 0$$

$$\frac{1 - \ln x}{x^2} = 0$$

$$\ln x = 1$$

$$x = e$$

Maximum value is equal to $\frac{\ln}{e} = \frac{1}{e}$

Answer: (b)

30. The function $f(x) = |x| - x^3$ is

(a) odd (b) even (c) both even and odd (d) neither even nor odd

Solution:

if $f(x) = f(-x)$ then function is even.

if $f(x) = -f(-x)$ then function is odd.

$$f(x) = |x| - x^3$$

$$f(-x) = |-x| - (-x)^3 = |x| + x^3$$

$f(x) \neq f(-x)$ and $f(x) \neq -f(-x)$ So, function is neither odd and even.

Answer: (d)

31. What is

$$\int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\,dx$$

equal to?

(a) 8 (b) 4

(c) 2 (d) 0

Solution:

$$1 + \sin\frac{x}{2} = \sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}$$

$$= \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)^2$$

$$\int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\, \boldsymbol{dx} = \int_{\mathbf{0}}^{2\pi} \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right) dx$$

$$= \left. \frac{-\cos\frac{x}{4}}{\frac{1}{4}} + \frac{\sin\frac{x}{4}}{\frac{1}{4}} \right|_0^{2\pi} = \mathbf{8}$$

32. The area bounded by the curve $|x| + |y| = 1$ is

(a) 1 square unit

(b) $2\sqrt{2}$ square unit

(c) 2 square unit

(d) $2\sqrt{3}$ square unit

Solution:

The equation of curve represented by $|x| + |y| = 1$ is a square with vertex (1,0) (0,1) (-1, 0) and (0, -1).

Side of square = $\sqrt{1+1} = \sqrt{2}$

Area of square = $a^2 = 2$

Answer: (c)

33 Match List-I with List-II and select the correct answer using the code given below the lists:

List –I (Function)	List –II (Maximum value)
A. $\sin x + \cos x$	1.$\sqrt{10}$
B. $3\sin x + 4\cos x$	2.$\sqrt{2}$
C. $2\sin x + \cos x$	3. 5
D. $\sin x + 3\cos x$	4.$\sqrt{5}$

Code:

	A	B	C	D
(a)	2	3	1	4
(b)	2	3	4	1
(c)	3	2	1	4
(d)	3	2	4	1

Solution:

$\sin x + \cos x = \sqrt{2}\sin(x + 45^0)$

$3\sin x + 4\cos x = 5\left[\frac{3}{5}\sin x + \frac{4}{5}\cos x\right]$

$2\sin x + \cos x = \sqrt{5}\left[\frac{2}{\sqrt{5}}\sin x + \frac{1}{\sqrt{5}}\cos x\right]$

$\sin x + 3\cos x = \sqrt{10}\left[\frac{1}{\sqrt{10}}\sin x + \frac{3}{\sqrt{10}}\cos x\right]$

34. Geometrically $Re(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents

(a) circle

(b) ellipse

(c) rectangular hyperbola

(d)parabola

Solution:

$z = re^{i\theta}$

$z^2 = e^{i2\theta}$

$z^2 - i = r^2\cos 2\theta + i(r^2\sin 2\theta - 1)$

$Re(z^2 - i) = r^2\cos 2\theta = 2$

$(x^2 + y^2)\cos 2\theta = 2$

$\tan\theta = \frac{y}{x}$

$\cos 2\theta = cos^2\theta - sin^2\theta$

$= \frac{x^2}{x^2 + y^2} - \frac{y^2}{x^2 + y^2}$

$\cos 2\theta = \frac{x^2 - y^2}{x^2 + y^2}$

$(x^2 + y^2)\cos 2\theta = 2$

$x^2 - y^2 = 2$

Equation represent is rectangular hyperbola.

Answer: (c)

35. A committee of two persons is selected from two men and two women. The probability that the committee will have exactly one woman is

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Solution: Number of ways to selected two people = $C(4,2) = \frac{4\times3}{2} = 6$

Number of ways such a way that exactly one woman = $C(2,1)\times C(2,1) = 4$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

36. Let a die be loaded in such a way that even faces are twice likely to occur as the odd faces. What is the probability that a prime number will show up when the die is tossed?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Solution : P(Even number) = 2 P(odd no)

$$P(Even) + P(0dd) = 1$$

$$3P(Odd) = 1$$

$$P(odd) = \frac{1}{3}$$

$$P(Even) = \frac{2}{3}$$

prime number are { 1, 3, 5}

probability of odd = 1/3

37. For two events A and B, let P(A) = $\frac{1}{2}$, $P(A\cup B) = \frac{2}{3}$ and $P(A\cap B) = \frac{1}{6}$. What is $P(\bar{A}\cap B)$ equal to?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Solution:

$$P(A\cup B) = P(A) + PB) - P(A\cap B)$$

$$\frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{6}$$

$$P(B) = \frac{1}{3}$$

$$P(\bar{A}\cap B) = P(B) - P(A\cap B) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$$

Answer: (a)

38. Let A and B be two events with P(A) = $\frac{1}{3}$, $P(B) = \frac{1}{6}$ and P(A∩ B) = $\frac{1}{12}$. What is $P(B|\bar{A})$ equal to ?

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{10}$

Solution: P(A) = $\frac{1}{3}$, $P(B) = \frac{1}{6}$, $P(A\cap B) = \frac{1}{12}$

$$P(B|\bar{A}) = \frac{P(B\cap\bar{A})}{P(\bar{A})}$$

$$n(B\cap\bar{A}) = n(B) - n(A\cap B)$$

$$P(B\cap\bar{A}) = P(B) - P(A\cap B) = \frac{1}{6} - \frac{1}{12}$$

$$= \frac{1}{12}$$

$$P(B|\bar{A}) = \frac{P(B\cap\bar{A})}{P(\bar{A})}$$

$$P(B|\bar{A}) = \frac{\frac{1}{12}}{\frac{2}{3}} = \frac{1}{8}$$

Answer: (c)

39. If x_1 and x_2 are positive quantities, then the condition for the difference between the arithmetic mean and the geometric mean to be greater than 1 is

(a) $x_1 + x_2 > 2\sqrt{x_1x_2}$

(b) $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$

(c) $|\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$

(d) $x_1 + x_2 < 2(\sqrt{x_1x_2} + 1)$

Solution:

$$A.M. = \frac{x_1 + x_2}{2}$$

$G.M. = \sqrt{x_1x_2}$

$A.M. - G.M. > 1$

$\frac{x_1 + x_2}{2} - \sqrt{x_1x_2} > 1$

$\frac{x_1 + x_2 - 2\sqrt{x_1x_2}}{2} > 1$

$\left|\sqrt{x_1} - \sqrt{x_2}\right|^2 > 2$

$\left|\sqrt{x_1} - \sqrt{x_2}\right| > \sqrt{2}$

Answer: (c)

40. Five sticks of length 1, 3, 5, 7 and 9 feet are given. Three of these sticks are selected at random. What is the probability that the selected sticks can form a triangle?

(a) 0.5 (b) 0.4

(c) 0.3 (d) 0

Solution: Total number of ways = $C(5,3)$

= 10

To form a triangle then sides obey this two rule.

(1) Sum of any two side is always greater than third side.

(2) Difference any any two side is always less than third side.

Sample space S = $\{(1, 3, 5), (1,3,7), (1,3,9), (3,5,7), (1, 5, 7), (1,5,9), (3,5,9), (3,7,9), (5,7,9)\}$

To form a triangle (3, 5, 7) and (5, 7, 9) need to selected.

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{10}$$

Answer: (a)

41. If $A = \{x: x \text{ is a multiple of } 2\}$,

$B = \{x: x \text{ is a multiple of } 5\}$ and

$C = \{x: x \text{ is a muliple of } 10,\}$

then $A \cap (B \cap C)$ is equal to

(a) A

(b) B

(c) C

(d) $\{x: x \text{ is a muliple of } 100,\}$

Solution: $A = \{x: x \text{ is a multiple of } 2\}$

$A = \{2, 4, 6, 8, \ldots\ldots\}$

$B = \{x: x \text{ is a multiple of } 5\}$

$B = \{5, 10, 15, 20, \ldots\}$

$C = \{x: x \text{ is a muliple of } 10,\}$

$C = \{10, 20, 30, 40\}$

$B \cap C = \{10, 20, 30, 40 \ldots.\}$

$A \cap (B \cap C) = \{\{10, 20, 30, 40 \ldots.\}\}$

$A \cap (B \cap C) = C$

42. if α and β are the roots of the equation $1 + x + x^2 = 0$, then the matrix product

$$\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix}$$

is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$

Solution: if α and β are the roots of the equation $1 + x + x^2 = 0$ then

$$1 + \alpha + \alpha^2 = 0$$

$$1 + \beta + \beta^2 = 0$$

$$\alpha + \beta = -\frac{b}{a} = -1$$

$$\alpha\beta = \frac{c}{a} = 1$$

$$\begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix} = \begin{bmatrix} \alpha + \beta & \beta + \beta^2 \\ \alpha^2 + \alpha & 2\alpha\beta \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$$

42. If |a| denotes the absolute value of an integer, then which of the following are correct?

1. $|ab| = |a||b|$
2. $|a + b| \leq |a| + |b|$
3. $|a - b| \geq \left||a| - |b|\right|$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Answer: (d)

43. How many different permutations can be made out of the letters of the word 'PERMUTATION'?

(a) 19958400 (b) 19954800

(c) 19952400 (d) 39916800

Solution:

Number of letter in word PERMUTATION is 11.

T is repeated two times

Number of ways to arrange this letter

$$= \frac{n!}{p!} = \frac{11!}{2!} = 19958400$$

44. If $A = \begin{bmatrix} 4i-6 & 10i \\ 14i & 6+4i \end{bmatrix}$ and $k = \frac{1}{2i}$, where $i = \sqrt{-1}$, then kA is equal to

(a) $\begin{bmatrix} 2+3i & 5 \\ 7 & 2-3i \end{bmatrix}$

(b) $\begin{bmatrix} 2-3i & 5 \\ 7 & 2+3i \end{bmatrix}$

(c) $\begin{bmatrix} 2-3i & 7 \\ 5 & 2+3i \end{bmatrix}$

(d) $\begin{bmatrix} 2+3i & 5 \\ 7 & 2+3i \end{bmatrix}$

Solution: $A = \begin{bmatrix} 4i-6 & 10i \\ 14i & 6+4i \end{bmatrix}$ and $k = \frac{1}{2i}$

$$kA = \frac{1}{2i}\begin{bmatrix} 4i-6 & 10i \\ 14i & 6+4i \end{bmatrix}$$

$$kA = \begin{bmatrix} \frac{4i-6}{2i} & \frac{10i}{2i} \\ \frac{14i}{2i} & \frac{6+4i}{2i} \end{bmatrix}$$

$$\frac{4i-6}{2i} = \frac{-3+2i}{i} = \frac{-3+2i}{i} \times \frac{i}{i} = \frac{2i^2-3i}{i^2}$$

$$= \frac{-2-3i}{-1} = 2+3i$$

$$\frac{6+4i}{2i} = \frac{3+2i}{i} \times \frac{i}{i} = \frac{3i+2i^2}{i^2} = \frac{-2+3i}{-1}$$

$$= 2-3i$$

$$kA = \begin{bmatrix} 2+3i & 5 \\ 7 & 2-3i \end{bmatrix}$$

45. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of P is

(a) $\frac{1}{27}$ (b) $\frac{1}{64}$

(c) $\frac{1}{81}$ (d) 1

Solution: the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real.

If discriminate of quadratic equation is greater than zero then roots are real.

D > 0

$$b^2 - 4ac \geq 0$$

a = 1, b = - 4 and c =$-\log_3 P$

$$16 + 4\log_3 P \geq 0$$

$$\log_3 P \geq -4$$

$$\log_3 P \geq \log_3 3^{-4}$$

Since base 3 is greater than 1 therefore

$$P \geq 3^{-4}$$

$$P \geq \frac{1}{81}$$

Minimum value of P is 1/81.

46. The value of the product $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots$ up to infinite terms is

(a) 6 (b) 36

(c) 216 (d) 512

Solution:

$$6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \ldots = 6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$$6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)} = 6$$

47. The value of the determinant

$$\begin{vmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{vmatrix}$$

for all values of θ , is

(a) 1 (b) $\cos\theta$

(c) $\sin\theta$ (d) $\cos 2\theta$

Solution: $\begin{vmatrix} \cos^2\frac{\theta}{2} & \sin^2\frac{\theta}{2} \\ \sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{vmatrix} = \cos^4\frac{\theta}{2} - \sin^4\frac{\theta}{2}$

$$= \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)$$

$$= \cos\theta$$

48. The number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

(a) 202 (b) 101

(c) 51 (d) 50

Solution:

$$(x+y)^n = \sum_{r=0}^{n} n_{C_r} x^r y^{n-r}$$

Number of term is n+1.

$$(x+a)^{100} = n_{C_0} a^{100} + n_{C_1} a^{99} x + n_{C_2} a^{99} x^2 + ..$$

$$(x-a)^{100} = n_{C_0} a^{100} - n_{C_1} a^{99} x + n_{C_2} a^{99} x^2 - ..$$

$$(x+a)^{100} + (x-a)^{100} = 2(n_{C_0} a^{100} + n_{C_2} a^{98} + n_{C_4} a^{96} + .. + n_{C_n} a^{100}$$

where n = 100

49. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is

(a) 2^{26} (b) 2^{49}

(c) 2^{50} (d) 2^{51}

Solution:

$(1+x)^{50} = C(50,0) + C(50,1)x + C(50,2)x^2 + C(50,3)x^3 + C(50,4)x^4$

$(1-x)^{50} = C(50,0) - C(50,1)x + C(50,2)x^2 - C(50,3)x^3 + C(50,4)x^4$

$(1+x)^{50} - (1-x)^{50} = 2(C(50,1)x + C(50,3)x^3 + +C(50,5)x^5)$

put x =1

$$2^{50} = 2(C(50,1) + C(50,3) + +C(50,5) + \cdots.)$$

$$(C(50,1) + C(50,3) + +C(50,5) + \cdots.) = 2^{49}$$

50. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

(a) 1 (b) 4

(c) 8 (d) 16

Solution:

$$1 + i = \sqrt{2}e^{i\frac{\pi}{4}}$$

$$1 - i = \sqrt{2}e^{-i\frac{\pi}{4}}$$

$$\left(\frac{1+i}{1-i}\right) = e^{i\frac{\pi}{2}}$$

$$\left(\frac{1+i}{1-i}\right)^n = e^{i\frac{n\pi}{2}} = \cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}$$

if n =1 $\left(\frac{1+i}{1-i}\right)^n = i$

if n = 4 $\left(\frac{1+i}{1-i}\right)^n = \cos 2\pi + i\sin 2\pi = 1$

51. A **man** running round a racecourse notes that the sum of the distances of two flag-posts from him is always 10 m and the distance between the flag-posts is 8 m. The area of the path he encloses is

(a) 18π square meters

(b) 15π square meters

(c) 12π square meters

(d) 8π square meters

Solution: If a man running a racecourse notes that sum of the distances of two flagposts from him is always 10m then equation of path of a man is ellipse. Flagposts are foci of the ellipse.

Distance between foci = 2ae = 8m

2a = 10

$e = \frac{8}{10} = \frac{4}{5}$

$b^2 = a^2(1-e^2) = 5^2\left(1 - \frac{16}{25}\right)$

$$b = 3$$

$$a = 5$$

Area of the ellipse $= \pi ab = 15\pi$

52. The equation of the ellipse whose centre is at origin, major axis is along x-axis with eccentricity $\frac{3}{4}$ and latus rectum 4 units is

(a) $\frac{x^2}{1024}+\frac{7y^2}{64}=1$

(b) $\frac{49x^2}{1024}+\frac{7y^2}{64}=1$

(c) $\frac{7x^2}{1024}+\frac{49y^2}{64}=1$

(d) $\frac{x^2}{1024}+\frac{y^2}{64}=1$

Solution:

Equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Latus rectum of ellipse is a straight line passing through the foci of ellipse and perpendicular to the major axis of ellipse

x-coordinate of focus = ae

$$\frac{(ae)^2}{a^2}+\frac{y^2}{b^2}=1$$

$$y=b\sqrt{1-e^2}$$

Length of latus rectum = $2y=2b\sqrt{1-e^2}$

$$2b\sqrt{1-e^2}=4$$

eccentricity $e=\frac{3}{4}$

$$2b\sqrt{1-\frac{9}{16}}=4$$

$$b=\frac{8}{\sqrt{7}}$$

$$b^2=a^2(1-e^2)$$

$$a^2=\frac{64}{7\left(1-\frac{9}{16}\right)}=\frac{64\times 16}{7\times 7}=\frac{1024}{49}$$

Equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{49x^2}{1024}+\frac{7y^2}{64}=1$$

53. The equation of the plane passing the planes $x+y+z=1$, $2x+3y+4z=7$, and perpendicular to the plane $x-5y+3z=5$ is given by

(a) $x+2y+3z-6=0$

(b) $x+2y+3z+6=0$

(c) $3x+4y+5z-8=0$

(d) $3x+4y+5z+8=0$

Solution: Equation of plane passing through point of intersection of Plane P_1 and Plane P_2

$P\equiv P_1+\lambda P_2=0$

$x+y+z-1+\lambda(2x+3y+4z-7)=0$

$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-7\lambda=0$

If two plane are perpendicular to each other then dot product of direction ratio of normal is equal to zero.

Direction ratio of normal of plane P

$(1+2\lambda, 1+3\lambda, 1+4\lambda)$

Direction ratio of normal of plane $x-5y+3z=5$

(1, -5, 3)

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(1+2\lambda)\times 1+(1+3\lambda)\times(-5)+(1+4\lambda)\times 3=0$$

$$1+2\lambda-5-15\lambda+3+12\lambda=0$$

$$-1-\lambda=0$$

$$\lambda=-1$$

Equation of plane

$$(1-2)x+(1-3)y+(1-4)z-1+7=0$$
$$x+2y+3z-6=0$$

54. Consider the following:

1. $x+x^2$ is continuous at $x=0$
2. $x+\cos\frac{1}{x}$ is discontinuous at x =0
3. $x^2+\cos\frac{1}{x}$ is continuous at x =0

Which of the above are correct?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

Solution: Function f(x) = x and g(x) =x^2 are continuous function.

$h(x)=f(x)+g(x)$ is also continuous function.

Function $f(x)=\cos\frac{1}{x}$ at x = 0 function is discontinuous function.

So f(x) = $x+\cos\frac{1}{x}$ is discontinuous at x =0 is discontinuous function.

55. If x is any real number, then $\frac{x^2}{1+x^4}$ belongs to which one of the following intervals?

(a) $(0,\ 1)$

(b) $(0,\frac{1}{2}]$

(c) $\left(0,\ \frac{1}{2}\right)$

(d) $[0,1]$

Solution:

$$y=\frac{x^2}{1+x^4}$$

$$y=\frac{1}{x^2+\frac{1}{x^2}}$$

y is maximum when $x^2+\frac{1}{x^2}$ is minimum.

Minimum value of $x^2+\frac{1}{x^2}$ is 2

Range of $y\ (0\ ,\frac{1}{2}]$

56. What is the distance between the straight lines $3x+4y=9$ and $6x+8y=15$?

(a) $3/2$ (b) $3/10$
(c) 6 (d) 5

Solution:

Lines $3x+4y=9$ and $6x+8y=15$ are parallel to each other.

distance between parallel line

$$=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=\frac{|-9+7.5|}{\sqrt{3^2+4^2}}=\frac{1.5}{5}=\frac{3}{10}$$

57. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}|=2, |\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{\imath}+2\hat{\jmath}+6\hat{k}$, then what is the acute angle between $\vec{a}$ and $\vec{b}$?

(a) 30^0 (b) 45^0
(c) 60^0 (d) 90^0

Solution: $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

$$|\vec{a}\times\vec{b}|=\sqrt{3^2+2^2+6^2}=\sqrt{9+4+36}=7$$

$$|\vec{a}||\vec{b}|\sin\theta=7$$

$$\sin\theta=\frac{7}{2\times 7}=\frac{1}{2}$$

Acute angle between $\vec{a}$ and $\vec{b}$ are vectors is 30^0.