1. What is the value of $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$ equal to?

(a) $3\log_2 7$

(b) $1 - 3\log_2 7$

(c) $1 - 3\log_7 2$

(d) $\frac{7}{8}$

Solution:

$$x = \log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$$

$$\log_7 \sqrt{7\sqrt{7\sqrt{7}}} = \frac{1}{2}\log_7 7\sqrt{7\sqrt{7}}$$

$$= \frac{1}{2}\left(\log_7 7 + \log_7 \sqrt{7\sqrt{7}}\right)$$

$$= \frac{1}{2}\left(1 + \frac{1}{2}\log_7 7\sqrt{7}\right)$$

$$= \frac{1}{2}\left(1 + \frac{1}{2}\log_7 7 + \frac{1}{2}\log_7 \sqrt{7}\right)$$

$$= \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{7}{8}$$

$$x = \log_7 \frac{7}{8} = 1 - 3\log_7 2$$

Answer: (c)

2. If an infinite GP has the first term x and the sum 5, then which one of the following is correct?

(a) $x < -10$ (b) $-10 < x < 0$

(c) $0 < x < 10$ (d) $x > 10$

Solution:

Sum of infinite series S = $\frac{a}{1-r}$

$$5 = \frac{x}{1-r}$$

$$x = 5(1-r)$$

If $|r| < 1$, then sum of infinite series be defined.

$0 < 5(1-r) < 10$

$0 < x < 10$

Answer: (c)

3. Consider the following expression:

1. $x + x^2 - \frac{1}{x}$
2. $\sqrt{ax^2 + bx + x - c + \frac{d}{x} - \frac{e}{x^2}}$
3. $3x^2 - 5x + ab$
4. $\frac{2}{x^2 - ax + b^3}$
5. $\frac{1}{x} - \frac{2}{x+5}$

Which of the above are rational expressions?

(a) 1, 4 and 5 only

(b) 1, 3, 4 and 5 only

(c) 2, 4 and 5 only

(d) 1 and 2 only

4. A square matrix A is called orthogonal if

(a) $A = A^2$

(b) $A' = A^{-1}$

(c) $A = A^{-1}$

(d) $A = A'$

Solution: An orthogonal matrix is a matrix whose transpose is equal to the inverse of the matrix.

$$A^T = A^{-1}$$

Answer: (b)

4. The sum of the series $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$ equal to

(a) $\frac{20}{9}$ (b) $\frac{9}{20}$

(c) $\frac{9}{4}$ (d) $\frac{4}{9}$

Solution:

Series is GP. First term is 3 and common ratio is -1/3.

$$S_n = \frac{a}{1-r} = \frac{3}{1-(-\frac{1}{3})} = \frac{9}{4}$$

Answer: (c)

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students. like to play tennis. 32 students like to play exactly two games out of the three games.

5. How many students like to play all the three games?

(a) 14 (b) 21

(c) 28 (d) 35

Solution:

Total number of students play cricket n(A) = 125

Total number of students play football n(B) = 145

Total number of students plays tennis n(C) = 9

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) - n(A \cap B \cap C)$$

$$n(A \cup B \cup C) = 125 + 145 + 90 - 32 - n(A \cap B \cap C)$$

$$n(A \cup B \cup C) = 300$$

$$n(A \cap B \cap C) = 28$$

Answer: (c)

6. What is the coefficient of the middle term is the binomial expansion of $(2 + 3x)^4$?

(a) 6 (b) 12

(c) 108 (d) 216

Solution:

In the binomial expansion of $(x + y)^n$, number of term is n +1.

r^th term of binomial expansion $(x + y)^n$is $C(n, r)x^r y^{n-r}$.

If n is even then middle term is $\frac{n}{2} + 1$.

$$r = \frac{n}{2} + 1$$

n = 4 then r = 3

Coefficient of middle term = $C(4,3)2^3(3x)^{4-3}$

= 216

Answer: (d)

7 Which one of the following factor does the expansion of the determinant

$\begin{vmatrix} x & y & 3 \\ x^2 & 5y^3 & 9 \\ x^3 & 10y^5 & 27 \end{vmatrix}$ contain ?

(a) $x - 3$ (b) $x - y$

(c) $y - 3$ (d) $x - 3y$

Solution:

If x = 3 then $\begin{vmatrix} 3 & y & 3 \\ 9 & 5y^3 & 9 \\ 27 & 10y^5 & 27 \end{vmatrix}$ Since column 1 and column two is same. therefore determinant of the matrix is equal to zero.

x – 3 is a factor of the determinant.

Answer: (a)

8. What is the adjoint of the matrix.

$\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}$?

(a) $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Solution:

Cofactor matrix = $\begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$

Transpose of Cofactor matrix

= $\begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$

9. What is the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$ where = $\sqrt{-1}$?

(a) 3 (b) 2

(c) 1 (d) 0

Solution:

$$\frac{-1 + i\sqrt{3}}{2} = \omega$$

$$\frac{-1 - i\sqrt{3}}{2} = \omega^2$$

ω, ω^2 are complex roots of cubic equation.

$$1 + \omega + \omega^2 = 1$$

$$\omega^3 = 1$$

$$z = \omega^{3n} + \omega^{6n}$$

put n = 1

$$z = \omega^3 + \omega^6 = 1 + 1 = 2$$

Answer: (b)

10. There are 17 cricket players, out of which 5 players can bowl. In how many ways can a team of 11 players be selected so as to include 3 bowlers?

(a) C(17, 11)

(b) C(12, 8)

(c) C(17, 5) x C(5, 3)

(d) C(5,3) x C(12, 8)

Solution:

No of ways = C(12, 8) x C (5,3)

Answer: (d)

11. What is the value of $\log_9 27 + \log_8 32$?

(a) $\frac{7}{2}$ (b) $\frac{19}{6}$

(c) 4 (d) 7

Solution:

$$\log_9 27 = \log_9 9 + \log_9 3$$

$$= 1 + \log_9 9^{(1/2)} = \frac{3}{2}$$

$$\log_8 32 = \log_8 8 + \log_8 4 = 1 + \log_8 \frac{8}{2}$$

$$= 1 + \log_8 8 - \log_8 2 = 1 + 1 - \log_8 8^{\frac{1}{3}}$$

$$= 2 - \frac{1}{3} = \frac{5}{3}$$

$$\log_9 27 + \log_8 32 = \frac{3}{2} + \frac{5}{3} = \frac{19}{6}$$

Answer: (b)

12. If A and B are two invertible square matrices of same order, then what is $(AB)^{-1}$equal to ?

(a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$

(c) $B^{-1}A$ (d) $A^{-1}B$

Answer: (a)

13. If $a + b + c = 0$, then one of the solutions of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a) $x = a$

(b) $x = \sqrt{\frac{3(a^2+b^2+c^2)}{2}}$

(c) $x = \sqrt{\frac{2(a^2+b^2+c^2)}{3}}$

(d) $x = 0$

Solution:

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

put x = 0

$$\begin{vmatrix} a & c & b \\ c & b & a \\ b & a & c \end{vmatrix} = 0$$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c & b & a \\ b & a & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 0 & 0 \\ c & b & a \\ b & a & c \end{vmatrix} = 0$$

Since elements of first rows is zero.

So x = 0 is the solution of the equation.

Answer: (d)

14. What should be the value of x so that the matrix $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$ does not have an inverse?

(a) 16 (b) -16

(c) 8 (d) -8

Solution:

Let A = $\begin{pmatrix} 2 & 4 \\ -8 & x \end{pmatrix}$. If $\det(A) = 0$ then matrix A is singular matrix. For singular matrix , inverse doesn't exist.

$$\begin{vmatrix} 2 & 4 \\ -8 & x \end{vmatrix} = 0$$

$$2x + 32 = 0$$

$$x = -16$$

Answer: (b)

15. The system of equations

$$2x + y - 3z = 5$$

$3x - 2y + 2z = 5$ and

$$5x - 3y - z = 16$$

(a) is inconsistent

(b) is consistent, with a unique solution

(c) is consistent, with infinitely many solutions.

(d) has its solution lying along x-axis in three –dimensional space.

Solution:

Solution of three linear equations is x = 1, y =-3, and z = -2. So solution is consistent, with a unique solution.

Answer: (b)

16. Which one of the following is correct in respect of the cube roots of unity?

(a) They are collinear

(b) They lie on a circle of radius

(c) They form an equilateral triangle

(d) None of the above

Solution:

$\boldsymbol{x^3 = 1}$ **.**

Roots of cubic equation 1, ω , and ω^2

$$\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

If A (1,0), B(-1/2 , $\sqrt{3}/2$), C(-1/2, -$\sqrt{3}/2$)

$$AB = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

$$BC = \sqrt{\left(\sqrt{3}\right)^2} = \sqrt{3}$$

$$CA = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

Answer: (c)

17. If u, v and w (all positive) are the p^{th}, q^{th} and r^{th} terms of a GP, then the determinant of the matrix $\begin{pmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{pmatrix}$ is

(a) 0

(b) 1

(c) $(p-q)(q-r)(r-p)$

(d) $\ln u \times \ln v \times \ln w$

Solution:

$u = ar^{p-1}$

$v = ar^{q-1}$

$w = ar^{r-1}$

$\ln u = \ln a + (p-1)\ln r$

$= \log\frac{a}{r} + p \ln r$

$\ln v = \ln\frac{a}{r} + q \ln r$

$\ln w = \ln\frac{a}{r} + r \ln r$

$$\begin{Bmatrix} \ln u \\ \ln v \\ \ln w \end{Bmatrix} = \ln\frac{a}{r} \times \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} + \ln r \times \begin{Bmatrix} p \\ q \\ r \end{Bmatrix}$$

So determinant of the matrix is equal to zero.

Answer: (a)

18 Let T_r be the r^{th} term of an AP for $r = 1, 2, 3, \ldots$.. If for some distinct positive integers m and n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then what is T_{mn} equal to?

(a) $(mn)^{-1}$ (b) $m^{-1} + n^{-1}$

(c) 1 (d) 0

Solution: $T_m = T_1 + (m-1)d$

$T_n = T_1 + (n-1)d$

$T_1 + (m-1)d = \frac{1}{n}$

$T_1 + (n-1)d = \frac{1}{m}$

$T_1 = d = \frac{1}{mn}$

$$T_{mn} = T_1 + (mn-1)d = \frac{1}{mn} + \frac{mn-1}{mn} = 1$$

19. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is

(a) 45360 (b) 30240

(c) 27216 (d) 15120

Solution: Total number of numbers formed of 5 digit number = $9 \times 9 \times 8 \times 7 \times 6 = 27216$

20. What is the determinant of the matrix $\begin{pmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{pmatrix}$?

(a) $(x-y)(y-z)(z-x)$

(b) $(x-y)(y-z)$

(c) $(y-z)(z-x)$

(d) $(z-x)^2(x+y+z)$

Solution:

$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ y & z & x+y \end{vmatrix} =$

$\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ x+y+z & x+y+z & 2(x+y+z) \end{vmatrix} =$

$(x+y+z)\begin{vmatrix} x & y & y+z \\ z & x & z+x \\ 1 & 1 & 2 \end{vmatrix}$

21. What is $\frac{2\tan\theta}{1+tan^2\theta}$ equal to?

(a) $\cos 2\theta$ (b) $\tan 2\theta$

(c) $\sin 2\theta$ (d) $cosec\ \theta$

Solution:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 + \tan A \tan B}$$

A = B = θ

$$\tan 2\theta = \frac{2\tan\theta}{1+tan^2\theta}$$

Answer: (b)

22. If $+B+C = 180^0$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to ?

(a) $-4\sin A \sin B\ \sin C$

(b) $-4\cos A \sin B \cos C$

(c) $-4\cos A \cos B \sin C$

(d) $-4\sin A \cos B \cos C$

Solution: $\sin 2A - \sin 2B - \sin 2C$

$$= 2sin\ ACos\ A - 2\sin\frac{2B+2C}{2}\cos\frac{2B-2C}{2}$$

$$= 2\sin A\cos A - 2\sin(B+C)\cos(B-C)$$

$$= 2\sin A\,(\cos A - \cos(B-C))$$

$$= 2\sin A\left(-\cos(B+C) - Cos(B-C)\right)$$

$$= -2\sin A\,(\cos(B+C) + \cos(B-C))$$

$$= -2\sin A\, 2\cos\frac{B+C+B-C}{2}\cos\frac{B+C-B+C}{2}$$

$$= -4sinA\cos B\cos C$$

23. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\frac{\pi}{6}$ and $\frac{\pi}{3}$ respectively. What is the height of the hill?

(a) 2h (b) $\frac{3h}{2}$

(c) h (d) $\frac{h}{2}$

Solution: Let distance between building and hill is B

H is height of the hill

$$\tan 60^0 = \frac{H}{B}$$

$$\tan 30^0 = \frac{H-h}{B}$$

$$\frac{\tan 60^0}{\tan 30^0} = \frac{H}{H-h}$$

$$\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{H}{H-h}$$

$$3 = \frac{H}{H-h}$$

$$H = \frac{3h}{2}$$

24. What is/are the solution(s) of the trigonometric equation $cosec\ x + cot\ x = \sqrt{3}$, where $0 < x < 2\pi$?

(a) $\frac{5\pi}{3}$ only **(b) $\frac{\pi}{3}$ only**

(c) π only **(d) $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$**

Solution:

$$\text{cosec x} + \cot x = \sqrt{3}$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{3}$$

$$1 + \cos x = \sqrt{3}\sin x$$

$$1 = \sqrt{3}\sin x - \cos x$$

$$\frac{1}{2} = \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$$

$$\frac{1}{2} = \sin(x - 30^0)$$

x = 180^0 , 60^0

Answer: (*)

25. if $\cos\alpha$ and $\cos\beta$ ($0 < \alpha < \beta < \pi$) are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$?

(a) $-4/3$ **(b)** $4/3$

(c) $3/4$ **(d)** $-3/4$

Solution:

$$\cos\alpha + \cos\beta = -\frac{b}{a} = 0$$

$$\cos\alpha\cos\beta = \frac{c}{a} = -\frac{3}{4}$$

$$\sec\alpha\sec\beta = -\frac{4}{3}$$

Answer: (a)

26. The equation of a circle whose end points of a diameter are (x_1, y_1) and (x_2, y_2) is

(a) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = x^2 + y^2$

(b) $(x - x_1)^2 + (y - y_1)^2 = x_2y_2$

(c) $x^2 + y^2 + 2x_2x_1 + 2y_1y_2 = 0$

(d) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Solution:

The equation of the circle

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

End points of a diameter are

(x_1, y_1) and (x_2, y_2)

Centre of the circle is midpoint of end points of diameter.

$$x_0 = \frac{x_1 + x_2}{2}$$

$$y_0 = \frac{y_1 + y_2}{2}$$

$$R^2 = \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\left(x - \frac{x_1 + x_2}{2}\right)^2 + \left(y - \frac{y_1 + y_2}{2}\right)^2 = \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}$$

$$(2x - x_1 - x_2)^2 + (2y - y_1 - y_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$(2x - x_1 - x_2)^2 - (x_1 - x_2)^2 + (2y - y_1 - y_2)^2 - (y_1 - y_2)^2 = 0$$

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Answer: (d)

27. The second degree equation $x^2 + 4y^2 - 2x - 4y + 2 = 0$ represents

(a) A point

(b) An ellipse of semi-major axis 1

(c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$

(d) None of the above

Solution:

$$x^2 + 4y^2 - 2x - 4y + 2 = 0$$

$$(x - 1)^2 + (2y - 1)^2 = 0$$

Above equation satisfy at x =1 and y = 1/2

Answer: (a)

28. The angle between the two lines $lx + my + n = 0$ and $l'x + m'y + n' = 0$ is given by $\tan^{-1}\theta$. What is θ equal to?

(a) $\left|\frac{lm' - l'm}{ll' - mm'}\right|$

(b) $\left|\frac{lm' + l'm}{ll' + mm'}\right|$

(c) $\left|\frac{lm' - l'm}{ll' + mm'}\right|$

(d) $\left|\frac{lm' + l'm}{ll' - mm'}\right|$

Solution:

Slope of line L_1: $lx + my + n = 0$

$$m_1 = -\frac{l}{m}$$

Slope of line L_2: $l'x + m'y + n' = 0$

$$m_2 = -\frac{l'}{m'}$$

Angle between two lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\frac{l}{m} + \frac{l'}{m'}}{\frac{l}{m} \times \frac{l'}{m'} + 1}\right|$$

$$\tan\theta = \left|\frac{l'm - lm'}{ll' + mm'}\right|$$

$$\theta = \tan^{-1}\left|\frac{l'm - lm'}{ll' + mm'}\right|$$

Answer: (c)

29. Consider the following

1. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_1 - c_2|}{\sqrt{1+m^2}}$.
2. The distance between the lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$
3. The distance between the lines $x = c_1$ and $x = c_2$ is $|c_1 - c_2|$.

Which of the above statements are correct?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

Solution:

Line L$_1$: $y = mx + c_1$

Line L$_2$: $y = mx + c_2$

Line L$_1$ and Line L$_2$ are parallel to each other.

Perpendicular distance from origin to Line L$_1$ and L$_2$ are

$$d_1 = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d_1 = \frac{|a \times 0 + b \times 0 + c|}{\sqrt{a^2 + b^2}}$$

$$d_1 = \frac{|c|}{\sqrt{a^2 + b^2}}$$

$$d_1 = \frac{|c_1|}{\sqrt{1 + m^2}}$$

Similarly $d_2 = \frac{|c_2|}{\sqrt{1+m^2}}$

Perpendicular distance between line L$_1$ and L$_2$ is equal to $|d_1 - d_2|$

$$|d_1 - d_2| = \left|\frac{|c_1|}{\sqrt{1 + m^2}} - \frac{|c_2|}{\sqrt{1 + m^2}}\right|$$

$$|d_1 - d_2| = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$$

Answer: (d)

30. What is the equation of straight line passing through the point of intersection of the lines $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$, and parallel to the line $4x + 5y - 6 = 0$?

(a) $20x + 25y - 54 = 0$
(b) $25x + 20y - 54 = 0$
(c) $4x + 5y - 54 = 0$
(d) $4x + 5y - 45 = 0$

Solution:

Point of intersection of lines $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$ is $\left(\frac{6}{5}, \frac{6}{5}\right)$

Equation of line parallel to $4x + 5y - 6 = 0$ is $4x + 5y + c = 0$.

Line $4x + 5y + c = 0$ is passes through $\left(\frac{6}{5}, \frac{6}{5}\right)$.

$$4 \times \frac{6}{5} + 5 \times \frac{6}{5} + c = 0$$

$$c = -\frac{54}{5}$$

Equation of line $4x + 5y - \frac{54}{5} = 0$

$20x + 25y - 54 = 0$

Answer: (a)

31. What is the distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$

(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

Solution:

Equation of the plane

$$3x - 6y + 2z + 11 = 0$$

Direction ratio of normal to the plane P is (3, -6, 2)

Direction ratio of the plane P is (3, -6, 2)

Equation of line passing through (2, 3, 4) and normal to the given plane Pis $\frac{x-2}{l} = \frac{y-3}{m} = \frac{z-4}{n} = k$

$$l = \frac{3}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3}{7}$$

$$m = \frac{-6}{\sqrt{3^2+(-6)^2+2^2}} = \frac{-6}{7}$$

$$n = \frac{2}{\sqrt{3^2+(-6)^2+2^2}} = \frac{2}{7}$$

$$\frac{x-2}{3} = \frac{y-3}{-6} = \frac{z-4}{2}$$

$$x = 3k+2$$

$$y = -6k+3$$

$$z = 2k+4$$

point x, y, z lie on the plane P

$$3x - 6y + 2z + 11 = 0$$

$$3(3k+2) - 6(-6k+3) + 2(2k+4) + 11 = 0$$

$$k = \frac{1}{7}$$

$$x = 3 \times \frac{1}{7} + 2 = \frac{3}{7} + 2$$

$$y = -\frac{6}{497} + 3$$

$$z = \frac{2}{7} + 4$$

Distance D = $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (y_1-y_2)^2}$

$$= \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$$

= 1

Answer: (a)

32 .Consider the following statements

1. The angle between the planes $2x - y + z = 1$ and $x + y + 2z = 3$ is $\frac{\pi}{3}$

2. The distance between the planes $6x - 3y + 6z + 2 = 0$ and $2x - y + 2z + 4 = 0$ is $\frac{10}{9}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

General equation of plane is

$$\boldsymbol{ax + by + cz + d = 0} \quad .$$

Direction ratio of normal to plane is (a, b, c).

Direction ratio of plane $2x - y + z = 1$ is (2, -1, 1)

Direction ratio of plane $x + y + 2z = 3$ is

(1, 1, 2)

$\cos\theta$

$$= \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1)^2 + (b_1)^2 + (c_1)^2}\sqrt{(a_2)^2 + (b_2)^2 + (c_{21})^2}}$$

$$\cos\theta = \frac{2-1+2}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\theta = 60^0$$

Distance of plane P_1 from origin is

$$d_1 = \frac{6\times0-3\times0+6\times0+2}{\sqrt{6^2+3^2+6^2}} = \frac{2}{9}$$

Distance of plane P_2 from origin is

$$d_2 = \frac{4}{3}$$

$$Distance\ between\ plane\ P_1 and\ P_2$$

$$= |d_1 - d_2| = \frac{10}{9}$$

33. Consider the following statements:

Statement I : If the line segment joining the points P(m,n) and Q(r,s) subtends an angle α at the origin, then

$$\cos\alpha = \frac{ms - nru}{\sqrt{(m^2+n^2)(r^2+s^2)}}$$

Statement II : In any triangle ABC, it is true that $a^2 = b^2 + c^2 - 2bc\cos A$.

Which one of the following is correct in respect of the above two statements?

(a) Both Statement I and Statements II are true and statement II is the coruuruect explanation of statement I

(b) Both Statement I and Statement II are true, but Statement II is not the correcty explanation of statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true.

Solution:

Position vector $\overrightarrow{OP} = m\hat{\imath} + n\hat{\jmath}$

Position vector $\overrightarrow{OQ} = r\hat{\imath} + s\hat{\jmath}$

Angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is

$$\cos\alpha = \frac{Dot\ product\ of\ \overrightarrow{OP} \text{ and } \overrightarrow{OQ}}{|\overrightarrow{OP}||\overrightarrow{OQ}|}$$

$$\cos\alpha = \frac{mr + ns}{\sqrt{(m^2+n^2)(r^2+s^2)}}$$

Answer: (d)

34. Let $|\vec{a}| \neq 0, |\vec{b}| \neq 0$. $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ holds if and only if

(a) $\vec{a}$ and $\vec{b}$ are perpendicular

(b) $\vec{a}$ and $\vec{b}$ are parallel

(c) $\vec{a}$ and $\vec{b}$ are inclined at an angle of 45^0

(d) $\vec{a}$ and $\vec{b}$ are anti-parallel

Solution:

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$

Given $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$\vec{a}.\vec{b} = 0$

$\vec{a}$ and $\vec{b}$ are perpendicular.

Answer: (a)

35. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, what is $\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k})$ equal to?

(a) x

(b) $x + y$

(c) $-(x+y+z)$

(d) $(x+y+z)$

Solution:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k}) = x + y + z$$

Answer: (d)

36.A unit vector perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - \hat{k}$ is

(a) $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$

(b) $\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{1}{2}\hat{k}$

(c) $\frac{1}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$

(d) $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

Solution:

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 5\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$$

$$|\vec{a}\times\vec{b}| = 5\sqrt{3}$$

$$\hat{n} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$$

Answer: (a)

37. if $|\vec{a}| = 3$. $|\vec{b}| = 4$ and $|\vec{a}-\vec{b}| = 5$, then what is the value of $|\vec{a}+\vec{b}|$?

(a) 8 (b) 6

(c) $5\sqrt{2}$ (d) 5

Solution:

$|\vec{a}-\vec{b}| = 5$

$|\vec{a}-\vec{b}|^2 = 25$

$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} = 25$

$3^2 + 4^2 - 2\vec{a}.\vec{b} = 25$

$\vec{a}.\vec{b} = 0$

Vectors are perpendicular to each other. Vector $\vec{a}+\vec{b}$ and vector $\vec{a}-\vec{b}$ are diagonal of rectangle formed by vector $\vec{a}$ and $\vec{b}$.

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| = 5$$

38. .Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three mutually perpendicular vectors each of unit magnitude. If $\vec{A} = \vec{a}+\vec{b}+\vec{c}$, $\vec{B} = \vec{a}-\vec{b}+\vec{c}$ and

$\vec{C} = \vec{a} - \vec{b} - \vec{c}$, then which one of the following is correct?

(a) $|\vec{A}| > |\vec{B}| > |\vec{C}|$

(b) $|\vec{A}| = |\vec{B}| \neq |\vec{C}|$

(c) $|\vec{A}| = |\vec{B}| = |\vec{C}|$

(d) $|\vec{A}| \neq |\vec{B}| \neq |\vec{C}|$

$$|\vec{A}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c})$$
$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$$

$|\vec{A}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 \quad (\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0)$

$|\vec{A}|^2 = 3$

Answer: (c)

39. What is $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ equal to?

(a) $\vec{0}$

(b) $\vec{a} \times \vec{b}$

(c) 2($\vec{a} \times \vec{b}$)

(d) $|\vec{a}|^2 - |\vec{b}|^2$

Solution:

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = \vec{a} \times (\vec{a} + \vec{b}) - \vec{b} \times (\vec{a} + \vec{b})$$
$$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} + \vec{b} \times \vec{b} = 2(\vec{a} \times \vec{b})$$

Answer: (c)

40. A spacecraft located at $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is subjected to a force $\lambda\hat{k}$ by firing a rocket. The spacecraft is subjected to a moment of magnitude

(a) λ (b) $\sqrt{3}\lambda$

(c) $\sqrt{5}\lambda$ (d) None of the above

Solution:

Moment = $\vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 0 & \lambda \end{vmatrix} = (2\lambda)\hat{\imath} - \lambda\hat{\jmath}$$

Magnitude of moment = $\sqrt{5}\lambda$

41 If $f(x) = \frac{\sqrt{x-1}}{x-4}$ defines a function on R, then what is its domain

(a) $(-\infty, 4) \cup (4, \infty)$

(b) $[4, \infty)$

(c) $(1, 4) \cup (4, \infty)$

(d) $[1, 4) \cup (4, \infty)$

Solution:

$f(x) = \frac{\sqrt{x-1}}{x-4}$

Denominator should not equal to zero.

$x - 4 \neq 0$

$x \neq 4$

Domain: $(-\infty, 4) \cup (4, \infty)$

Answer: (a)

42. Consider the function

$$f(x) = \begin{cases} \frac{\sin}{5x} & if\ x \neq 0 \\ \frac{2}{15} & if\ x = 0 \end{cases}$$

Which one of the following is correct in respect of the function?

(a) It is not continuous at x = 0

(b) It is continuous at every x

(c) It is not continuous at x = π

(d) It is continuous at x = 0

Solution:

$\lim_{x\to 0} \frac{\sin 2x}{5x} = \lim_{x\to 0} \frac{2x}{5x} = \frac{2}{5}$

$\lim_{x\to 0} f(x) \neq f(0)$

Function is discontinuous at x =0

Answer: (a)

43. For the function $f(x) = |x - 3|$, which one of the following is not correct?

(a) The function is not continuous at $x = -3$

(b) The function is continuous at x =3

(c) The function is differentiable at x =0

(d) The function is differentiable at x = -3

Answer: (a)

44. If the function

$$\mathrm{f(x)} = \frac{2\mathrm{x} - \sin^{-1}\mathrm{x}}{2\mathrm{x} + \tan^{-1}\mathrm{x}}$$

is continuous at each point in its domain, then what is the value of f(0)?

(a) $-1/3$ (b) $1/3$

(c) $2/3$ (d) 2

Solution:

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} =$

$\lim_{x\to 0}\frac{2x-x}{2x+x} = \lim_{x\to 0}\frac{x}{3x} = \frac{1}{3}$

45. If $(x) = \sqrt{25-x^2}$, then what is $\lim_{x\to 1}\frac{j(x)-j(1)}{x-1}$ equal to?

(a) $-\frac{1}{\sqrt{24}}$ (b) $\frac{1}{\sqrt{24}}$

(c) $-\frac{1}{4\sqrt{3}}$ (d) $\frac{1}{4\sqrt{3}}$

Solution:

$$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1} = f'(1)$$

$$f(x) = \sqrt{25-x^2}$$

$$f^{/}(x) = \frac{-x}{\sqrt{25-x^2}}$$

$$f^{/}(1) = \frac{-1}{\sqrt{24}}$$

Answer: (b)

46. Which one of the following is correct in respect of the function $f(x) = x\sin x + \cos x + \frac{1}{2}cos^2x$?

(a) It is increasing in the interval $(0, \frac{\pi}{2})$

(b) It remains constant in the interval $(0, \frac{\pi}{2})$

(c) It is decreasing in the interval $(0, \frac{\pi}{2})$

(d) It is decreasing in the interval $(\frac{\pi}{4}, \frac{\pi}{2})$

Solution:

$$f(x) = x\sin x + \cos x + \frac{1}{2}cos^2x$$

$$f(x) = x\sin x + \cos x + \frac{1+\cos 2x}{4}$$

$$f'(x) = \sin x + xcosx - \sin x + \frac{1}{4}(-\sin 2x) \times 2$$

$$f'(x) = xcosx - \sin x\cos x$$

$$f'(x) = \cos x\,(x - \sin x)$$

$$f'(x) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}$$

$$x - \sin x = 0$$

$$x = 0$$

In interval $(0, \frac{\pi}{2})$ cos x and x – sin x are positive . So $f'(x)$ is positive in this interval. So function f(x) is increasing function in interval $(0, \frac{\pi}{2})$

Answer: (a)

47. What is $\lim_{\theta\to 0}\frac{\sqrt{1-\cos}}{\theta}$ equal to?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2\sqrt{2}}$

Solution:

$$\lim_{\theta\to 0^+}\frac{\sqrt{1-\cos\theta}}{\theta}$$

$$= \lim_{\theta\to 0}\frac{\sqrt{2sin^2\left(\frac{\theta}{2}\right)}}{\theta}$$

$$= \lim_{\theta\to 0^+}\frac{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}{\theta}$$

$$= \frac{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}{2\times\frac{\theta}{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\lim_{\theta\to 0^-}\frac{\sqrt{1-\cos\theta}}{\theta}$$

$$= \lim_{\theta\to 0^+}\frac{-\sqrt{2}\sin\left(\frac{\theta}{2}\right)}{\theta}$$

$$= \frac{-\sqrt{2}\sin\left(\frac{\theta}{2}\right)}{2\times\frac{\theta}{2}}$$

$$= -\frac{1}{\sqrt{2}}$$

Limit doesn’t exist.

Answer: (*)

48. A function $f : A \to R$ is defined by the equation $f(x) = x^2 - 4x + 5$ where A = (1, 4). What is the range of the function?

(a) (2, 5) (b) (1, 5)

(c) [1, 5) (d) [1,5]

Solution:

$f(x) = x^2 - 4x + 5$

$f(x) = (x-2)^2 + 1$

$f(1) = 2$

$f(4) = 5$

$f(2) = 1$

Range of function f(x) is [1, 5)

Answer: (d)

49. What is $\int_2^8 |x-5|\,dx$ equal to?

(a) 2 (b) 3

(c) 4 (d) 9

Solution:

$$I = \int_2^5 5 - x\,dx + \int_5^8 x - 5\,dx$$

$$I = 5x - \frac{x^2}{2}\Big|_2^5 + \frac{x^2}{2} - 5x\Big|_5^8$$

$$I = 5(5-2) - \frac{5^2 - 2^2}{2} + \frac{8^2 - 5^2}{2} - 5(8-5)$$

$$I = 9$$

Answer: (d)

50. In which one of the following intervals is the function $f'(x) = x^2 - 5x + 6$ decreasing?

(a) $(-\infty, 2]$ (b) $[3, \infty)$

(c) $(-\infty, \infty)$ (d) $(2, 3)$

Solution:

$f'(x) = x^2 - 5x + 6$

$f'(x) = (x-3)(x-2)$

If $2 \le x \le 3$, then $f'(x) < 0$

f(x) is decreasing in interval (2, 3).

Answer: (d)

51. The differential equation of the family of curves $y = p\cos(ax) + q\sin(ax)$, where p, q are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} - a^2y = 0$

(b) $\frac{d^2y}{dx^2} - ay = 0$

(c) $\frac{d^2y}{dx^2} + ay = 0$

(d) $\frac{d^2y}{dx^2} + a^2y = 0$

Solution:

$\mathrm{y = p\cos(ax) + q\sin(ax)}$

$$\mathrm{\frac{dy}{dx} = -pa\sin(ax) + qa\cos(ax)}$$

$$\mathrm{\frac{d^2y}{dx^2} = -pa^2\cos(ax) - qa^2\sin(ax)}$$

$$\mathrm{\frac{d^2y}{dx^2} + a^2y = 0}$$

Answer: (d)

52. The equation of the curve passing through the point (-1, -2) which satisfies $\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$, is

(a) $17x^2y - 6x^2 + 3x^5 - 2 = 0$

(b) $6x^2y + 17x^2 + 2x^5 - 3 = 0$

(c) $6xy - 2x^2 + 17x^5 + 3 = 0$

(d) $17x^2y + 6xy - 3x^5 + 5 = 0$

Solution:

$$\frac{dy}{dx} = -x^2 - \frac{1}{x^3}$$

$$y = -\frac{x^3}{3} - \frac{x^{-3+1}}{-3+1} = -\frac{x^3}{3} + \frac{1}{2x^2} + c$$

Curve y passes through $(-1, -2)$

$$-2 = \frac{1}{3} + \frac{1}{2} + c$$

$$c = -\frac{17}{6}$$

$$y = -\frac{x^3}{3} + \frac{1}{2x^2} - \frac{17}{6}$$

$6x^2y + 2x^5 + 17x^2 - 3 = 0$

Answer: (c)

53. What is the solution of the differential equation $\ln\left(\frac{dy}{dx}\right) = ax + by$?

(a) $ae^{ax} + be^{by} = c$

(b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{by} = c$

(c) $ae^{ax} + be^{-by} = c$

(d) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = c$

where c is an arbitrary constant.

Solution:

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\int e^{-by}dy = \int e^{ax}dx$$

$$\frac{e^{-b}}{-b} + c = \frac{e^{ax}}{a}$$

$$\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = c$$

Answer: (d)

54. What is the solution $e^{ax}\sin bx$ and $v = e^{ax}\cos bx$, then what is $u\frac{du}{dx} + v\frac{dv}{dx}$ equal to?

(a) ae^{2ax} (b) $(a^2+b^2)e^{ax}$

(c) abe^{2ax} (d) $(a+b)e^{ax}$

Solution:

$u = e^{ax}\sin bx$

$v = e^{ax}\cos bx$

$u^2 + v^2 = e^{2ax}$

$$2u\frac{du}{dx} + 2v\frac{dv}{dx} = 2ae^{2ax}$$

$$u\frac{du}{dx} + v\frac{dv}{dx} = ae^{2ax}$$

Answer: (a)

55. If $= \sin(\ln x)$, then which one of the following is correct?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(d) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Solution:

$y = \sin(\ln x)$

$$\frac{dy}{dx} = \frac{\cos(\ln x)}{x}$$

$$x\frac{dy}{dx} = \cos(\ln x)$$

$$\frac{dy}{dx} + x\frac{d^2y}{dx^2} = -\frac{\sin(\ln x)}{x}$$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Answer: (c)

56. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. if the flower – bed has the greatest possible area then what is the radius of the sector?

(a) 25 m (b) 20 m

(c) 10 m (d) 5 m

Solution:

Let r is radius of the sector and l is length of arc.

Given $2r + l = 40$

Area of the sector A = $\frac{rl}{2}$

$$A = \frac{rl}{2} = \frac{r}{2}(40 - 2r) = r(20 - r)$$

$$\frac{dA}{dr} = 20 - 2r$$

Find the critical point $\frac{dA}{dr} = 0$

$20 - 2r = 0$

$r = 10$

Answer: (c)

57. What is the minimum value of $[x(x-1) + 1]^{\frac{1}{3}}$, where $0 \le x \le 1$?

(a) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$ (b) 1

(c) $\frac{1}{2}$ (d) $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

Solution:

$$y = [x(x-1) + 1]^{\frac{1}{3}}$$

$$y = [x^2 - x + 1]^{\frac{1}{3}}$$

$$y = \left[\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}\right]^{\frac{1}{3}}$$

Minimum value of y

$$y = \left[\frac{3}{4}\right]^{\frac{1}{3}}$$

Answer: (a)

58 If $y = |\sin x|^{|x|}$, then what is the value of $\frac{dy}{dx}$ at $= -\frac{\pi}{6}$?

(a) $\frac{2^{-\frac{\pi}{6}}(6\ln 2-\sqrt{3}\pi)}{6}$

(b) $\frac{2^{\frac{\pi}{6}}(6\ln 2\quad \sqrt{3}\pi)}{6}$

(c) $\frac{2^{-\frac{\pi}{6}}(6\ln 2+\sqrt{3}\pi)}{6}$

(d) $\frac{2^{\frac{\pi}{6}}(6\ln 2-\sqrt{3}\pi)}{6}$

Solution:

$y = |\sin x|^{|x|}$

$\ln y = |x| \ln|\sin x|$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d|x|}{dx}\ln|\sin x| + |x|\frac{d(\ln|\sin x|)}{dx}$$

At $x = -\frac{\pi}{6}$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d(-x)}{dx}\ln|\sin x| + |x|\frac{1}{|\sin x|}\frac{d|\sin x|}{dx}$$

$y = |\sin x|^{|x|}$

$$y = \left|\sin\left(-\frac{\pi}{6}\right)\right|^{\left|-\frac{\pi}{6}\right|}$$

$$y = \left(\frac{1}{2}\right)^{\frac{\pi}{6}} = 2^{-\frac{\pi}{6}}$$

$$\frac{dy}{dx} = 2^{-\frac{\pi}{6}}\left[-\ln\frac{1}{2} + \frac{\pi}{6}\times 2\times(-\cos\left(-\frac{\pi}{6}\right))\right]$$

$$\frac{dy}{dx} = 2^{-\frac{\pi}{6}}\left[\ln 2 - \frac{\pi\sqrt{3}}{6})\right]$$

$$= \frac{2^{-\frac{\pi}{6}}(6\ln 2 - \sqrt{3}\pi)}{6}$$

59. What is

$$\frac{d\sqrt{1-\sin 2x}}{dx}$$

equal to, where

$\frac{\pi}{4} < x < \frac{\pi}{2}$?

(a) $\cos x + \sin x$

(b) $-(\cos x + \sin x)$

(c) $\pm(\cos x + \sin x)$

(d) None of the above

Solution:

$$\sqrt{1-\sin 2x} = \sqrt{sin^2 x + cos^2 x - 2\sin x\cos x}$$

$$= \sqrt{(\sin x - \cos x)^2}$$

$$= |\sin x - \cos x|$$

$$\frac{d\sqrt{1-\sin 2x}}{dx} = \frac{d\,|\sin x - \cos x|}{dx}$$

$= \pm(\cos x + \sin x)$

Answer: (c)

60. What is $\log_{x\to\frac{\pi}{6}}\frac{2sin^2x+\sin x-1}{2si\;\;{}^2x-3\sin x+1}$ equal to ?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$

(c) -2 (d) -3

Solution:

$$\log_{x\to\frac{\pi}{6}}\frac{2sin^2x+\sin x-1}{2sin^2x-3\sin x+1}$$

$2\mathrm{sin}^2\mathrm{x} + \mathrm{sin\,x} - 1$

$= 2\mathrm{sin}^2\mathrm{x} + 2\,\mathrm{sin\,x} - \mathrm{sin\,x} - 1$

$= (2\sin x - 1)(\sin x + 1)$

$2sin^2x - 3\sin x + 1$

$= 2\mathrm{sin}^2\mathrm{x} - 2\,\mathrm{sin\,x} - \mathrm{sin\,x} + 1$

$= (2\sin x - 1)(\sin x - 1)$

$$\log_{x\to\frac{\pi}{6}}\frac{2sin^2x+\sin x-1}{2sin^2x-3\sin x+1}$$

$$= \log_{x\to\frac{\pi}{6}}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$$

$$= \log_{x\to\frac{\pi}{6}}\frac{(\sin x+1)}{(\sin x-1)} = \frac{\sin 30^0+1}{\sin 30^0-1} = -3$$

Answer: (d)

61. The average age of a combined group of men and women is 25 years. If the average age of the group of men is 26 years and that of the group of women is 21 years, then the % of men and women in the group is respectively.

(a) 20 , 80 (b) 40, 60

(c) 60, 40 (d) 80, 20

Solution:

Let number of men = x

number of women = y

Average age of men = 26 yrs

Average age of women = 21 yrs

Average age of group = 25 yrs

Average age of group = $\frac{26x+21y}{x+y}$

$$25 = \frac{26x + 21y}{x + y}$$

$x = 4y$

$$\%men = \frac{x}{x+y} \times 100 = \frac{x}{x + \frac{x}{4}} \times 100$$

$$= \frac{4x}{5x} \times 100 = 80$$

$\%\ women = 100 - 80 = 20$

62. 8 coins are tossed simultaneously. The probability of getting at least 6 heads is

(a) $7/64$ (b) $57/64$

(c) $37/256$ (d) $229/256$

Solution:

Total number of sample space = 2^8

Total number of Event

= $C(8,6) + C(8,7) + C(8,8) = 37$

Probability of Event = $\frac{n(E)}{n(S)} = \frac{37}{256}$

Answer: (c)

$$= (x+y+z)\left[\begin{vmatrix} y & y+z \\ x & z+x \end{vmatrix} - \begin{vmatrix} x & y+z \\ z & z+x \end{vmatrix} + 2\begin{vmatrix} x & y \\ z & x \end{vmatrix}\right]$$

$$= (x+y+z)(yz + xy - xy - zx - xz - x^2 + yz + z^2 + 2x^2 - 2yz)$$

$$= (x+y+z)(z-x)^2$$

63. Let the slope of the curve $y = \cos^{-1}(\sin x)$ be $\tan\theta$. Then the value of θ in the interval $(0,\pi)$

(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Solution:

$$\cos^{-1}(\sin x) + \sin^{-1}(\sin x) = \frac{\pi}{2}$$

$$\cos^{-1}(\sin x) = \frac{\pi}{2} - \sin^{-1}(\sin x)$$

$$y = \frac{\pi}{2} - x$$

slope of curve y is equal to -1

$$\tan\theta = -1$$

$$\theta = \frac{3\pi}{4}$$