1. How many four-digit natural numbers are there such that all of the digits are odd?

(a) 625
(b) 400
(c) 196
(d) 120

Solution: Odd number from 0 to 9 are 1, 3, 5, 7, 9. Total number of numbers are 5.

Number of ways to formed four-digit natural numbers such that all of the digits are odd

= $5\ \times 4\ \times 3 \times 2 = 120$

2. What is $\sum_{r=0}^{r=n} 2^r C(n, r)$ equal to?

(a) 2^n
(b) 3^n
(c) 2^{2n}
(d) 3^{2n}

Solution: $(1+x)^n = \sum_{r=0}^{r=n} x^r C(n, r)$

put x = 2 we get $(1+2)^n = \sum_{r=0}^{r=n} 2^r C(n, r)$

$$\sum_{r=0}^{r=n} 2^r C(n, r) = 3^n$$

Answer: (b)

3. If different permutations of the letters of the word 'MATHEMATICS' are listed as in a dictionary, how many words (with or without meaning) are there in the list before the first word that starts with C?

(a) 302400
(b) 403600
(c) 907200
(d) 1814400

Solution: Set of letters in the word MATHEMATICS = $\{A, A, C, E, H, I, M, M, S, T, T\}$

Word begin with letter A= $\frac{10!}{2!2!} = 907200$

Answer: (c)

4. For how many quadratic equations, the sum of roots is equal to the product of roots?

(a) 0 (b)1
(b) 2 (d) Infinitely many

Solution:

$$ax^2 + bx + c = 0$$

Let α and β are roots of the quadratic equation.

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = \alpha\beta$$

$$-\frac{b}{a} = \frac{c}{a}$$

$$\frac{b+c}{a} = 0$$

$$b + c = 0$$

Number of quadratic equation is equal to infinity.

5. Consider the following statements:

1. The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{5}$ is an infinite set.

2. The set of all odd integers less than 100 is a finite set.

Which of the statements given above is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution:

The set of all odd integers less than 100 is a finite set.

Set A = $\{-\infty, \ -2, -1, 0,\ 1,\ 2,\ 3,\ \ldots, 99\ \}$

Number of element in set A is infinity

6. Let p,q (p > q) be the roots of the quadratic equation $x^2 + bx + c = 0$ where $c > 0$. If $p^2 + q^2 - 11pq = 0$, then what is p –q equal to ?

(a) $3\sqrt{c}$
(b) 3c
(c) $9\sqrt{c}$
(d) 90

Solution:

if p, q are the roots of the quadratic equation $x^2 + bx + c = 0$

$p + q = -b$

$pq = c$

$p^2 + q^2 - 11pq = 0$

$(p - q)^2 - 9pq = 0$

$p - q = 3\sqrt{pq}$

$= 3\sqrt{c}$

Answer: (a)

7. What is the diameter of a circle inscribed in a regular polygon of 12 sides, each of length 1 cm?

(a) $1 + \sqrt{2}$ cm

(b) $2 + \sqrt{2}$ cm

(c) $2 + \sqrt{3}$ cm

(d) $3 + \sqrt{3}$ cm

Solution:

$$\tan 15^0 = \frac{1/2}{r}$$

$$\tan 15^0 = \tan(45^0 - 30^0)$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\tan 15^0 = \frac{1}{2r}$$

$$2r = \frac{1}{\tan 15^0} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

Consider the following for the next three (03) items that follow:

Let $z = \frac{1 + i\,\text{si}}{1 - i \sin\theta}$ where $i = \sqrt{-1}$

8 What is the modulus of z ?

(a) 1 (b) $\sqrt{2}$

(c) $1 + sin^2\theta$ (d) $\frac{1 + sin^2\theta}{1 - sin^2\theta}$

Solution:

$$z = \frac{1 + i \sin\theta}{1 - i \sin\theta}$$

$$|z| = \frac{|1 + i \sin\theta|}{|1 - i \sin\theta|}$$

$$|z| = \frac{\sqrt{1 + sin^2\theta}}{\sqrt{1 + (-sin)^2\theta}} = 1$$

Answer: (a)

9. What is angle θ such that z is purely real?

(a) $\frac{n\pi}{2}$

(b) $\frac{(2n+1)\pi}{2}$

(c) $n\pi$

(d) $2n\pi$ only

Where n is an integer

Solution:

$$z = \frac{1 + i \sin\theta}{1 - i \sin\theta}$$

$$z = \frac{1 + i \sin\theta}{1 - i \sin\theta} \times \frac{1 + i \sin\theta}{1 + isin\,\theta}$$

$$z = \frac{1 - sin^2\theta + 2isin\,\theta}{1 + sin^2\theta}$$

$$z = \frac{1 - sin^2\theta}{1 + sin^2\theta} + 2i\,\frac{\sin\theta}{1 + sin^2\theta}$$

z is purely real then imaginary part is equal to zero.

$$\frac{2\sin\theta}{1 + sin^2\theta} = 0$$

$$\sin\theta = 0$$

$$\theta = n\pi$$

10. What is angle θ such that z is purely imaginary?

(a) $\frac{n\pi}{2}$

(b) $\frac{(2n+1)\pi}{2}$

(c) $n\pi$

(d) $2n\pi$

Where n is an integer

Solution:

$$z = \frac{1 - sin^2\theta}{1 + sin^2\theta} + 2i\,\frac{\sin\theta}{1 + sin^2\theta}$$

z is purely imaginary then real part is equal to zero.

$$\frac{1 - sin^2\theta}{1 + sin^2\theta} = 0$$
$$1 - sin^2\theta = 0$$
$$sin^2\theta = 1$$
$$\theta = \frac{(2n+1)\pi}{2}$$

Consider the following for the next **three (03)** items that follow:

Let P be the sum of first n positive terms of an increasing arithmetic progression A. Let Q be the sum of first n positive terms of another increasing arithmetic progression B. Let P : Q = (5 n +4) : (9n + 6)

11. What is the ratio of the first term of A to that of B?

(a) $1/3$

(b) 2/5

(c) ¾

(d) 3/5

Solution:

$$P = a_1 + (n-1)d_1$$
$$Q = a_2 + (n-1)d_2$$
$$\frac{P}{Q} = \frac{5n+4}{9n+6}$$
$$\frac{a_1 - d_1 + nd_1}{a_2 - d_2 + nd_2} = \frac{5n+4}{9n+6}$$

$a_1 - d_1 = 4$ and $d_1 = 5$

$a_2 - d_2 = 6$ and $d_2 = 9$

$a_1 = 9$ and $a_2 = 15$

$\frac{a_1}{a_2} = \frac{9}{15} = \frac{3}{5}$

12. What is the ratio of their 10th terms?

(a) 11/29

(b) 22/49

(c) 33/59

(d) 44/69

Solution:

$$\frac{a_{10}}{a'_{10}} = \frac{a_1 + (10-1)d_1}{a_2 + (10-1)d_2} = \frac{9 + 9 \times 5}{15 + 9 \times 9} = \frac{54}{96}$$

Answer: (*)

13. if d is the common difference of A, and D is the common difference of B, then which one of the following is always correct?

(a) D > d

(b) $D < d$

(c) $7D > 12d$

(d) None of the above

Solution: d = d_1 = 5

D = d_2 = 9

D > d

Answer: (a)

Consider the following for the next three (03) items that follow:

Consider the binomial expansion of $(p + qx)^9$:

14. What is the value of q if the coefficients of x^3 and x^6 are equal?

(a) p (b) 9p

(c) $\frac{1}{p}$ (d) p^2

Solution:

$$(x+y)^n = \sum C(n,r)x^r y^{n-r}$$

Coefficient of x^6 = C(9,3)p^3q^6

Coefficient of x^3 = C(9,6)p^6q^3

Coefficient of x^6 = Coefficient of x^3

$C(9,3)p^3q^6 = C(9,6)p^6q^3$

$q^3 = p^3$

$p = q$

Answer: (a)

15. What is the ratio of the coefficients of middle terms in the expansion (when expanded in ascending powers of x) ?

(a) pq (b) p/q

(c) 4p/5q (d) 1/(pq)

Solution: Number of terms in $(p + qx)^9$ is 10.

Middle terms are fifth and sixth terms.

Coefficient of fifth term = $C(9,4)p^4q^5$

Coefficient of sixth term = $C(9,5)p^5q^4$

$$\frac{\text{Coefficient of fifth term}}{\text{Coefficient of sixth term}} = \frac{C(9,4)p^4q^5}{C(9,5)p^5q^4} = \frac{q}{p}$$

Answer: (*)

Consider the following for the next three (03) items that follow:

Let $A = \begin{pmatrix} 0 & sin^2\theta & cos^2\theta \\ cos^2\theta & 0 & sin^2\theta \\ sin^2\theta & cos^2\theta & 0 \end{pmatrix}$ and

$A = P + Q$ where P is symmetric matrix and Q is skew-symmetric matrix.

16. What is P equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $cos2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $cos2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

17. What is Q equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $cos2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $cos2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

Solution: Any matrix can be written as sum of symmetric matrix and skew symmetric matrix.

$$A = \frac{A + A^T}{2} + \frac{A - A^T}{2}$$

$$A = \begin{pmatrix} 0 & sin^2\theta & cos^2\theta \\ cos^2\theta & 0 & sin^2\theta \\ sin^2\theta & cos^2\theta & 0 \end{pmatrix}$$

$$\frac{A + A^T}{2}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & sin^2\theta & cos^2\theta \\ cos^2\theta & 0 & sin^2\theta \\ sin^2\theta & cos^2\theta & 0 \end{pmatrix}$$

$$+ \frac{1}{2}\begin{pmatrix} 0 & cos^2\theta & sin^2\theta \\ sin^2\theta & 0 & cos^2\theta \\ cos^2\theta & sin^2\theta & 0 \end{pmatrix}$$

$$\frac{A + A^T}{2} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\frac{A - A^T}{2}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & sin^2\theta & cos^2\theta \\ cos^2\theta & 0 & sin^2\theta \\ sin^2\theta & cos^2\theta & 0 \end{pmatrix}$$

$$- \frac{1}{2}\begin{pmatrix} 0 & cos^2\theta & sin^2\theta \\ sin^2\theta & 0 & cos^2\theta \\ cos^2\theta & sin^2\theta & 0 \end{pmatrix}$$

$$= cos2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$$

18. What is the value of $\tan\left(\frac{3\pi}{8}\right)$?

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $1 - \sqrt{2}$

(d) $-(\sqrt{2} + 1)$

Solution: Let

$$\theta = \frac{\pi}{8} = \frac{180}{8} = \frac{45}{2}$$

$$\tan\left(\frac{3\pi}{8}\right) = \frac{3\tan\left(\frac{\pi}{8}\right) - tan^3\left(\frac{\pi}{8}\right)}{1 - 3tan^2\left(\frac{\pi}{8}\right)}$$

$$\tan(2\theta + \theta) = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$$

$$\tan 2\theta = \tan 45^0 = 1$$

$$\tan(2\theta + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - tan^2\theta}$$

$$1 = \frac{2\tan\theta}{1 - tan^2\theta}$$

$$1 - tan^2\theta = 2\tan\theta$$

$$tan^2\theta + 2\tan\theta - 1 = 0$$

$$\tan\theta = \sqrt{2} - 1$$

$$\tan(2\theta+\theta) = \frac{1+\sqrt{2}-1}{2-\sqrt{2}} = \sqrt{2}+1$$

19. What is $\tan^{-1}\cot(\text{cosec}^{-1} 2)$ equal to ?

(a) $\frac{\pi}{8}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{3}$

Solution: $\tan^{-1}\cot(\text{cosec}^{-1} 2)$

$$\text{cosec}^{-1} 2 = \theta$$

$$cosec\ \theta = 2$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^0$$

$$\cot\theta = \cot 30^0 = \sqrt{3}$$

$$\tan^{-1}\sqrt{3} = 60^0$$

Answer : (d)

20. In a triangle ABC, a =4, b =3, c =2. What is cos3C equal to?

(a) $\frac{7}{128}$

(b) $\frac{11}{128}$

(c) $\frac{7}{64}$

(d) $\frac{11}{64}$

Solution:

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{4^2+3^2-2^2}{2\times4\times3} = \frac{7}{8}$$

$$Cos\ 3C = 4cos^3C - 3\cos C = \frac{7}{128}$$

21. What is $\cos 36^0 - \cos72^0$ equal to?

(a) $\frac{\sqrt{5}}{2}$

(b) $-\frac{\sqrt{5}}{2}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

Solution: $\cos \mathrm{A} - \cos \mathrm{B} = 2\sin\frac{\mathrm{A+B}}{2}\sin\frac{\mathrm{B-A}}{2}$

$$\cos 36^0 - \cos72^0 = 2\sin\frac{36+72}{2}\sin\frac{72-36}{2}$$

$$= 2sin54\sin 18$$

$$\sin 18^0 = \frac{\sqrt{5}-1}{4}$$

$$\sin 54^0 = \frac{\sqrt{5}+1}{4}$$

$$= 2sin54\sin 18 = 2\,\frac{5-1}{4\times4} = \frac{1}{2}$$

22. If $\sec x = \frac{25}{24}$ and x lies in the fourth quadrant, then what is the value of $\tan x + \sin x$?

(a) $-\frac{625}{168}$

(b) $-\frac{343}{600}$

(c) $\frac{625}{168}$

(d) $\frac{343}{600}$

Solution:

$$\sec x = \frac{25}{24} = \frac{H}{B}$$

$$P^2 + B^2 = H^2$$

$$P^2 = (25k)^2 - (24k)^2 = 49k^2$$

$$P = 7k$$

if x in fourth quadrant then sin x and tanx are negative.

$$\sin x = \frac{P}{H} = -\frac{7}{25}$$

$$\tan x = \frac{P}{B} = -\frac{7}{24}$$

$$\tan x + \sin x = -\frac{7}{25} - \frac{7}{24} = -\frac{343}{600}$$

23. What is the value of $tan^2 165^0 + cot^2 165^0$?

(a) 7

(b) 14

(c) $4\sqrt{3}$

(d) $8\sqrt{3}$

Solution:

$\tan 15^0 = \tan(45^0 - 30^0)$

$$= \frac{\tan 45^0 - \tan 30^0}{1+\tan 45^0 \tan 30^0}$$

$$= \frac{\mathbf{1 - \frac{1}{\sqrt{3}}}}{\mathbf{1 + \frac{1}{\sqrt{3}}}} = \frac{\mathbf{\sqrt{3} - 1}}{\mathbf{\sqrt{3} + 1}}$$

$$\mathbf{\tan 165^0} = \mathbf{\tan(180^0 - 15^0)} = \mathbf{-\tan 15^0}$$

$$tan^2 165^0 = \left(\frac{\mathbf{\sqrt{3} - 1}}{\mathbf{\sqrt{3} + 1}}\right)^{\mathbf{2}} = \left(2 - \sqrt{3}\right)^2$$

$$tan^2 165^0 + cot^2 165^0$$

$$= \left(2 - \sqrt{3}\right)^2 + \frac{1}{\left(2 - \sqrt{3}\right)^2}$$

$$= \mathbf{14}$$

24 Consider the following statements in respect of the line passing through origin and inclining at an angle of 75^0 with the positive direction of x-axis:

1. The line passes through the point $\left(1, \frac{1}{2-\sqrt{3}}\right)$.
2. The line entirely lies in first and third quadrants.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

Equation of line passing through origin

$y = mx$

Slope of the line $m = \tan 75^0$

$\tan 75^0 = \tan(45^0 + 30^0)$

$$= \frac{\tan 45^0 + \tan 30^0}{1 - \tan 45^0 \tan 30^0}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

At x =1 $y = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-1}{3+1-2\sqrt{3}} = \frac{1}{2-\sqrt{3}}$

Answer: (c)

25. If P(3,4) is the mid-point of a line segement between the axes, then what is the equation of the line?

(a) $3x + 4y - 25 = 0$

(b) $4x + 3y - 24 = 0$

(c) $4x - 34y = 0$

(d) $3x - 4y + 7 = 0$

Solution:

Let Equation of Line $\frac{x}{a} + \frac{y}{b} = 1$

Line intercept x-axes at A(a, 0) and intercept y-axes at B(0, b).

If P (3, 4) is the midpoint of AB, then

$$3 = \frac{a + 0}{2}$$

$a = 6$

$$4 = \frac{0 + b}{2}$$

$b = 8$

Equation of line:

$$\frac{x}{6} + \frac{y}{8} = 1$$

$$\frac{4x + 3y}{24} = 1$$

$4x + 3y - 24 = 0$

Answer: (b)

26 The base AB of an equilateral triangle ABC with side 8 cm lies along the y-axis such that the mid-point of AB is at the origin and B lies above the origin. What is the equation of line passing through (8, 0) and parallel to the side AC?

(a) $x - \sqrt{3}y - 8 = 0$

(b) $x + \sqrt{3}y - 8 = 0$

(c) $\sqrt{3}x + y - 8\sqrt{3} = 0$

(d) $\sqrt{3}x - y - 8\sqrt{3} = 0$

Solution ABC is an equilateral triangle and AB is base of equilateral triangle lies on y-axis. Midpoint of AB is at origin. Side AB is 8 cm and B lies above the origin. Co-ordinate A(0, -4). Point C lies on x-axis. Co-ordinate of C(x, 0)

AC = 8

$$\sqrt{(x - 0)^2 + (0 + 4)^2} = 8$$

$$x^2 + 16 = 64$$

$$x = 4\sqrt{3}$$

Slope of line AC : $m = \frac{-4-0}{-4\sqrt{3}} = \frac{1}{\sqrt{3}}$

Equation of line parallel to AC and passing through (8, 0)

$$y = \frac{1}{\sqrt{3}}(x-8)$$

$$x - \sqrt{3}y - 8 = 0$$

Answer: (a)

(e)

27. A Plane cuts intercepts 2, 2, 1 on the coordinate axes. What are the direction cosines of the normal to the plane?

(a) $< \frac{2}{3}, \frac{2}{3}, \frac{1}{3} >$

(b) $< \frac{1}{3}, \frac{2}{3}, \frac{2}{3} >$

(c) $< \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} >$

(d) $< \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} >$

Solution: Equation of plane

$$\frac{x}{2} + \frac{y}{2} + \frac{z}{1} = 1$$

Direction ratio of normal to the plane

a = $\frac{1}{2}$, $b = \frac{1}{2}$ and $c = 1$

$$l^2 + m^2 + n^2 = 1$$

$l = ka$, $m = kb$ and $n = k\,c$

$$\frac{k^2}{4} + \frac{k^2}{4} + k^2 = 1$$

$$k = \pm\sqrt{\frac{2}{3}}$$

$$l = \frac{1}{\sqrt{6}}, m = \frac{1}{\sqrt{6}} and\ n = \frac{2}{\sqrt{6}}$$

28. Consider the following statements:

1. The direction ratios of y-axis can be < 0, 4, 0 >

2. The direction ratios of a line perpendicular to z-axis can be < 5, 6, 0>

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

29. Let $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular. What is the angle between $\vec{a}$ and $\vec{b}$?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

Solution: If vector $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then dot product is equal to zero.

$(\vec{a} + 2\vec{b}).(5\vec{a} - 4\vec{b}) = 0$

$5|\vec{a}|^2 - 4\vec{a}.\vec{b} + 10\vec{b}.\vec{a} - 8|\vec{b}|^2 = 0$

Since $\vec{a}$ and $\vec{b}$ are two unit vectors therefore

$|\vec{a}| = |\vec{b}| = 1$

$6\vec{a}.\vec{b} = 3$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$6|\vec{a}||\vec{b}|\cos\theta = 3$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Answer: (c)

30. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors lying on the same plane. What is $\{(3\vec{a} + 2\vec{b}) \times (5\vec{a} - 4\vec{c})\}.(\vec{b} + 2\vec{c})$ equal to?

(a) -8

(b) -32

(c) 8

(d) 0

Solution: If $\vec{a}$, $\vec{b}$ and $\vec{c}$ lying on the same plane. Then the vectors $3\vec{a} + 2\vec{b}$, $5\vec{a} - 4\vec{c}$ and $\vec{b} + 2\vec{c}$ lie on same plane. Direction of cross product of vectors $3\vec{a} + 2\vec{b}$ and

$5\vec{a}-4\vec{c}$ perpendicular to plane containing vector $\vec{a}$, $\vec{b}$ and $\vec{c}$. Dot product of $(3\vec{a}+2\vec{b})\times(5\vec{a}-4\vec{c})$ and $(\vec{b}+2\vec{c})$ be equal to zero.

31. What are the values of x for which the angle between the vectors $2x^2\hat{\imath}+3x\hat{\jmath}+\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+x^2\hat{k}$ is obtuse?

(a) $0<x<2$

(b) $x<0$

(c) $x>2$

(d) $0\le x\le 2$

Solution: $\vec{a}.\vec{b}<0$

$$2x^2-6x+x^2<0$$
$$3x(x-2)<0$$
$$0<x<2$$

32 The position vectors of vertices A, B and C of triangle ABC are respectively $\hat{\jmath}+\hat{k}$, $3\hat{\imath}+\hat{\jmath}+5\hat{k}$ and $3\hat{\jmath}+3\hat{k}$. What is angle C equal to?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Solution:

$$\overrightarrow{OA}=\hat{\jmath}+\hat{k}$$
$$\overrightarrow{OB}=3\hat{\imath}+\hat{\jmath}+5\hat{k}$$
$$\overrightarrow{OC}=3\hat{\jmath}+3\hat{k}$$

$$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$$
$$=(3\hat{\jmath}+3\hat{k})-(3\hat{\imath}+\hat{\jmath}+5\hat{k})$$
$$=-3\hat{\imath}+2\hat{\jmath}-2\hat{k}$$
$$\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}$$
$$=(\hat{\jmath}+\hat{k})-(3\hat{\jmath}+3\hat{k})$$
$$=-2\hat{\jmath}-2\hat{k}$$
$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$
$$=(3\hat{\imath}+\hat{\jmath}+5\hat{k})-(\hat{\jmath}+\hat{k})$$
$$=3\hat{\imath}+4\hat{k}$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

33. What is the area of the region bounded by $x-|y|=0$ and $x-2=0$?

(a) 1 (b) 2

(c) 4 (d) 8

Solution: Area = $\frac{1}{2}\times 4\times 2=4$

34. What is $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos 4x}}$ equal to ?

(a) $\frac{1}{2\sqrt{2}}$

(b) $-\frac{1}{2\sqrt{2}}$

(c) $\sqrt{2}$

(d) Limit does not exist

Solution:

$\cos 4x = 1-2sin^2 2x$

$1-\cos 4x = 2sin^2 2x$

$$\lim_{x\to 0^+}\frac{x}{\sqrt{1-\cos 4x}}=\lim_{x\to 0^+}\frac{x}{\sqrt{2}\sin 2x}=\frac{1}{2\sqrt{2}}$$
$$\lim_{x\to 0^-}\frac{x}{\sqrt{1-\cos 4x}}=\lim_{x\to 0^-}\frac{x}{-\sqrt{2}\sin 2x}$$
$$=-\frac{1}{2\sqrt{2}}$$

Answer: (d)

35. What is

$$\lim_{x\to\frac{\pi}{2}}\frac{4x-2\pi}{\cos x}$$

equal to?

(a) -4

(b) -2

(c) 2

(d) 4

Solution:

$$\lim_{x\to\frac{\pi}{2}}\frac{4x-2\pi}{\cos x}=4\frac{x-\frac{\pi}{2}}{\sin\left(\frac{\pi}{2}-x\right)}=\frac{-4}{\frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)}}$$
$$=-4$$

Answer: (a)

36 If

$$f(x)=\frac{x^2+x+|x|}{x}$$

, then what is $\lim_{x\to 0}f(x)$ equal to?

(a) 0

(b) 1

(c) 2

(d) $\lim_{x\to 0} f(x)$ does not exist

Solution:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x^2 + x + |x|}{x}$$
$$= \lim_{x\to 0^+} \frac{x^2 + x + x}{x}$$
$$= \lim_{x\to 0^+} \frac{x^2 + 2x}{x} = 2$$
$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2 + x + |x|}{x}$$
$$= \lim_{x\to 0^-} \frac{x^2 + x - x}{x}$$
$$= \lim_{x\to 0^-} \frac{x^2}{x} = 0$$
$$\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$$

$\lim_{x\to 0} f(x)$ does not exist

Answer: (d)

37. What is $\lim_{h\to 0} \frac{sin^2(x+h) - sin^2 x}{h}$ equal to ?

(a) $sin^2 x$

(b) $cos^2 x$

(c) $sin2x$

(d) $cos2x$

Solution:

$$\lim_{h\to 0} \frac{sin^2(x+h) - sin^2 x}{h} = \frac{d(sin^2 x)}{dx}$$
$$= 2\sin x \cos x = \sin 2x$$

Answer: (c)

38. The centre of the circle passing through origin and making positive intercepts 4 and 6 on the coordinate axes, lies on the line

(a) $2x - y + 1 = 0$

(b) $3x - 2y - 1 = 0$

(c) $3x - 4y + 6 = 0$

(d) $2x + 3y - 26 = 0$

Solution: Circle passes through origin and cut x-intercept at point A (4, 0) and cut y-intercept at point B(0, 6).

Centre of circle is mid point of AB.

$$x_0 = \frac{x_A + x_B}{2} = \frac{4 + 0}{2} = 2$$
$$y_0 = \frac{y_A + y_B}{2} = \frac{0 + 6}{2} = 3$$

Centre C = (2, 3)

Point C satisfy $3x - 4y + 6 = 0$

$$3 \times 2 - 4 \times 3 + 6 = 6 - 12 + 6 = 0$$

39. The centre of an ellipse is at (0, 0), major axis is on the y-axis. If the ellipse passes through (3, 2) and (1, 6), then what is its eccentricity?

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$

(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{5}$

Solution: If major axes lies on y-axis then equation of the ellipse is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$
$$\frac{9}{b^2} + \frac{4}{a^2} = 1$$
$$\frac{1}{b^2} + \frac{36}{a^2} = 1$$
$$a^2 = 40$$
$$b^2 = 10$$
$$b^2 = a^2(1 - e^2)$$
$$10 = 40(1 - e^2)$$
$$e^2 = 1 - \frac{1}{4} = \frac{3}{4}$$
$$e = \frac{\sqrt{3}}{2}$$

40. If $y = (x^x)^x$, then which one of the following is correct?

(a) $\frac{dy}{dx} + xy(1 + 2\ln x) = 0$

(b) $\frac{dy}{dx} - xy(1 + 2\ln x) = 0$

(c) $\frac{dy}{dx} - 2xy(1 + \ln x) = 0$

(d) $\frac{dy}{dx} + 2xy(1 + \ln x) = 0$

Solution: $y = (x^x)^x$

$$\ln y = x^2 \ln x$$
$$\frac{1}{y}\frac{dy}{dx} = 2x\ln x + \frac{x^2}{x} = 2x\ln x + x$$

$$\frac{dy}{dx} = xy(1 + 2\ln x)$$
$$\frac{dy}{dx} - xy(1 + 2\ln x) = 0$$

41. What is the maximum value of $3(\sin x - \cos x) + 4(cos^3x - sin^3x)$?

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

Solution:

$f(x) = 3(\sin x - \cos x) + 4(\cos^3 x - \sin^3 x)$

$\sin 3x = 3\sin x - 4sin^3x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$f(x) = \sin 3x + \cos 3x$

Maximum value of $f(x)$ is $\sqrt{2}$

42. What is the area of the region (in the first quadrant) bounded by $y = \sqrt{1-x^2}$, $y = x$ and $y = 0$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$

Solution: Intersection of curve $y = \sqrt{1-x^2}$ and $y = x$.

$x = \sqrt{1-x^2}$

$x^2 = 1 - x^2$

$2x^2 = 1$

$x = \frac{1}{\sqrt{2}}$

$$Area = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-y^2}\, dy$$

$$I = \int \sqrt{1-y^2}\, dy$$

$y = \sin\theta$

$dy = \cos\theta\, d\theta$

$$I = \int cos^2\theta\, d\theta = \int \frac{1+\cos 2\theta}{2} d\theta$$
$$= \frac{\theta}{2} + \frac{\cos 2\theta}{4}$$

$$\int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-y^2}\, dy = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} cos^2\theta\, d\theta$$
$$= \frac{\frac{\pi}{2} - \frac{\pi}{4}}{2} + \frac{\cos\pi - \cos\frac{\pi}{2}}{4}$$
$$= \frac{\pi}{8} - \frac{1}{4}$$
$$Area = \frac{1}{4} + \frac{\pi}{8} - \frac{1}{4} = \frac{\pi}{8}$$

43. What is the value of $\sin\left(2n\pi + \frac{5\pi}{6}\right)\sin\left(2n\pi - \frac{5\pi}{6}\right)$ where $n \in Z$?

(a) $-\frac{1}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Solution:

$\sin(2n\pi - \theta) = -\sin\theta$

$\sin(2n\pi + \theta) = \sin\theta$

$$\sin\left(2n\pi + \frac{5\pi}{6}\right)\sin\left(2n\pi - \frac{5\pi}{6}\right)$$
$$= -\sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{5\pi}{6}\right)$$
$$= -\sin^2 150^0 = -\left(\frac{1}{2}\right)^2 = -\frac{1}{4}$$

44. If $1 + 2(sinx + cosx)(sinx - cosx) = 0$ where $0 < x < 360^0$, then how many values does x take?

(a) Only one value

(b) Only two values

(c) Only three values

(d) Four values

Solution:

$1 + 2(sinx + cosx)(sinx - cosx) = 0$

$1 + 2(sin^2x - cos^2x) = 0$

$1 - 2\cos 2x = 0$

$\cos 2x = \frac{1}{2}$

Period of $\cos 2x$ is .

Number of values x which satisfy equation is four values.

Consider the following for the next three (03) items that follow:

Let $f(x)$ be a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$ such that $(1) = 2$:

45. If $\sum_{x=2}^{n} f(x) = 2044$, then what is the value of n ?

(a) 8

(b) 9

(c) 10

(d) 11

Solution: $\sum_{x=2}^{n} f(x) = 2044$

$$f(2) + f(3) + \ldots + f(n) = 2044$$

$$f(x+y) = f(x)f(y)$$

$$f(2) = f(1+1) = f(1)f(1) = f^2(1)$$

$$f(3) = f(2+1) = f(2)f(1) = f^3(1)$$

$$f(n) = f^n(1)$$

$$f^2(1) + f^3(1) + \cdots + f^n(1) = 2044$$

$$2^2 + 2^3 + \cdots + 2^n = 2044$$

$$\frac{a(r^n - 1)}{r - 1} = \frac{4(2^{n-1} - 1)}{2 - 1} = 2^{n+1} - 4 = 2044$$

$$2^{n+1} = 2048 = 2^{11}$$

$$n = 10$$

46. What is $\sum_{x=1}^{5} f(2x-1)$ equal to ?

(a) 341

(b) 682

(c) 1023

(d) 1364

Solution:

$$\sum_{x=1}^{5} f(2x-1)$$

$$= f(1) + f(3) + f(5) + f(7) + f(9)$$

$$= f(1) + f^3(1) + f^5(1) + f^7(1) + f^9(1)$$

$$= 2 + 2^3 + 2^5 + 2^7 + 2^9$$

$$= \frac{2(4^5 - 1)}{4 - 1} = \frac{2(2^{10} - 1)}{3} = 682$$

47. What is $\sum_{x=1}^{6} 2^x f(x)$ equal to ?

(a) 1365

(b) 2730

(c) 4024

(d) 5460

Solution:

$$\sum_{x=1}^{6} 2^x f(x)$$

$$= 2f(1) + 2^2 f(2) + 2^3 f(3) + 2^4 f(4) + 2^5 f(5) + 2^6 f(6)$$

$$= 2^2 + 2^4 + 2^6 + 2^8 + 2^{10} + 2^{12}$$

$$= \frac{4(4^6 - 1)}{4 - 1} = 5460$$