1. Let A ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing two or three elements is

(a) 45 (b) 120

(c) 165 (d) 330

Solution:

Number of ways = $C(10,2) + C(10,3)$

$= \frac{10!}{8!\times 2!} + \frac{10!}{7!\times 3!}$

$= 165$

Answer: (c)

2. The value of $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$, where $i = \sqrt{-1}$, is

(a) 0 (b) 1

(c) i (d) $-i$

Solution:

$i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$

$= i^{2n}(1 + i + i^2 + i^3)$

$= i^{2n}(1 + i - 1 - i)$

= 0

Answer: (a)

3. If the difference between the roots of the equation $x^2 + kx + 1 = 0$ is strictly less than $\sqrt{5}$, where $|k| \geq 2$, then k can be any element of the interval

(a) $(-3, -2) \cup [2, 3)$

(b) $(-3, 3)$

(c) $[-3, -2] \cup [2,3]$

(d) None of the above

Solution:

Let α and β are roots of equation $x^2 + kx + 1 = 0$

$\alpha + \beta = -k$

$\alpha\beta = 1$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\alpha - \beta = \sqrt{k^2 - 4}$

$\sqrt{k^2 - 4} < \sqrt{5}$

$k^2 - 4 < 5$

$k^2 < 9$

$-3 < k < 3$

Given $|k| \geq 2$

$(-3, -2] \cup [2, 3)$

Answer: (a)

4. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct?

(a) $p^2m = l^2q$

(b) $m^2p = l^2q$

(c) $m^2p = q^2l$

(d) $m^2p^2 = l^2q$

Solution:

Let α and β are the roots of quadratic equation $x^2 + px + q = 0$.

$\alpha + \beta = -p$

$\alpha\beta = q$

Let α' and β' are the roots of quadratic equation $x^2 + lx + m = 0$.

$\alpha' + \beta' = -l$

$\alpha'\beta' = m$

$\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} = k$

$\alpha = \beta k$

$\alpha' = \beta' k$

$\alpha + \beta = -p$

$\beta k + \beta = -p$

$\beta = \frac{-p}{1+k}$

$\alpha\beta = q$

$\beta k \times \beta = q$

$\beta^2 k = q$

$\frac{p^2}{(1+k)^2} \times k = q$

$\alpha' + \beta' = -l$

$\beta' k + \beta' = -l$

$\beta' = \frac{-l}{1+k}$

$\alpha'\beta' = m$

$\beta' k \times \beta' = m$

$$(\beta')^2 k = m$$

$$\frac{l^2}{(1+k)^2} k = m$$

$$p^2 m = l^2 q$$

Answer: (a)

5. The value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$ where n is not a multiple of 3 and $= \sqrt{-1}$, is

(a) 1 (b) -1

(c) i (d) $-i$

Solution:

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^n + \left(\frac{-1-\sqrt{3}i}{2}\right)^n$$

$$\omega^n + \omega^{2n}$$

If n = 1

$$\omega^1 + \omega^2 = -1$$

Answer: (b)

6. What is the sum of the series $0.3 + 0.33 + 0.333 + \ldots$ n terms?

(a) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

(b) $\frac{1}{3}\left[n - \frac{2}{9}\left(1 - \frac{1}{10^n}\right)\right]$

(c) $\frac{1}{3}\left[n - \frac{1}{3}\left(1 - \frac{1}{10^n}\right)\right]$

(d) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 + \frac{1}{10^n}\right)\right]$

Solution:

$$S_n = 0.3 + 0.33 + 0.333 + \ldots \text{n terms}$$

$$S_n = \frac{3}{10} + \frac{33}{100} + \frac{333}{1000} + \ldots$$

$$S_n = \frac{1}{3}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\right]$$

$$S_n = \frac{1}{3}\left[1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \ldots\right]$$

$$S_n = \frac{1}{3}\left[n - \frac{a(1-r^n)}{(1-r)}\right] \quad \left(a = \frac{1}{10}, r = \frac{1}{10}\right)$$

$$S_n = \frac{1}{3}\left[n - \frac{(1/10)\left(1 - \left(\frac{1}{10}\right)^n\right)}{\left(1 - \frac{1}{10}\right)}\right]$$

$$S_n = \frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

Answer: (a)

7. If 1, ω , ω^2 are the cube roots of unity, then $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$ is equal to

(a) -2 (b) -1

(c) 0 (d) 2

Solution:

$$1 + \omega + \omega^2 = 0$$

Answer: (c)

8. if the sum of m terms of an AP is n and the sum of n terms is m, then the sum of $(m+n)$ terms is

(a) mn (b) $m+n$

(c) $2(m+n)$ (d) $-(m+n)$

Solution:

$$S_n = \frac{n}{2}(\text{first term} + \text{last term})$$

If $S_m = n$ (Number of terms is m)

$$n = \frac{m}{2}(2a + (m-1)d)$$

$$m = \frac{n}{2}(2a + (n-1)d)$$

$$S_{m+n} = \frac{m+n}{2}(2a + (m+n-1)d)$$

Answer: (b)

9. The modulus and principal argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$

are respectively

Solution:

$$z = \frac{1+2i}{1-(1-i)^2}$$

$$z = \frac{1+2i}{1-(1+i^2-2i)}$$

$$z = \frac{1+2i}{1+2i} = 1$$

$$|z| = 1$$

$$\arg(z) = 0^0$$

Answer: (a)

10. If the graph of a quadratic polynomial lies entirely above x-axis, then which one of the following is correct?

(a) Both the roots are real

(b) One root is real and the other is complex

(c) Both the roots are complex

(d) Cannot say

Answer: (c)

11. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is

(a) 0 (b) 4

(c) 6 (d) 10

Solution:

$|z+4| \leq 3$

Complex z lies inside a circle with centre $(-4, 0)$ and radius 3 units.

$|z+1|$ is the distance between complex number z and -1.

For maximum value of $|z+1|$, z = $(-7,0)$

$|z+1| = |-7+1| = 6$

Answer: (c)

12. The number of z of the equation $z^2 = 2\bar{z}$ is

(a) 2 (b) 3

(c) 4 (d) zero

Solution:

$z^2 = 2\bar{z}$

$(x+iy)^2 = 2(x-iy)$

$x^2 - y^2 + 2xyi = 2x - 2yi$

$x^2 - y^2 = 2x$

$2xy = -2y$

$2y(x+1) = 0$

$y = 0$ or $x = -1$

If y = 0,

$x^2 - y^2 = 2x$

$x^2 - 0^2 = 2x$

x = 0, 2

If x = -1

$1 - y^2 = -2$

$3 = y^2$

$y = \pm\sqrt{3}$

Solution: (0, 0) ,(2, 0), (-1, √3) and (-1, -√3)

Answer: (c)

13. If $\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of $\cot(\alpha+\beta)$ is

(a) $\frac{c-1}{b}$ (b) $\frac{1-c}{b}$

(c) $\frac{b}{c-1}$ (d) $\frac{b}{1-c}$

Solution:

$\cot\alpha$ and $\cot\beta$ are the roots of the equation $x^2 + bx + c = 0$

Sum of roots of quadratic equation:

$\cot\alpha + \cot\beta = -b$

$\cot\alpha\cot\beta = c$

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta} = \frac{c-1}{-b}$$

$$= \frac{1-c}{b}$$

Answer: (b)

14. The sum of the roots of the equation $x^2 + bx + c = 0$ (where b and c are non-zero) is equal to the sum of the reciprocals of their squares. Then $\frac{1}{c}, b, \frac{c}{b}$ are in

(a) AP (b) GP

(c) HP (d) None of the above

Solution:

Let α and β are roots of the equation $x^2 + bx + c = 0$.

$\alpha + \beta = -b$

$\alpha\beta = c$

Given

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$-b = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$-b = \frac{b^2 - 2c}{c^2}$$

$-bc^2 = b^2 - 2c$

$2c = b^2 + bc^2$

$$2\frac{c}{b} = b + c^2$$

$$\frac{2}{b} = \frac{b}{c} + c$$

c, $\frac{1}{b}$ and $\frac{b}{c}$ are in AP. Therefore $\frac{1}{c}, b, \frac{c}{b}$ are in HP.

15. The sum of the roots of the equation $ax^2 + x + c = 0$(where a and c are non-zero) is equal to the sum of the reciprocals of their squares. Then a, ca^2, c^2 are in

(a) AP (b) GP

(c) HP (d) None of the above

Solution:

Let α and β are roots of the equation $ax^2 + x + c = 0$

$$\alpha + \beta = -\frac{1}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Given

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$-\frac{1}{a} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$-\frac{1}{a} = \frac{\frac{1}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$-\frac{1}{a} = \frac{\frac{1-2ca}{a^2}}{\frac{c^2}{a^2}}$$

$$-\frac{1}{a} = \frac{1-2ac}{c^2}$$

$$-c^2 = a - 2a^2c$$

$$a^2c - c^2 = a - a^2c$$

$$\mathbf{2}a^2c = a + c^2$$

a, ca^2, c^2 are in AP

Answer: (a)

16.The value of $[C(7,0) + C(7,1) + C(7,2) + \ldots + C(7,6) + C(7,7)]$ is

(a) 254 (b) 255

(c) 256 (d) 257

Solution:

$$(1+1)^7 = C(7,0) + C(7,1) + \ldots . + C(7,7)$$

$$2^7 = C(7,0) + C(7,1) + \ldots . + C(7,7)$$

$$[C(7,0) + C(7,1) + C(7,2) + \ldots + C(7,6) + C(7,7)]$$

$$= 2 \times 2^7 - 2 = 254$$

17. The number of different words(eight-letter words) ending and beginning with a consonant which can be made out of the letters of the word 'EQUATION' is

(a) 5200 (b) 4320

(c) 3000 (d) 2160

Solution: Total number of vowel is 5 and total number of consonant is 3.

Number of word = $3 \times 6! \times 2 = 4320$

Answer: (b)

18. The fifth term of an AP of n terms, whose sum is $n^2 - 2n$, is

(a) 5 (b) 7

(c) 8 (d) 15

Solution: $S_n = n^2 - 2n$

$$S_5 = 25 - 10 = 15$$

$$S_4 = 16 - 8 = 8$$

$$t_5 = S_5 - S_4 = 15 - 8 = 7$$

Answer: (b)

19. The sum of all the two-digit odd numbers is

(a) 2475 (b) 2530

(c) 4905 (d) 5049

Solution: First term of two digit odd number = 11

Last term of two digit odd number of odd number = 99

All two digit odd numbers are 11, 13, . . 99 form an AP series.

$$a_n = a_1 + (n-1)d$$

$$99 = 11 + (n-1)2$$

$$n = 45$$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{45}{2}(11 + 99) = 2475$$

Answer: (a)

20. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then what is AA^T equal to (where A^T is the transpose of A)?

(a) Null matrix

(b) identity matrix

(c) A

(d) – A

Solution:

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$AA^T = \begin{bmatrix} cos^2\alpha + sin^2\alpha & -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha & sin^2\alpha + cos^2\alpha \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Answer: (b)

21. The equations $\begin{matrix} x + 2y + 3z = 1 \\ 2x + y + 3z = 2 \\ 5x + 5y + 9z = 4 \end{matrix}$

(a) have the unique solution

(b) have infinitely many solutions

(c) are inconsistent

(d) None of the

22. What is the value of the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}?$$

(a) $1 + x + y + z$

(b) $2xyz$

(c) $x^2y^2z^2$

(d) $2x^2y^2z^2$

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}$$

$$= \begin{vmatrix} 1+xyz & 1 \\ 1 & 1+xyz \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 1+xyz \end{vmatrix} + \begin{vmatrix} 1 & 1+xyz \\ 1 & 1 \end{vmatrix}$$

$$= (1+xyz)^2 - 1 - (1 + xyz - 1) + (1 - 1 - xyz$$

$$= 1 + x^2y^2z^2 + 2xyz - 2xyz - 1 = x^2y^2z^2$$

Answer: (c)

23. If $= \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$, then what is A^3 equal to?

(a) $\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

(b) $\begin{bmatrix} cos^3\,\theta & sin^3\,\theta \\ -sin^3\,\theta & cos^3\,\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

(d) $\begin{bmatrix} cos^3\,\theta & -sin^3\,\theta \\ sin^3\,\theta & cos^3\,\theta \end{bmatrix}$

Solution: $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

$$A^2 = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \sin\theta\cos\theta & -\sin\theta\sin\theta + \cos\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta\, cos\theta - \sin 2\theta\, sin\theta & \cos 2\theta\, sin\theta + \sin 2\theta\, cos\theta \\ -\sin 2\theta\, cos\theta - \cos 2\theta\, sin\theta & -\sin 2\theta\, sin\theta + \cos 2\theta\, cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

Answer: (a)

24. What is the order of

$$[x \quad y \quad z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}?$$

(a) 3 x 1 (b) 1 x 1

(c) 1 x 3 (d) 3 x 3

Solution: Let A = $[x \quad y \quad z]$

B = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

C = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Dimension of matrix A = [1 x 3]

Dimension of matrix B = [3 x 3]

Dimension of matrix C = [3 x 1]

Dimension of matrix AB = [1 x 3] [3 x 3]

= [1 x 3]

Dimesnion of matrix ABC = [1 x 3] [3 x 1]

= [1 x 1]

Answer: (b)

25. What is $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$ equal to ?

(a) 0 (b) 1

(c) 2 (d) 4

Solution:

$$= \frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$$

$$= \frac{\cos 10^0 - \sqrt{3}\sin 10^0}{\sin 10^0 \cos 10^0}$$

$$= 4\,\frac{\left(\frac{1}{2}\cos 10^0 - \frac{\sqrt{3}}{2}\sin 10^0\right)}{2\sin 10^0 \cos 10^0}$$

$$= 4\,\frac{(\cos 60^0 \cos 10^0 - \sin 60^0 \sin 10^0)}{2\sin 10^0 \cos 10^0}$$

$$= 4\,\frac{\cos(60^0 + 10^0)}{\sin 20^0}$$

$$= 4\,\frac{\cos 70^0}{\sin 20^0} = 4\,\frac{\cos(90^0 - 20^0)}{\sin 20^0} = 4\,\frac{\sin 20^0}{\sin 20^0}$$

$$= 4$$

Answer: (d)

26. If a vertex of a triangle is (1, 1) and the midpoints of two sides of the triangle through this vertex are (-1, 2) and (3, 2), then the centroid of the triangle is

(a) $\left(-\frac{1}{3}, \frac{7}{3}\right)$

(b) $\left(-1, \frac{7}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$

(d) $\left(1, \frac{7}{3}\right)$

Solution:

Let Vertex A is (1, 1) and other vertex is B and C.

Co-ordinate of midpoint of AB is (-1, 2).

$$-1 = \frac{x_A + x_B}{2}$$

$$-1 = \frac{1 + x_B}{2}$$

$$x_B = -3$$

$$2 = \frac{y_A + y_B}{2}$$

$$2 = \frac{1 + y_B}{2}$$

$$y_B = 3$$

Co-ordinate of midpoint of AC is (3, 2)

$$3 = \frac{x_A + x_c}{2}$$

$$3 = \frac{1 + x_c}{2}$$

$$x_c = 5$$

$$2 = \frac{y_A + y_c}{2}$$

$$2 = \frac{1 + y_c}{2}$$

$$y_c = 3$$

Co-ordinate of centroid G

$$x_G = \frac{x_A + x_B + x_C}{3} = \frac{1 - 3 + 5}{3} = 1$$

$$y_G = \frac{y_A + y_B + y_c}{3} = \frac{1 + 3 + 3}{3} = \frac{7}{3}$$

Answer: (d)

27. The incentre of the triangle with vertices A(1, $\sqrt{3}$), B(0, 0) and C(2, 0) is

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

Solution: AB = $\sqrt{(1-0)^2 + \left(\sqrt{3} - 0\right)^2} = 2$

$$BC = \sqrt{(0-2)^2 + (0-0)^2} = 2$$

$$CA = \sqrt{(2-1)^2 + \left(\sqrt{3}\right)^2} = 2$$

Triangle ABC is an equilateral triangle . So incentre , circum-centre and centroid lie on same point.

Centroid = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

28. If the three consecutive vertices of a parallelogram are (-2, -1), (1, 0) and (4, 3), then what are the coordinates of the fourth vertex?

(a) (1, 2) (b) (1, 0)

(c) (0, 0) (d) (1, -1)

Solution: In parallelogram diagonal bisects each other.

A = (-2, -1) , B = (1,0) , C = (4,3) and D(x, y)

Mid point of AC = $\left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2}\right) = \left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$

$= (1, 1)$

$Mid\ point\ of\ BD = \frac{x+1}{2}, \frac{y+0}{2}$

$\frac{x+1}{2} = 1$

$\frac{y+0}{2} = 1$

x = 1 and y =2

28 The two circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points. Then which one of the following is correct?

(a) $2 < r < 8$

(b) $r = 2\ or\ r = 8$

(c) $r < 2$

(d) $r > 2$

Solution: Centre and radius of circle $x^2 + y^2 - 10x + 16 = 0$

$$(x-5)^2 + y^2 = 9$$

centre = (5, 0) and radius is 3.

Center of circle $x^2 + y^2 = r^2$ is (0, 0) and radius is r.

if r =2 circles touches externally and if r = 8 circles touches internally . if r lies between 2 and 8 circles intersect at two distinct points.

29. A straight line with direction cosines (0, 1, 0) is

(a) parallel to x-axis

(b) parallel to y-axis

(c) parallel to z-axis

(d) equally inclined to all the axes

Solution:

Direction cosine of x-axis is (1, 0, 0)

Direction cosine of y-axis is (0, 1, 0)

Direction cosine of z-axis is (0, 0, 1)

Direction cosine of line equally inclined to all the axes $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

30. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 3\hat{\imath} + 2\hat{\jmath} - \lambda\hat{k}$ are perpendicular, then what is the value of λ?

(a) 2 (b) 3

(c) 4 (d) 5

Solution:

Vector $\vec{a}$ and $\vec{b}$ are perpendicular to each other. Dot product of vector $\vec{a}$ and $\vec{b}$ is equal to zero. $\vec{a}.\vec{b} = 0$

$$\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right).\left(3\hat{\imath} + 2\hat{\jmath} - \lambda\hat{k}\right) = 0$$

$$6 + 6 - 4\lambda = 0$$

$$\lambda = 3$$

Answer: (b)

31. If $f(x) = \frac{x}{x-1}$, then what is $\frac{f(a)}{f(a+1)}$ equal to ?

(a) $f(-\frac{a}{a+1})$ (b) $f(a^2)$

(c) $f(\frac{1}{a})$ (d) $f(-a)$

Solution: $f(x) = \frac{x}{x-1}$

$$f(a) = \frac{a}{a-1}$$

$$f(a+1) = \frac{a+1}{a}$$

$$\frac{f(a)}{f(a+1)} = \frac{a^2}{a^2-1} = f(a^2)$$

32. What is $\int \frac{(x^{e-1}+e^{x-1})dx}{x^a+e^x}$ equal to?

(a) $\frac{x^2}{2}+c$

(b) $\ln(x+e)+c$

(c) $\ln(x^e+e^x)+c$

(d) $\frac{1}{e}\ln(x^e+e^x)+c$

33. If

$F(x)=\sqrt{9-x^2}$,

then what is

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1}$$

equal to?

(a) $-\frac{1}{4\sqrt{2}}$ (b) 1/8

(c) $-\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

Solution:

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1} = F'(1)$$

$$F(x)=\sqrt{9-x^2}$$

$$F'(x) = \frac{-2x}{2\sqrt{9-x^2}}$$

$$F'(1)=\frac{-2}{2\sqrt{9-1}} = -\frac{1}{2\sqrt{2}}$$

Answer: (c)

34. What is the length of the longest interval in which the function $f(x)=3\sin x-4sin^3x$ is increasing?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

Solution: $f(x)=3\sin x-4sin^3x=\sin 3x$

The interval of increasing function sin 3x is $\frac{\pi}{3}$.

35. If $xdy=y(dx+ydy); y(1)=1$ and $y(x)>0$, then what is $y(-3)$ equal to?

(a) 3 (b) 2

(c) 1 (d) 0

Solution: $xdy=y(dx+ydy)$

$$\frac{xdy-ydx}{y^2}=dy$$

$$d\left(-\frac{x}{y}\right)=dy$$

$$-\frac{x}{y}=y+c$$

$$-\frac{1}{y(1)}=y(1)+c$$

$$-1=1+c$$

$$c=-2$$

$$-\frac{x}{y}=y-2$$

x= -3

$$\frac{3}{y(-3)}=y(-3)-2$$

$$3=y^2-2y$$

$$y^2-2y-3=0$$

$$y=3,-1$$

36. What is the maximum value of the function $f(x)=4sin^2x+1$?

(a) 5 (b) 3

(c) 2 (d) 1

Solution:

$y=4sin^2x+1$

$0\le sin^2x\le 1$

Maximum value of y occur, when sin^2x is maximum.

$y_{max}=4\times 1+1=5$

Answer: (a)

37. Let f(x) be an indefinite integral of $\sin^2 x$. Consider the following statement :

Statement 1: The function f(x) satisfies f(x+π) =f(x) for all real x.

Statement 2: $sin^2(x+\pi)=sin^2x$ for real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

38 What are the degree and order respectively of the differential equation $y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2$?

(a) 1, 2 (b) 2, 1

(c) 1, 4 (d) 4, 1

Solution:

The differential equation

$$y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2$$

$$x\left(\frac{dy}{dx}\right)^4 - y\left(\frac{dy}{dx}\right)^2 + 1 = 0$$

Order of differential equation is equal to highest derivative in differential equation.

Degree of differential equation is equal to power of highest derivative in differential equation.

So order = 1 and degree = 4.

Answer: (d)

39. What is the general solution of the differential equation $ydx - (x + 2y^2)dy = 0$?

(a) $x = y^2 + cy$

(b) $x = 2cy^2$

(c) $x = 2y^2 + cy$

(d) None of the above

Solution:

The differential equation is

$$ydx - (x + 2y^2)dy = 0$$

$$ydx - xdy = 2y^2dy$$

$$\frac{ydx - xdy}{y^2} = 2dy$$

$$d\left(\frac{x}{y}\right) = 2dy$$

Integrate both sides of the equation we get,

$$\frac{x}{y} = 2y + c$$

$$x = 2y^2 + cy$$

Answer: (c)

40. What is the solution of the differential equation

$$ln\left(\frac{dy}{dx}\right) - a = 0?$$

(a) $y = xe^a + c$

(b) $x = ye^a + c$

(c) $y = \ln x + c$

(d) $x = \ln y + c$

Solution: $ln\left(\frac{dy}{dx}\right) - a = 0$

$$\frac{dy}{dx} = e^a$$

$$y = xe^a + c$$

Answer: (a)

41 .Let f(x) be defined as follows:

$$f(x) = \begin{cases} 2x + 1, & -3 < x < -2 \\ x - 1, & -2 \le x < 0 \\ x + 2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point.

(b) It is continuous only in the interval (-3, -2).

(c) It is discontinuous at x =0 but continuous at every other point.

(d) It is discontinuous at every point.

Solution:

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} 2x + 1 = -3$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} x - 1 = -3$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = f(-2) = -3$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x - 1 = -1$$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x + 2 = 2$

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

f(x) is discontinuous at x =0.

Answer: (c)

42. Which one of the following functions is neither even nor odd?

(a) $x^2 - 1$

(b) $x + \frac{3}{x}$

(c) $|x|$

(d) $x^2(x-3)$

Solution:

$x^2 - 1$ is a even function

$x + \frac{3}{x}$ is odd function

Even function: $f(x) = f(-x)$

Odd function: $f(x) = -f(-x)$

Answer: (d)

43. What is the derivative of $\log_{10}(5x^2+3)$ with respect to x?

(a) $\frac{x\log_{10} e}{5x^2+3}$ (b) $\frac{2x\log_{10} e}{5x^2+3}$

(c) $\frac{10x\log_{10} e}{5x^2+3}$ (d) $\frac{10x\log_{e} 10}{5x^2+3}$

Solution:

$y = \log_{10}(5x^2+3) = \log_{10} e \log_e(5x^2+3)$

$$\frac{dy}{dx} = \frac{d(\log_{10} e \log_e(5x^2+3))}{dx}$$

$$\frac{dy}{dx} = \frac{10x\log_{10} e}{5x^2+3}$$

Answer: (c)

44. Let $f(a) = \frac{a-1}{a+1}$ Consider the following:

1. $f(2a) = f(a) + 1$
2. $f\left(\frac{1}{a}\right) = -f(a)$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

$$f(a) = \frac{a-1}{a+1}$$

$$f(2a) = \frac{2a-1}{2a+1}$$

$$f(a) + 1 = \frac{a-1}{a+1} + 1$$

$$f(a) + 1 = \frac{a-1+a+1}{a+1} = \frac{2a}{a+1}$$

$$f\left(\frac{1}{a}\right) = \frac{\frac{1}{a}-1}{\frac{1}{a}+1} = \frac{1-a}{1+a} = -f(a)$$

Answer: (b)

45. Let $f(x) = x + \frac{1}{x}$, where $x \in (0,1)$. Then Which one of the following is correct?

(a) f(x) fluctuates in the interval

(b) f(x) increases in the interval

(c) f(x) decreases in the interval

(d) None of the above

Solution:

$$f(x) = x + \frac{1}{x}$$

$$f'(x) = 1 - \frac{1}{x^2}$$

In interval $x \in (0,1)$. $f'(x)$ is negative. So function f(x) is decreasing function in this interval.

Answer: (c)

46. What is $\int_{e^{-1}}^{e^2}\left|\frac{lnx}{x}\right| dx$ equal to ?

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) 3 (d) 4

Solution:

$$\int_{e^{-1}}^{e^2}\left|\frac{lnx}{x}\right| dx = \int_{e^{-1}}^{e} -\frac{lnx}{x}dx + \int_{e}^{e^2}\frac{lnx}{x}dx$$

$$= -\frac{(\ln x)^2}{2}\Big|_{e^{-1}}^{e} + \frac{(\ln x)^2}{2}\Big|_{e}^{e^2}$$

$$= \frac{3}{2}$$

Answer: (a)

47. The mean of a group of 100 observations was found to be 20. Later it was found that four observations were incorrect, which were recorded as 21, 21, 18 and 20. What is the mean if the incorrect observation are omitted?

(a) 18 (b) 20

(c) 21 (d) 22

Solution:

$$\frac{\sum_{i=0}^{100} x_i}{100} = 20$$

$$\sum_{i=0}^{100} x_i = 2000$$

Let $x_{97}, x_{98}, x_{99}, x_{100}$ are incorrect observations

$$\sum_{i=0}^{96} x_i = 2000 - (x_{97} + x_{98} + x_{99} + x_{100})$$

$$\sum_{i=0}^{96} x_i = 2000 - (21 + 21 + 18 + 20) = 1960$$

$$Mean = \frac{\sum_{i=0}^{96} x_i}{200 - 4} = \frac{1960}{96} = 20$$

Answer: (b)

48. A committee of two persons is constituted from two men and two women. What is the probability that the committee will have only women?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Solution:

Number of ways to select two person = $C(4,2) = \frac{4!}{2!2!} = 6$

Number of ways to select only women = 1

$$P(E) = \frac{1}{6}$$

Answer: (a)

49. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70 kg and that of girls is 55 kg. What is the number of boys in the class?

(a) 50 (b) 55

(c) 60 (d) 100

Solution: Let number of boys is x and number of girls is y.

Average weight of class = $\frac{\text{Sum of weight of students}}{\text{number of students}}$

$$60 = \frac{\text{Sum of weight of students}}{150}$$

Sum of weight of students

$= 60 \times 150 = 9000$ kg

Average weight of boys = $\frac{\text{Sum of weight of boys}}{\text{number of boys}}$

$$70 = \frac{\text{Sum of weight of boys}}{\text{x}}$$

Sum of weight of boys $= 70$x

Average weight of girls = $\frac{\text{Sum of weight of girls}}{\text{number of boys}}$

$$55 = \frac{\text{Sum of weight of girls}}{\text{y}}$$

Sum of weight of girls $= 55$y

$$\text{Sum of weight of students} = \text{Sum of weight of boys} + \text{Sum of weight ofgirls}$$

$$70x + 55y = 9000$$

$$x + y = 150$$

$$x = \frac{9000 - 55 \times 150}{15} = 50$$

Number of boys = 50.

Answer: (a)

50. A point is chosen at random inside a circle. What is the probability that the point is closer to the centre of the circle than to its boundary?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Solution: If point selected lies inside the circle of radius r/2.

Probability of point close to centre =

$$\frac{Area\ of\ circle\ of\ radius\ r/2}{Area\ of\ circle\ of\ radius\ r} = \frac{\frac{\pi r^2}{4}}{\pi r^2} = \frac{1}{4}$$

Answer: (b)

51. In an examination, 40% of candidates got second class. When the data are represented by a pie chart, what is the angle corresponding to second class?

(a) 40^0 (b) 90^0

(c) 144^0 (d) 320^0

Solution: Angle corresponding to 40% students in pi chart = $\frac{40}{100} \times 360 = 144^0$

Answer: (c)

52. If two fair dice are thrown, then what is the probability that the sum is neither 8 nor 9?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{5}{6}$

Solution:

Total number of sample space n(S) = 36

Let E is the event of occurrence of 8 and 9.

E= $\{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6),(4,5),(5,4),(6,3)\}$

$n(E) = 9$

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

$$P(E^c) = 1 - \frac{1}{4} = \frac{3}{4}$$

Answer: (c)

53. Let A and B are two mutually exclusive events with P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{4}$. What is the value of $(\bar{A} \cap \bar{B})$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

Solution:

For mutually exclusive events,

$$n(A \cup B) = n(A) + n(B)$$

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \frac{7}{12} = \frac{5}{12}$$

Answer: (d)

54. The mean and standard deviation of a binomial distribution are 12 and 2 respectively. What is the number of trials?

(a) 2 (b) 12

(c) 18 (d) 24

Solution:

Mean $\mu = n \times p = 12$

Standard deviation

$$\sigma_x = \sqrt{Variance} = \sqrt{np(1-p)} = 2$$

$$\sqrt{12(1-p)} = 2$$

$$p = \frac{2}{3}$$

$$np = 12$$

$$n \times \frac{2}{3} = 12$$

$$n = 18$$

Answer: (c)

55. What is the ratio in which the point C$\left(-\frac{2}{7}, -\frac{20}{7}\right)$ divides the line joining the points $A(-2,-2)$ and $B(2,-4)$?

(a) 1 : 3 (b) 3 : 4

(c) 1 : 2 (d) 2 : 3

Solution: Let C divide AB in ratio of m : n

$$\mathrm{x_c} = \frac{\mathrm{mx_B + nx_A}}{\mathrm{m+n}}$$

$$-\frac{2}{7} = \frac{m \times 2 + n \times -2}{m+n}$$

$$-\frac{1}{7} = \frac{m-n}{m+n}$$

$$-m - n = 7m - 7n$$

$$7n - n = 7m + m$$

$$6n = 8m$$

$$\frac{m}{n} = \frac{6}{8} = \frac{3}{4}$$

56. What is the equation of the ellipse having foci $(\pm 2, 0)$ and the eccentricity $\frac{1}{4}$?

(a) $\frac{x^2}{64} + \frac{y^2}{60} = 1$

(b) $\frac{x^2}{60} + \frac{y^2}{64} = 1$

(c) $\frac{x^2}{20}+\frac{y^2}{24}=1$

(d) $\frac{x^2}{24}+\frac{y^2}{20}=1$

Solution: Focus of ellipse F = $(\pm ae, 0)$

F = $(\pm 2, 0)$

Eccentricity e = $\frac{1}{4}$

$$ae = 2$$

$$a \times \frac{1}{4} = 2$$

$$a = 8$$

$$b^2 = a^2(1-e^2) = 8^2\left(1-\left(\frac{1}{4}\right)^2\right) = 64 \times \frac{15}{16}$$

$$b^2 = 60$$

Equation of ellipse $\frac{x^2}{a^2}+\frac{y^a}{b^2}=1$

$$\frac{x^2}{64}+\frac{y^a}{60}=1$$

57. What is the equation of the straight line parallel to $2x+3y+1=0$ and passes through the point (-1, 2) ?

(a) $2x+3y-4=0$

(b) $2x+3y-5=0$

(c) $x+y-1=0$

(d) $3x-2y+7=0$

Solution: Equation of line parallel to $2x+3y+1=0$ is

$$2x+3y+c=0$$

Line passes through (-1, 2) , so it satisfy this equation of line.

$$2\times -1+3\times 2+c=0$$

$$c=-4$$

Equation of line L:

$$2x+3y-4=0$$

58. What is the acute angle between the pair of straight lines $\sqrt{2}x+\sqrt{3}y=1$ and $\sqrt{3}x+\sqrt{2}y=2$?

(a) $\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(c) $\tan^{-1}(3)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Solution: Slope of line L_1: $\sqrt{2}x+\sqrt{3}y=1$

$$m_1 = -\frac{\sqrt{2}}{\sqrt{3}}$$

Slope of line L_2 : $\sqrt{3}x+\sqrt{2}y=2$

$$m_2 = -\frac{\sqrt{3}}{\sqrt{2}}$$

Angle between line L_1 and L_2

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{-\frac{\sqrt{2}}{\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{2}}}{1+\left(-\frac{\sqrt{2}}{\sqrt{3}}\right)\left(-\frac{\sqrt{3}}{\sqrt{2}}\right)}\right|$$

$$\tan\theta = \frac{1}{2\sqrt{6}}$$

$$\theta = \tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$$

59. If the centroid of a triangle formed by (7, x), (y, -6) and (9, 10) is (6,3), then the values of x and y are respectively

(a) 5, 2 (b) 2, 5

(c) 1, 0 (d) 0 , 0

Solution:

Coordinate of centroid of a triangle

$$x_G = \frac{x_1+x_2+x_3}{3}$$

$$6 = \frac{7+y+9}{3}$$

$$y = 2$$

$$y_G = \frac{y_1+y_2+y_3}{3}$$

$$3 = \frac{x-6+10}{3}$$

$$x = 5$$

60. (0, 0, 0), (a, 0 ,0), (0, b, 0) and (0 , 0, c) are four distinct points. What are the coordinates of the point which is equidistant from the four points?

(a) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$

(b) (a, b, c)

(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

(d) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Solution:

Equation of sphere

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$$

Sphere passes through origin (0, 0, 0). SO it must satisfy it.

$(0-x_0)^2+(0-y_0)^2+(0-z_0)^2=R^2$

$R^2 = x_0^2 + y_0^2 + z_0^2$

$(a-x_0)^2+(0-y_0)^2+(0-z_0)^2$

$= x_0^2 + y_0^2 + z_0^2$

$(a-x_0)^2 = x_0^2$

$a-x_0 = x_0$

$x_0 = \frac{a}{2}$

$(0-x_0)^2+(b-y_0)^2+(0-z_0)^2$

$= x_0^2 + y_0^2 + z_0^2$

$(b-y_0)^2 = y_0^2$

$b-y_0 = y_0$

$y_0 = \frac{b}{2}$

$(0-x_0)^2+(0-y_0)^2+(c-z_0)^2$

$= x_0^2 + y_0^2 + z_0^2$

$c-z_0 = z_0$

$z_0 = \frac{c}{2}$

$$(x_0, \quad y_0, \quad z_0) = \left(\frac{a}{2}, \ \frac{b}{2}, \frac{c}{2}\right)$$

61. What is the equation of the circle which passes through the points (3, -2) and (-2, 0) and having its centre on the line $2x-y-3=0$?

(a) $x^2+y^2+3x+2=0$

(b) $x^2+y^2+3x+12y+2=0$

(c) $x^2+y^2+2x=0$

(d) $x^2+y^2=5$

Solution: Equation of the circle

$$x^2+y^2+2gx+2fy+c=0$$

Circle passes through (3, -2) and (-2, 0)

$$3^2+(-2)^2+2g\times 3+2f\times(-2)+c=0$$

$$6g-4f+c+13=0$$

$$-c=6g-4f+13$$

$$(-2)^2+0^0+2g\times(-2)+2f\times(0)+c=0$$

$$-4g+c+4=0$$

$$-c=4-4g$$

$$6g-4f+13=-4g+4$$

$$10g-4f+9=0$$

Centre of circle passes through line $2x-y-3=0$

$$2\times(-g)-(-f)-3=0$$

$$-2g+f-3=0$$

$$f=3+2g$$

$$10g-4f+9=0$$

$$10g-4(3+2g)+9=0$$

$$10g-8g-12+9=0$$

$$2g-3=0$$

$$g=\frac{3}{2}$$

$$f=3+3=6$$

$$c=4g-4=4\times\frac{3}{2}-4=2$$

$$x^2+y^2+2gx+2fy+c=0$$

$$x^2+y^2+2\times\frac{3}{2}x+2\times 6\times y+2=0$$

$$x^2+y^2+3x+12y+2=0$$

62. The points P(3, 2, 4), Q(4, 5, 2), R(5, 8, 0) and S(2, -1, 6) are

(a) vertices of a rhombus which is not a square
(b) non-coplanar
(c) collinear
(d) coplanar but not collinear

Solution:

$\overrightarrow{PQ} = (4-3)\hat{\imath} + (5-2)\hat{\jmath} + (2-4)\hat{k}$

$\overrightarrow{PQ} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\overrightarrow{QR} = (5-4)\hat{\imath} + (8-5)\hat{\jmath} + (0-2)\hat{k}$

$\overrightarrow{QR} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\overrightarrow{RS} = (2-5)\hat{\imath} + (-1-8)\hat{\jmath} + (6-0)\hat{k}$

$\overrightarrow{RS} = -3(\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

$\overrightarrow{PQ}, \overrightarrow{QR}$ and $\overrightarrow{RS}$ are parallel to each other.

So point P, Q, R and S are collinear points

63. The line passing through the points (1, 2, -1) and (3, -1, 2) meets the yz-plane at which of the following points?

(a) $\left(0, -\frac{7}{2}, \frac{5}{2}\right)$ (b) $\left(0, \frac{7}{2}, \frac{1}{2}\right)$

(c) $\left(0, -\frac{7}{2}, -\frac{5}{2}\right)$ (d) $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

Solution:

Let A = $(1, 2, -1)$ and B = (3, -1, 2) meet yz-plane at point P which divide AB in ratio of m : n.

X-coordinate of point P is equal to 0.

$$x_P = 0$$

$$\frac{mx_B + nx_A}{m+n} = 0$$

$$m \times 3 + n \times 1 = 0$$

$$\frac{m}{n} = -\frac{1}{3}$$

$$y_p = \frac{my_B + ny_A}{m+n} = \frac{-m+2n}{m+n} = \frac{-\frac{m}{n}+2}{\frac{m}{n}+1}$$

$$= \frac{\frac{1}{3}+2}{-\frac{1}{3}+1} = \frac{7}{2}$$

$$z_p = \frac{mz_B + nz_A}{m+n} = \frac{\frac{m}{n}z_B + z_A}{\frac{m}{n}+1} = \frac{-\frac{1}{3}\times 2 - 1}{-\frac{1}{3}+1}$$

$$= \frac{-\frac{5}{3}}{\frac{2}{3}} = -\frac{5}{2}$$

64. What is $\lim_{x\to 0} \frac{e^x-(1+x)}{x^2}$ equal to ?

(a) 0 (b) ½

(c) 1 (d) 2

Solution: Apply L Hospital rule

$$\lim_{x\to 0} \frac{e^x - 1}{2x} = \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$$

65. What is $\int_0^{\frac{\pi}{2}} \frac{d\theta}{1+cos\theta}$ equal to?

(a) ½ (b) 1

(c) $\sqrt{3}$ (d) None of the above

Solution: $\cos 2\theta = 2cos^2\theta - 1$

$1 + \cos 2\theta = 2cos^2\theta$

$1 + \cos\theta = 2cos^2\frac{\theta}{2}$

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{1+cos\theta} = \int_0^{\frac{\pi}{2}} \frac{1}{2cos^2\frac{\theta}{2}} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{sec^2\frac{\theta}{2}}{2} d\theta$$

$Let\ \frac{\theta}{2} = z$

$\frac{d\theta}{2} = dz$

$$= \int_0^{\frac{\pi}{4}} sec^2 z\, dz = \tan z|_0^{\frac{\pi}{4}} = \tan\frac{\pi}{4} - \tan 0$$

$$= 1$$

66. If $\sin A = \frac{3}{5}$, where $450^0 < A < 540^0$, then $\cos\frac{A}{2}$ is equal to

(a) $\frac{1}{\sqrt{10}}$ (b) $-\sqrt{\frac{3}{10}}$

(c) $\sqrt{\frac{3}{10}}$ (d) None of the above

Solution: $\sin A = \frac{3}{5}$

$$\cos A = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$$

$$225^0 < \frac{A}{2} < 270^0$$

$$\cos\frac{A}{2} < 0$$

$$2cos^2\frac{A}{2} = 1+\cos A = 1-\frac{4}{5} = \frac{1}{5}$$

$$\cos\frac{A}{2} = -\frac{1}{\sqrt{10}}$$

67. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0\,,\,\frac{\pi}{2}\right)$ is attained at

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Solution:

$$y = \sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$$

$$y = \sqrt{2}\sin\left(x+\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$x+\frac{\pi}{6}+\frac{\pi}{4} = \frac{\pi}{2}$$

$$x = \frac{\pi}{12}$$

68. If $A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, then the value of A^4 is

(a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$

(d) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Solution: $A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$$A^2 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$A^4 = A^2A^2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

69. If $K = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then what is the value of K?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Solution: $\frac{\pi}{18} = 10^0$

$$\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$$

$$= \sin 10^0 \sin 50^0 \sin 70^0$$

$$= \frac{1}{2}(2\sin 10^0\sin 50^0\sin 70^0)$$

$$2\sin A\sin B = \cos(A-B)-\cos(A+B)$$

$$2\sin 10^0\sin 50^0$$

$$= \cos(10^0-50^0) - \cos(10^0+50^0)$$

$$= \cos(-40^0)-\cos 60^0$$

$$= \cos 40^0 - \frac{1}{2}$$

$$\frac{1}{2}(2\sin 10^0\sin 50^0\sin 70^0)$$

$$= \frac{1}{2}\left(\left(\cos 40^0-\frac{1}{2}\right)\sin 70^0\right)$$

$$= \frac{1}{2}\left(\cos 40^0\sin 70^0 - \frac{1}{2}\sin 70^0\right)$$

$$= \frac{1}{4}\left(2\sin 70^0\cos 40^0 - \frac{2}{2}\sin 70^0\right)$$

$$2\sin A\cos B = \sin(A+B)+\sin(A-B)$$

$$2\sin 70^0\cos 40^0$$

$$= \sin(70^0+40^0) + \sin(70^0-40^0)$$

$$= \sin(110^0)+\sin 30^0$$

$$= \sin(90^0+20^0)+\frac{1}{2}$$

$$= \cos(20^0)+\frac{1}{2}$$

$$\frac{1}{4}(2\sin 70^0\cos 40^0 - \sin 70^0)$$

$$= \frac{1}{4}\left(\cos(20^0)+\frac{1}{2}-\sin(90^0-20^0)\right)$$

$$= \frac{1}{4}\left((\cos(20^0)+\frac{1}{2}-\cos(20^0)\right)$$

$$= \frac{1}{8}$$

70. If $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ and $\vec{c} = \hat{\imath} + m\hat{\jmath} + n\hat{k}$ are three coplanar vectors and $|\vec{c}| = \sqrt{6}$, then which one of the following is correct?

(a) $m = 2 \text{ and } n = \pm 1$

(b) $m = \pm 2 \text{ and } n = -1$

(c) $m = 2 \text{ and } n = -1$

(d) $m = \pm 2 \text{ and } n = 1$

Solution: if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar vectors then scalar triple product is equal to 0.

$$(\vec{a} \times \vec{b}).\vec{c} = 0$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} \hat{k}$$

$$= (-2-3)\hat{\imath} - (2-2)\hat{\jmath} + (3+2)\hat{k}$$

$$= -5\hat{\imath} + 5\hat{k}$$

$$(\vec{a} \times \vec{b}).\vec{c} = -5 \times 1 + 0 \times m + 5 \times n = 0$$

$$5(n-1) = 0$$

$$n = 1$$

$$|\vec{c}| = \sqrt{6}$$

$$\sqrt{1^2 + m^2 + n^2} = \sqrt{6}$$

$$m^2 = 4$$

$$m = \pm 2$$

71. If $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then which one of the following is correct?

(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}| = |\vec{c}|$ and $|\vec{b}| = 1$

(b) $\vec{a}, \vec{b}, \vec{c}$ are non-orthogonal to each other

(c) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{a}| \neq |\vec{c}|$

(d) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{b}| \neq 1$

Solution: $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

Vector $\vec{c}$ is perpendicular to vector $\vec{a}$ and $\vec{b}$

Vector $\vec{a}$ is perpendicular to vector $\vec{b}$ and $\vec{c}$

$\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs.

$$|\vec{a} \times \vec{b}| = |\vec{c}|$$

$$|\vec{a}||\vec{b}| \sin 90^0 = |\vec{c}|$$

$$|\vec{b} \times \vec{c}| = |\vec{a}|$$

$$|\vec{b}||\vec{c}| \sin 90^0 = |\vec{a}|$$

$$|\vec{b}|^2 |\vec{c}| = |\vec{c}|$$

$$|\vec{b}|^2 = 1$$

$$|\vec{b}| = 1$$

71. Let $f: [-6, 6] \to R$ be defined by $f(x) = x^2 - 3$. Consider the following:

1. $(fofof)(-1) = (fofof)(1)$
2. $(fofof)(-1) = -4(fofof)(1) = (fof)(0)$

Which of the above is /are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution: $f(x) = x^2 - 3$

$f(-1) = (-1)^2 - 3 = -2$

$f(1) = 1^2 - 3 = -2$

$f(f(-1)) = f(-2) = (-2)^2 - 3 = 1$

$f(f(f(-1)) = f(1) = -2$

$f(f(1)) = f(-2) = 1$

$f(f(f(1)) = f(1) = -2$

$f(f(f(-1)) = f(f(f(1))$

72. Let $f(x+y) = f(x)f(y)$ for all x and y. Then what is $f'(5)$ equal to [where $f'(x)$ is the derivative of (x)] ?

(a) $f(5)f'(0)$

(b) $f(5) - f'(0)$

(c) $f(5)f(0)$

(d) $f(5) + f'(0)$

Solution:

$$f(x+y) = f(x)f(y)$$
$$f(5+h) = f(5)f(h)$$
$$\lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$$
$$\lim_{h\to 0} \frac{f(5)f(h) - f(5)}{h}$$
$$\lim_{h\to 0} \frac{f(h) - 1}{h} \times f(5)$$
$$= f(5) \times f'(0)$$

73. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then what is $\int_0^a f(x)g(x)dx$ equal to?

(a) $\int_0^a g(x)dx$ (b) $\int_0^a f(x)dx$

(c) $2\int_0^a f(x)dx$ (d) 0

Solution:

$$\int_0^a f(x)g(x)dx = \int_0^a f(a-x)g(a-x)dx$$
$$= \int_0^a f(x)\big(2 - g(x)\big)dx$$
$$= 2\int_0^a f(x)\,dx - \int_0^a f(x)g(x)dx$$
$$2\int_0^a f(x)g(x)dx = 2\int_0^a f(x)\,dx$$
$$\int_0^a f(x)g(x)dx = \int_0^a f(x)\,dx$$

74. A question is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the question will be solved?

(a) $\frac{1}{24}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{23}{24}$

Solution:

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

Event A , B and C are independent event.

$$P(A\cap B) = P(A).P(B) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$$
$$P(B\cap C) = P(B).P(C) = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12}$$
$$P(C\cap A) = P(C).P(A) = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$
$$P(A\cap B\cap C) = P(A).P(B).P(C)$$
$$= \frac{1}{2}\times\frac{1}{3}\times\frac{1}{4} = \frac{1}{24}$$
$$P(A\cup B\cup C) = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{6} - \frac{1}{12} - \frac{1}{8} + \frac{1}{24}$$
$$= \frac{12+8+6-4-2-3+1}{24} = \frac{18}{24} = \frac{3}{4}$$

75. What is the value of tan 18^0?

(a) $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$

(b) $\frac{\sqrt{5}-1}{\sqrt{10+\sqrt{5}}}$

(c) $\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$

(d) $\frac{\sqrt{10+\sqrt{5}}}{\sqrt{5}-1}$

Solution: Let $\theta = 18^0$

$$\tan 5\theta = \tan(3\theta + 2\theta)$$
$$= \frac{\tan(3\theta) + \tan(2\theta)}{1 - \tan(3\theta)\tan(2\theta)}$$
$$\tan 5\theta = \tan 90^0 = \infty$$
$$1 - \tan 3\theta \tan 2\theta = 0$$
$$\tan(3\theta)\tan(2\theta) = 1$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - tan^2\theta}$$
$$\tan(3\theta) = \frac{3\tan\theta - tan^3\theta}{1 - 3\tan^2\theta}$$
$$\frac{3\tan\theta - tan^3\theta}{1 - 3\tan^2\theta} \times \frac{2\tan\theta}{1 - tan^2\theta} = 1$$

Let $\tan\theta = t$

$$\frac{3t - t^3}{1 - 3t^2} \times \frac{2t}{1 - t^2} = 1$$
$$6t^2 - 2t^4 = (1 - t^2 - 3t^2 + 3t^4)$$
$$5z^2 - 10z + 1 = 0 \quad (z = t^2)$$
$$z = \frac{10 \pm \sqrt{100 - 4\times 5\times 1}}{2\times 5} = \frac{10 \pm \sqrt{80}}{10}$$

$$t^2 = \frac{10 \pm 4\sqrt{5}}{10} = \frac{5 \pm 2\sqrt{5}}{5}$$

$$t = \sqrt{\frac{5 - 2\sqrt{5}}{5}}$$

76. Consider the following for triangle ABC:

1. $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$
2. $\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right)$
3. $sin(B + C) = \cos A$
4. $\tan(B + C) = -cotA$

Which of the above are correct?

(a) 1 and 3
(b) 1 and 2
(c) 1 and 4
(d) 2 and 3

Solution: In triangle ABC

$\angle A + \angle B + \angle C = 180^0$

$\angle B + \angle C = 180^0 - \angle A$

$$\frac{\angle B + \angle C}{2} = 90^0 - \frac{\angle A}{2}$$

$$\sin\left(\frac{\angle B + \angle C}{2}\right) = \sin\left(90^0 - \frac{\angle A}{2}\right)$$

$\sin(90^0 - \theta) = \cos\theta$

$$\sin\left(90^0 - \frac{\angle A}{2}\right) = \cos\left(\frac{\angle A}{2}\right)$$

$$\sin\left(\frac{\angle B + \angle C}{2}\right) = \cos\left(\frac{\angle A}{2}\right)$$

$$\tan\left(\frac{\angle B + \angle C}{2}\right) = \tan\left(90^0 - \frac{\angle A}{2}\right)$$

$\tan(90^0 - \theta) = \cot\theta$

$$\tan\left(90^0 - \frac{\angle A}{2}\right) = \cot\left(\frac{\angle A}{2}\right)$$

$\sin(B + C) = sin\,(180^0 - A) = \sin A$

$\tan(B + C) = \tan(\pi - A) = -\tan A$

77. The expression

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta}$$

is equal to

(a) $\tan\left(\frac{\alpha+\beta}{2}\right)$
(b) $\cot\left(\frac{\alpha+\beta}{2}\right)$
(c) $\sin\left(\frac{\alpha+\beta}{2}\right)$
(d) $\cos\left(\frac{\alpha+\beta}{2}\right)$

Solution:

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \tan\left(\frac{\alpha+\beta}{2}\right)$$

78. From the top of a lighthouse, 100 m high, the angle of depression of a boat is $\tan^{-1}\left(\frac{5}{12}\right)$. What is the distance between the boat and the lighthouse?

(a) 120 m (b) 180 m
(c) 240 m (d) 360 m

Solution: Let θ is angle of depression.
Let x is the distance between the boat and the lighthouse

$$\tan\theta = \frac{100}{x}$$

$$\tan\tan^{-1}\left(\frac{5}{12}\right) = \frac{100}{x}$$

$$\frac{5}{12} = \frac{100}{x}$$

$x = 240\ m$

79. A card is drawn from a well-shuffled ordinary deck of 52 cards. What is the probability that it is an ace?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$
(c) $\frac{3}{13}$ (d) $\frac{1}{52}$

Solution: Number of ace in deck of 52 cards = 4

If a card drawn from pack of 52 cards then probability of occurrence of an ace.

$$P(E) = \frac{4}{52} = \frac{1}{13}$$

80. What is the maximum area of a triangle that can be inscribed in a circle of radius a?

(a) $\frac{3a^2}{4}$ (b) $\frac{a^2}{2}$

(c) $\frac{3\sqrt{3}a^2}{4}$ (d) $\frac{\sqrt{3}a^2}{4}$

Solution:

Suppose a base BC is at distance x from centre of circle of radius. For maximum area of triangle ABC, point A such that height of the triangle should be maximum.

Base = $2\sqrt{a^2 - x^2}$

Height = $a + x$

Area of triangle = $\frac{1}{2} \times (a + x) \times 2\sqrt{a^2 - x^2}$

Area of triangle = $(a + x)\sqrt{a^2 - x^2}$

$$\frac{dA}{dx} = \sqrt{a^2 - x^2} - \frac{(a + x)x}{\sqrt{a^2 - x^2}}$$

$$\frac{dA}{dx} = 0$$

$$\sqrt{a^2 - x^2} - \frac{(a + x)x}{\sqrt{a^2 - x^2}} = 0$$

$$\sqrt{a^2 - x^2} = \frac{(a + x)x}{\sqrt{a^2 - x^2}}$$

$$a^2 - x^2 = (a + x)x$$

$$a - x = x$$

$$x = \frac{a}{2}$$

Maximum area

$$= \left(a + \frac{a}{2}\right)\sqrt{a^2 - \frac{a^2}{4}}$$

$$= \frac{3a}{2} \times \frac{\sqrt{3}}{2}a = \frac{3\sqrt{3}a^2}{4}$$

81. If $\sin\theta = 3\,sin(\theta + 2\alpha)$, then the value of $tan\,(\theta + \alpha) + 2tan\,(\alpha)$ is equal to

(a) -1 (b) 0

(c) 1 (d) 2

Solution: $\sin\theta = 3\,sin(\theta + 2\alpha)$

$$\frac{\sin\theta}{\sin(\theta + 2\alpha)} = \frac{3}{1}$$

$$\frac{\sin\theta + \sin(\theta + 2\alpha)}{\sin\theta - \sin(\theta + 2\alpha)} = \frac{3 + 1}{3 - 1} = 2$$

$$\frac{2\sin\frac{\theta + \theta + 2\alpha}{2}\cos\frac{\theta - \theta - 2\alpha}{2}}{2\cos\frac{\theta + \theta + 2\alpha}{2}\sin\frac{\theta - \theta - 2\alpha}{2}} = 2$$

$$\frac{tan\,(\theta + \alpha)}{-\tan\alpha} = 2$$

$$tan\,(\theta + \alpha) + 2tan\,(\alpha) = 0$$

82. The expansion of $(x - y)^n, n \le 5$ is done in the descending powers of x. If the sum of the fifth and sixth terms is zero, then $\frac{x}{y}$ is equal to

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

Solution:

Fifth terms = $C(n, 4) \times (-y)^4 \times x^{n-4}$

Sixth term = $C(n, 5) \times (-y)^5 \times x^{n-5}$

$$C(n, 4) \times (-y)^4 \times x^{n-4} + C(n, 5) \times (-y)^5 \times x^{n-5} = 0$$

$$\frac{n!}{4!\,(n - 4)!}\,y^4x^{n-4} - \frac{n!}{5!\,(n - 5)!}y^5x^{n-5} = 0$$

$$\frac{n!\,y^4x^{n-5}}{4!\,(n - 5)!}\left(\frac{x}{n - 4} - \frac{y}{5}\right) = 0$$

$$\frac{x}{n - 4} - \frac{y}{5} = 0$$

$$\frac{x}{y} = \frac{n - 4}{5}$$

83. In the binary equation $(1p101)_2 + (10q1)_2 = (100r00)_2$ where p, q and r are binary digits, what are the possible values of p, q and r respectively?

(a) 0, 1, 0

(b) 1, 1, 0

(c) 0, 0, 1

(d) 1, 0, 1

Solution:

$(1p101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + p \times 2^3 + 1 \times 2^4 = 8p + 21$

$(10q1)_2 = 1 \times 2^0 + q \times 2^1 + 0 \times 2^2 + 1 \times 2^3$

$(10q1)_2 = 2q + 9$

$(100r00)_2 = r \times 2^2 + 1 \times 2^5 = 4r + 32$

$8p + 21 + 2q + 9 = 4r + 32$

$8p + 2q = 4r + 2$

$p = 0, q = 1 \; and \; r = 0$

84. If $S = \{x: x^2 + 1 = 0, x \; is \; real\}$, then S is

(a) $\{-1\}$
(b) $\{0\}$
(c) $\{1\}$
(d) an empty set

Solution: $S = \{x: x^2 + 1 = 0, x \; is \; real\}$

S = an empty set

85. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then what is AA^T equal to (where A^T is the transpose of A)?

(a) Null matrix (b) Identity matrix
(c) A (d) – A

Solution: $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

82. $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $and \; C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$. If $AB = C$,

then what is A^2 equal to?

(a) $\begin{bmatrix} 4 & 8 \\ -4 & -16 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -4 \\ 8 & -16 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -8 \\ 4 & 12 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

Solution: $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}$

$$B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$AB = C$$

$$\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3x+3y-2y \\ 3x-2x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$3x - y = 4$$

$$x - 2y = -2$$

After solving these two equations we get,

x = 2 and y = 2

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$