1. Suppose ω is a cube root of unity with $\neq 1$. Suppose P and Q are the points on the complex plane defined by ω and ω^2. If O is the origin, then what is the angle between OP and OQ?

(a) 60^0 (b) 90^0

(c) 120^0 (d) 150^0

Solution:

Cubic roots of unity is

$x^3 = 1$

$x^3 - 1 = 0$

$(x-1)(x^2+x+1) = 0$

$x - 1 = 0$

$x = 1$

$x^2 + x + 1 = 0$

$x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Angle made by ω from x-axis is 120^0 and Angle made by ω^2 from x-axis is 240^0. Therefore angle made by OP and OQ is 120^0.

Answer: (c)

2. If $x^2 - px + 4 > 0$ for all real values of x, then which one of the following is correct?

(a) $|p| < 4$ (b) $|p| \le 4$

(c) $|p| > 4$ (d) $|p| \ge 4$

Solution:

$x^2 - px + 4 > 0$

$$\left(x - \frac{p}{2}\right)^2 + \frac{16 - p^2}{4} > 0$$

IF $16 - p^2 > 0$ then above quadratic equation is always positive for all values of x.

$$16 > p^2$$

$$\sqrt{16} > \sqrt{p^2}$$

$$4 > |p|$$

Answer: (a)

3. If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$?

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Solution:

$$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} = re^{i\theta}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\tan\theta = \frac{y}{x} = -\frac{1/\sqrt{2}}{1/\sqrt{2}} = -1$$

$$\theta = -\frac{\pi}{4}$$

$$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} = 1e^{-i\frac{\pi}{4}}$$

$$\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25} = e^{-i\frac{\pi}{4}\times -25} = e^{i\frac{25\pi}{4}}$$

$$= \cos\left(\frac{25\pi}{4}\right) + i\sin\left(\frac{25\pi}{4}\right)$$

$$\cos\left(\frac{25\pi}{4}\right) = \cos\left(6\pi + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{25\pi}{4}\right) = \sin\left(6\pi + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25} = \frac{1+i}{\sqrt{2}}$$

$$\frac{z-\sqrt{2}}{z-i\sqrt{2}} = \frac{\frac{1+i}{\sqrt{2}} - \sqrt{2}}{\frac{1+i}{\sqrt{2}} - i\sqrt{2}} = \frac{\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) + \frac{i}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right) + i\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$\arg\left(\frac{z-\sqrt{2}}{z-i\sqrt{2}}\right) = \arg\left(\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) + \frac{i}{\sqrt{2}}\right) - \arg\left(\left(\frac{1}{\sqrt{2}}\right) + i\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)\right)$$

$$\arg\left(\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right)+\frac{i}{\sqrt{2}}\right)$$

$$= \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1/\sqrt{2}}{-1/\sqrt{2}}$$

$$= \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\arg\left(\left(\frac{1}{\sqrt{2}}\right)+i\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right)\right)$$

$$= \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1/\sqrt{2}}{1/\sqrt{2}}$$

$$= \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) = 0$$

Answer: (*)

4. What is the range of the function $y = \frac{x^2}{1+x^2}$ where $\in R$?

(a) $[0, 1)$ (b) $(0, 1)$

(c) $(0, 1]$ (d) $[0, 1]$

Solution:

$$y = \frac{x^2}{1+x^2}$$

Function y is even function $f(-x) = f(x)$.

Function y = $1 - \frac{1}{1+x^2}$

Range of function is [0 , 1).

Answer: (a)

5. A straight line intersects x and y axes. at P and Q respectively. If (3, 5) is the middle point of PQ, then what is the area of the triangle OPQ?

(a) 12 square units

(b) 15 square units

(c) 20 square units

(d) 30 square units

Solution: If P is x-intercept and Q is y-intercept of line L. Co-ordinate of point P (a, 0) and Q (0, b).

Coordinate of Midpoint of PQ is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

$OP = a = 6$ and $OQ = b = 10$

Area of right angles triangle OPQ = $\frac{1}{2} \times OP \times OQ = \frac{1}{2} \times 6 \times 10 = 30$ square units.

Answer: (d)

6. If a circle of radius b units with center at (0 , b) touches the line $y = x - \sqrt{2}$, then what is the value of b ?

(a) $2 + \sqrt{2}$ (b) $2 - \sqrt{2}$

(c) $2\sqrt{2}$ (d) $\sqrt{2}$

Solution:

Perpendicular distance from the centre of circle to line which touches the circle is equal to radius of the circle.

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

Given $d = b$, $x_1 = 0$ and $y_1 = b$

$$b = \frac{|x_1 - y_1 - \sqrt{2}|}{\sqrt{1+1}} = \frac{|0 - b - \sqrt{2}|}{\sqrt{2}} = \frac{b+\sqrt{2}}{\sqrt{2}}$$

$$\sqrt{2}b = b + \sqrt{2}$$

$$b = \frac{\sqrt{2}}{\sqrt{2}-1} = 2 + \sqrt{2}$$

Answer: (a)

Consider the function $f(\theta) = 4(sin^2\,\theta + cos^4\,\theta)$

7. What is the maximum value of the function f(θ) ?

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

$$\text{f}(\theta) = 4(\sin^2\theta + \cos^4\theta)$$

$$= 4(\cos^4\theta - \cos^2\theta + 1)$$

Let $\cos^2\theta = \text{x}$

$$\text{f(x)} = 4(\text{x}^2 - \text{x} + 1)$$

$$\text{f(x)} = 4\left(\left(\text{x} - \frac{1}{2}\right)^2 + \frac{3}{4}\right)$$

Maximum value of f(x) when $\left(\text{x}-\frac{1}{2}\right)^2$ is maximum. Value of x lies between 0 to 1. Maximum value of $\left(\text{x}-\frac{1}{2}\right)^2$ occur at x = 0 and x =1 .

$$f(x) = 4\left(\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}\right) = 4\left(\left(1 - \frac{1}{2}\right)^2 + \frac{3}{4}\right)$$
$$= 4$$

Answer: (d)

8. What is the minimum value of the function $f(\theta)$?

(a) 0 (b) 1

(c) 2 (d) 3

Solution:

$$f(x) = 4\left(\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}\right)$$

Minimum value of f(x) occur at x = ½.

Minimum value of f(x) = 3

Answer: (d)

9. Consider the following statements

1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Solution:

$$f(\theta) = 4(\sin^2\theta + \cos^4\theta)$$
$$= 4(\cos^4\theta - \cos^2\theta + 1)$$

Since f(θ) is continuous function. Minimum value of f(θ) is 3 and maximum value of $f(\theta)$ is 4.

So f(θ) = 2 has no solution and $f(\theta) = \frac{7}{2}$ has more than one solution.

Answer: (a)

For the next two (2) items that follow:

Consider the curves $f(x) = x|x| - 1$ and $g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \le 0 \end{cases}$

10. Where do the curves intersect?

(a) At (2, 3) only

(b) At (-1, -2) only

(c) At (2, 3) and (-1, -2)

(d) Neither at (2, 3) not at (-1, -2)

Solution:

$$f(x) = g(x)$$
$$x|x| - 1 = \frac{3x}{2}$$
$$x^2 - 1 - \frac{3x}{2} = 0$$
$$2x^2 - 3x - 2 = 0$$
$$x = 2, -\frac{1}{2}$$

But x > 2 therefore x =2, y = 3

$$f(x) = g(x)$$
$$x|x| - 1 = 2x$$
$$-x^2 - 1 = 2x$$
$$x^2 + 2x + 1 = 0$$
$$x = -1$$
$$y = -2$$

Answer:(c)

11. What is the area bounded by the curves?

(a) $\frac{17}{6}$ square units

(b) $\frac{8}{3}$ square units

(c) 2 square units

(d) $\frac{1}{3}$ square units

Solution: Area = $\left|\int_a^b (f(x) - g(x))\,dx\right|$

$$I = \left|\int_{-1}^{0}(-x^2 - 1 - 2x)dx\right| + \left|\int_0^2 \left(x^2 - 1 - \frac{3x}{2}\right)dx\right|$$

$$\int_{-1}^{0}(-x^2 - 1 - 2x)dx$$
$$= -\frac{x^3}{3} - x - \frac{2x^2}{2}\Big|_{-1}^{0}$$
$$= -\frac{0^3}{3} - 0 - 0^2 + \frac{(-1)^3}{3} + (-1) + (-1)^2 = -\frac{1}{3}$$

$$\int_0^2 \left(x^2 - 1 - \frac{3x}{2}\right)dx$$
$$= \frac{x^3}{3} - x - \frac{3x^2}{4}\Big|_0^2$$
$$= \frac{8}{3} - 2 - 3$$
$$= \frac{8-15}{3} = -\frac{7}{3}$$
$$I = \frac{1}{3} + \frac{7}{3} = \frac{8}{3}$$

For the next two (2) items that follow:

Consider the functions $f(x) = xg(x)$ and $g(x) = \left[\frac{1}{x}\right]$ where [.] is the greatest integer function.

12. What is

$$\int_{\frac{1}{3}}^{\frac{1}{2}} g(x)\,dx$$

equal to ?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{5}{18}$ (d) $\frac{5}{36}$

Solution: $g(x) = \left[\frac{1}{x}\right]$

$$\frac{1}{3} \le x \le \frac{1}{2}$$

$$2 \le \frac{1}{x} \le 3$$

$$g(x) = 2 \quad , 2 \le \frac{1}{x} < 3$$

$$\int_{\frac{1}{3}}^{\frac{1}{2}} g(x)\,dx = \int_{\frac{1}{3}}^{\frac{1}{2}} 2\,dx = \frac{1}{3}$$

13. What is

$$\int_{\frac{1}{3}}^{1} f(x)\,dx$$

(a) $\frac{37}{72}$ (b) $\frac{2}{3}$

(c) $\frac{17}{72}$ (d) $\frac{37}{144}$

Solution:

$$\int_{\frac{1}{3}}^{1} f(x)\,dx = \int_{\frac{1}{3}}^{1} xg(x)\,dx$$

$$= \int_{\frac{1}{3}}^{\frac{1}{2}} 2x dx + \int_{\frac{1}{2}}^{1} x dx$$

$$= x^2\Big|_{\frac{1}{3}}^{\frac{1}{2}} + \frac{x^2}{2}\Big|_{\frac{1}{2}}^{1} = \frac{37}{72}$$

For the next five (5) items that follow:

Consider the function $f(x) = |x-1| + x^2$ where $x \in R$.

14. Which one of the following statements is correct?

(a) f(x) is continuous but not differentiable at x =0

(b) f (x) is continuous but not differentiable at x =1

(c) f(x) is differentiable at x =1

(d) f(x) is not differentiable at x =0 and x =1

Solution:

$$f(x) = x^2 - x + 1 \quad x < 1$$

$$= x^2 + x - 1 \quad x \ge 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 - x + 1 \quad = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 + x - 1 \quad = 1$$

$$f(1) = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

f(x) is continuous function

$$f'(x) = 2x - 1\,, \; x < 1$$

$$= 2x + 1, \; x \ge 1$$

$$f'(1^-) = 1$$

$$f'(1^+) = 3$$

$f(x)$ is not differentiable at x = 1.

Answer: (b)

15. What is the area of the region bounded by x-axis, the curve $y = f(x)$ and the two ordiantes $x = \frac{1}{2}$ and $x = 1$?

(a) $\frac{5}{12}$ square unit

(b) $\frac{5}{6}$ square unit

(c) $\frac{7}{6}$ square units

(d) 2 square units

Solution:

$y = |x-1| + x^2$

If $\frac{1}{2} \le x \le 1$

$y = x^2 - x + 1$

Area

$$= \int_{\frac{1}{2}}^{1} x^2 - x + 1\,dx$$

$= \left.\frac{x^3}{3} - \frac{x^2}{2} + x\right|_{\frac{1}{2}}^{1}$

$= \frac{1}{3} - \frac{1}{2} + 1 - \frac{1}{24} + \frac{1}{8} - \frac{1}{2} = \frac{5}{12}$

Answer: (a)

16. What is the area of the region bounded by x-axis, the curve $y = f(x)$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$?

(a) $\frac{5}{12}$ square unit

(b) $\frac{7}{12}$ square unit

(c) $\frac{2}{3}$ square units

(d) $\frac{11}{12}$ square units

Solution: if $1 \le x \le \frac{3}{2}$

$f(x) = x - 1 + x^2 = x^2 + x - 1$

Area = $\int_1^{3/2} x^2 + x - 1\, dx = \frac{11}{12}$

Answer: (d)

For the next two (2) items that follow:

Consider the lines

$y = 3x, y = 6x$ and $y = 9$

17. What is the area of the triangle formed by these lines?

(a) $\frac{27}{4}$ square units

(b) $\frac{27}{2}$ square units

(c) $\frac{19}{4}$ square units

(d) $\frac{19}{2}$ square units

Solution:

Vertex of triangle are A(0, 0) , B(3 , 9) and C(3/2 , 9).

Area enclosed by lines $y = 3x, y = 6x$ and $y = 9$

$= \frac{1}{2} \times 9 \times 3 - \frac{1}{2} \times 9 \times \frac{3}{2} = \frac{27}{4}$

Answer: (a)

18. The centroid of the triangle is at which one of the following points?

(a) $(3, 6)$

(b) $\left(\frac{3}{2}, 6\right)$

(c) $(3,3)$

(d) $\left(\frac{3}{2}, 9\right)$

Solution:

Vertex of triangle are A(0, 0) , B(3 , 9) and C(3/2 , 9).

Centroid of the triangle G

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \equiv \left(\frac{0+3+\frac{3}{2}}{3}, \frac{0+9+9}{3}\right)$

$= \left(\frac{3}{2}, 6\right)$

Answer: (b)

For the next two (2) items that follow:

Consider the two circles

$(x-1)^2 + (y-3)^2 = r^2$ and

$x^2 + y^2 - 8x + 2y + 8 = 0$

19.What is the distance between the centres of the two circles?

(a) 5 units (b) 6 units

(c) 8 units (d) 10 units

Solution: The equation of the two circles are $(x-1)^2 + (y-3)^2 = r^2$ and

$x^2 + y^2 - 8x + 2y + 8 = 0$

$C_1 \equiv (1, 3)$ and $C_2 \equiv (4, -1)$

Distance between the circles is

$= \sqrt{(4-1)^2 + (-1-3)^2} = \sqrt{9+16} = 5$

20. If the circles intersect at two distinct points, then which one of the following is correct?

(a) $r = 1$ (b) $1 < r < 2$

(c) $r = 2$ (d) $2 < r < 8$

Solution: radius of circle-2 $R_2 = 3$

if r = 2 two circles touches internally and if r = 8 two circles touches externally.

Two circles intersect at two distinct point when r lies between 2 to 8.

For the next two (2) items that follow:

Consider the two lines $x + y + 1 = 0$ and $3x + 2y + 1 = 0$

21. What is the equation of the line passing through the point of intersection of the given lines and parallel to x-axis?

(a) $y+1=0$

(b) $y-1=0$

(c) $y-2=0$

(d) $y+2=0$

Solution:

Intersection of lines $x+y+1=0$ and $3x+2y+1=0$ is (1, -2)

Line passing through point of intersection and parallel to x-axis is y = -2 .

Equation of line is y + 2 =0

Answer: (d)

22. What is the equation of the line passing through the point of intersection of the given lines and parallel to y-axis?

(a) $x+1=0$ (b) $x-1=0$

(c) $x-2=0$ (d) $x+2=0$

Solution:

Intersection of lines $x+y+1=0$ and $3x+2y+1=0$ is (1, -2)

Line passing through point of intersection and parallel to y-axis is x = 1 .

Equation of line is x - 1 =0

Answer: (b)

For the next three (2) items that follow:

A plane P passes through the line of intersection of the planes $2x-y+3z=2$, $x+y-z=1$ and the point (1, 0, 1).

23. What are the direction ratios of the line of intersection of the given planes?

(a) $(2,-5,-3)$

(b) $(1,-5,-3)$

(c) $(2,5,3)$

(d) $(1,3,5)$

Solution: Let direction ratio of line is (a, b, c)

Equation of line passing through intersection of the planes $2x-y+3z=2$, $x+y-z=1$

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

For the next two (2) items that follow:

Let $\hat{a},\hat{b}$ be two unit vectors and θ be the angle between them.

24. What is $\cos\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a}-\hat{b}|}{2}$ (b) $\frac{|\hat{a}+\hat{b}|}{2}$

(c) $\frac{|\hat{a}-\hat{b}|}{4}$ (d) $\frac{|\hat{a}+\hat{b}|}{4}$

Solution: if $\hat{a},\hat{b}$ be two unit vectors

$$|\hat{a}|=|\hat{b}|=1$$

Dot product of vectors $\hat{a},\hat{b}$ is

$$\hat{a}.\hat{b}=|\hat{a}||\hat{b}|\cos\theta=1\times1\times\cos\theta$$

$$|\hat{a}+\hat{b}|^2=(\hat{a}+\hat{b}).(\hat{a}+\hat{b})$$

$$=|\hat{a}|^2+|\hat{b}|^2+2\hat{a}.\hat{b}=2(1+\hat{a}.\hat{b})$$

$$=2(1+\cos\theta)=4cos^2\frac{\theta}{2}$$

$$\cos\frac{\theta}{2}=\frac{|\hat{a}+\hat{b}|}{2}$$

Answer: (b)

25. What is $\sin\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a}-\hat{b}|}{2}$ (b) $\frac{|\hat{a}+\hat{b}|}{2}$

(c) $\frac{|\hat{a}-\hat{b}|}{4}$ (d) $\frac{|\hat{a}+\hat{b}|}{4}$

Solution:

$|\hat{a}-\hat{b}|^2$

$=(\hat{a}-\hat{b}).(\hat{a}-\hat{b})$

$=|\hat{a}|^2+|\hat{b}|^2-2\hat{a}.\hat{b}$

$=2(1-\hat{a}.\hat{b})$

$=2(1-\cos\theta)$

$=4sin^2\frac{\theta}{2}$

$\sin\frac{\theta}{2}=\frac{|\hat{a}-\hat{b}|}{2}$

Answer: (b)

26. What is

$$\int_{-2}^{2} x dx - \int_{-2}^{2} [x] dx$$

equal to, where [.] is the greatest integer function?

(a) 0 (b) 1

(c) 2 (d) 4

Solution: $\int_{-2}^{2} x dx = 0$

$$\int_{-2}^{2} [x] dx = \int_{-2}^{-1} [x]\, dx + \int_{-1}^{0} [x]\, dx + \int_{0}^{1} [x]\, dx + \int_{1}^{2} [x]\, dx$$

$$\int_{-2}^{-1} [x]\, dx = \int_{-2}^{-1} (-2) dx = -2$$

$$\int_{-1}^{0} [x]\, dx = \int_{-2}^{-1} (-1) dx = -1$$

$$\int_{0}^{1} [x]\, dx = \int_{0}^{1} (0) dx = 0$$

$$\int_{1}^{2} [x]\, dx = \int_{1}^{2} 1\, dx = 1$$

$$\int_{-2}^{2} [x] dx = -2 - 1 + 0 + 1 = -2$$

$$\int_{-2}^{2} x dx - \int_{-2}^{2} [x] dx = 2$$

27. What is $\lim_{x \to 0} e^{-\frac{1}{x^2}}$ equal to?

(a) 0

(b) 1

(c) -1

(d) Limit does not exist.

Solution: $\lim_{x \to 0} e^{-\frac{1}{x^2}} = e^{-\infty} = \frac{1}{e^{\infty}}$

$$= \frac{1}{\infty}$$

$$= 0$$

Answer: (a)

28. What is $\int_0^{4\pi} |\cos x|\, dx$ equal to?

(a) 0 (b) 2

(c) 4 (d) 8

Solution: $\boldsymbol{I} = \int_0^{4\pi} |\cos x|\, dx$

$|\cos x|$ is a periodic function with period π

$$\boldsymbol{I} = \int_0^{4\pi} |\cos x|\, dx = \mathbf{4} \int_0^{\pi} |\cos x|\, dx$$

$$= 4 \int_0^{\frac{\pi}{2}} \cos x\, dx + 4 \int_{\frac{\pi}{2}}^{\pi} - \cos x\, dx$$

$$= 8$$

Answer: (d)

29. (a, 2b) is the mid-point of the line segment joining the points (10, -6) and (k, 4). If $a - 2b = 7$, then what is the value of k?

(a) 2 (b) 3

(c) 4 (b) 5

Solution: $a = \frac{10+k}{2}$ $2b = \frac{-6+4}{2} = -1$

$$a - 2b = 7$$

$$a + 1 = 7$$

$$a = 6$$

$$\frac{10 + k}{2} = 6$$

$$k = 12 - 10 = 2$$

Answer: (a)

30. if $\log_a(ab) = x$, then what is $\log_b(ab)$ equal to?

(a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$

(c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

Solution: $\log_a(ab) = x$

$$\log_a a + \log_a b = x$$

$$1 + \log_a b = x$$

$$\log_b a = \frac{1}{\log_a b} = \frac{1}{x - 1}$$

$$\log_b(ab) = \log_b a + \log_b b$$

$$= \log_b a + 1 = \frac{1}{x - 1} + 1 = \frac{x}{x - 1}$$

Answer: (d)

31. What is the number of different messages that can be represented by three 0's and two 1's?

(a) 10 (b) 9

(c) 8 (d) 7

Solution: Number of ways = $\frac{n!}{p!q!}$

Total number of numbers are 5.

0 are 3 times and 1 is two times.

p = 3 and q =2

n = 5

Number of ways = $\frac{5!}{3!2!} = \frac{5\times4}{2} = 10$

32. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

(a) $k \neq 1$ and $k \neq -2$

(b) $k \neq 1$ and $k \neq 2$

(c) $k \neq -1$ and $k \neq -2$

(d) $k \neq -1$ and $k \neq 2$

Solution: System of linear equation are

$kx + y + z = 1$

$x + ky + z = 1$

$x + y + kz = 1$

Simultaneous equation can be written in form of AX = B

A = $\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$

For unique solution inverse of A should exists. If matrix is non-singular matrix then inverse of matrix A exists.

$$\det(A) = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \neq 0$$

$$k \begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 1 \end{vmatrix} \neq 0$$

$k(k^2 - 1) - (k - 1) + (1 - k) \neq 0$

$k(k - 1)(k + 1) - 2(k - 1) \neq 0$

$(k - 1)(k^2 + k - 2) \neq 0$

$(k - 1)(k^2 + 2k - k - 2) \neq 0$

$(k - 1)(k + 2)(k - 1) \neq 0$

$(k - 1)^2(k + 2) \neq 0$

$k \neq 1, -2$

33. What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$?

(a) 30^0 (b) 45^0

(c) 60^0 (d) 75^0

Solution: Slope of Line L_1: $y - \sqrt{3}x - 5 = 0$ is $m_1 = \sqrt{3}$

Slope of Line L_1: $\sqrt{3}y - x + 6 = 0$ is $m_2 = \frac{1}{\sqrt{3}}$

Angle between two lines is θ

$$\tan\theta = \frac{|m_1 - m_2|}{|1 + m_1 m_2|} = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \frac{1}{\sqrt{3}}$$

$$\theta = 60^0$$

34. Which of the following determinants have value zero?

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

3. $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

Select the correct answer using the code given below

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Solution:

$$\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$$

$$Column - 1 = \begin{Bmatrix} 41 \\ 79 \\ 29 \end{Bmatrix}$$

$$Column - 2 = \begin{Bmatrix} 1 \\ 7 \\ 5 \end{Bmatrix}$$

$$\text{Column} - 3 = \begin{Bmatrix} 5 \\ 9 \\ 3 \end{Bmatrix}$$

$$\begin{Bmatrix} 41 \\ 79 \\ 29 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 7 \\ 5 \end{Bmatrix} + 8\begin{Bmatrix} 5 \\ 9 \\ 3 \end{Bmatrix}$$

$$\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 1+8\times 5 & 1 & 5 \\ 7+8\times 9 & 7 & 9 \\ 5+8\times 3 & 5 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix} + 8\begin{vmatrix} 5 & 1 & 5 \\ 9 & 7 & 9 \\ 3 & 5 & 3 \end{vmatrix}$$

$$= 0$$

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$= 0$$

$$\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix} = -c\begin{vmatrix} -c & a \\ -b & 0 \end{vmatrix} + b\begin{vmatrix} -c & 0 \\ -b & -a \end{vmatrix}$$

$$= -abc + abc = 0$$

35. Consider the following in respect of the matrix $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$:

1. $A^2 = -A$
2. $A^3 = 4A$

Which of the above is /are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution: $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

$A^2 = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} (-1\times -1)+(1\times 1) & (-1\times 1)+(1\times -1) \\ (1\times -1)+(-1\times 1) & 1+1 \end{pmatrix}$

$= \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = -2A$

$A^3 = A^2A = -2AA = -2A^2 = -2(-2A)$

$= 4A$

Answer: (b)

36. What is the area of the parallelogram having diagonals $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$?

(a) $5\sqrt{5}$ square units

(b) $4\sqrt{5}$ square units

(c) $5\sqrt{3}$ square units

(d) $15\sqrt{2}$ square units

Solution: If $\vec{a}$ and $\vec{b}$ are two vector representing adjacent side of parallelogram.

$\vec{a} + \vec{b} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{a} - \vec{b} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$2\vec{a} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

$2\hat{b} = 2\hat{\imath} + 4\hat{\jmath} - 6\hat{k}$

Area of the parallelogram = $|\vec{a}\times\vec{b}|$ = $\begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 2 & -3 \end{vmatrix} = \hat{\imath}\begin{vmatrix} -1 & -1 \\ 2 & -3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$

$= |5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}| = 5\sqrt{3}$

37. What is a vector of unit length orthogonal to both the vectors $\hat{\imath} + \hat{\jmath} + \hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$?

(a) $\frac{-4\hat{\imath}+3\hat{\jmath}-\hat{k}}{\sqrt{26}}$ (b) $\frac{-4\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{26}}$

(c) $\frac{-3\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{14}}$ (d) $\frac{-3\hat{\imath}+2\hat{\jmath}+\hat{k}}{\sqrt{14}}$

Solution: $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

Unit vector perpendicular to both vector $\vec{a}$ and $\vec{b}$

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$= \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}\hat{\imath} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}\hat{k}$

$= -4\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{4^2 + 3^2 + 1^2}$

$= \sqrt{16+9+1} = \sqrt{26}$

$$\hat{n} = \frac{-4\hat{\imath} + 3\hat{\jmath} + \hat{k}}{\sqrt{26}}$$

38. What is the number of four-digit decimal number (<1) in which no digit is repeated?

(a) 3024 (b) 4536

(c) 5040 (d) None of the above

Solution: Number of ways

= 8 x 8 x 7 x 6 = 2688

39. If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$ then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to?

(a) 10 (b) 2

(c) 1 (d) 0

Solution:

$$y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$$

$$\frac{dy}{dx} = \frac{d(\log_{10} x + \log_x 10 + \log_x x + \log_{10} 10)}{dx}$$

$$\frac{dy}{dx} = \frac{d\log_{10} x}{dx} + \frac{d\log_x 10}{dx} + 0 + 0$$

$$(\log_x x = \log_{10} 10 = 1)$$

$$\frac{dy}{dx} = \frac{1}{x\ln 10} - \frac{(\log_{10} x)^{-2}}{x\ln 10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10\ln 10} - \frac{1}{10\ln 10} = 0$$

40. Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then what is $(\omega_1 - \omega_2)^2$ equal to?

(a) 3 (b) 1

(c) -1 (d) -3

Solution: Cubic roots of equation $x^3 = 1$

$$(x-1)(x^2+x+1) = 0$$

$$(x^2+x+1) = 0$$

Let $\omega_1$and ω_2 are the roots of above quadratic equation

$$\omega_1 + \omega_2 = -1$$

$$\omega_1\omega_2 = 1$$

$$(\omega_1 - \omega_2)^2 = (\omega_1 + \omega_2)^2 - 4\omega_1\omega_2$$
$$= (-1)^2 - 4 \times 1 = -3$$

41. Three disc are thrown simultaneously. What is the probability that the sum on the three faces is at least 5?

(a) $\frac{17}{18}$ (b) $\frac{53}{54}$

(c) $\frac{103}{108}$ (d) $\frac{215}{215}$

Solution: Number of sample space = $6 \times 6 \times 6$

Let E is event of occurance of sum of three faces is atleast 5.

Let E' is event of occurance of sum of three faces is equal to 3 and 4.

$x + y + z = 3$

$x + y + z = 4$

$E' = \{(1,1,1), (2,1,1), (1,2,1), (1,1,2)\}$

$$P(E') = \frac{4}{6\times6\times6} = \frac{1}{3\times3\times6} = \frac{1}{54}$$

$$P(E) + P(E') = 1$$

$$P(E) = \frac{53}{54}$$

42. Two independent events A and B have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$ What is the probability that exactly one of the two events A or B occurs?

(a) $\frac{1}{4}$ (b) $\frac{5}{6}$

(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

Solution:

If A and B are independent events then

$P(A \cap B) = P(A)P(B) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

The probability that exactly one of the two events A or B occurs is equal to

$P(A \cup B) - P(A \cap B) = P(A) + P(B) - 2P(A \cap B) = \frac{1}{3} + \frac{3}{4} - \frac{2}{4} = \frac{7}{12}$

43. A coin is tossed three times. What is the probability of getting head and tail alternately?

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Solution:

Number of sample space $n(s) = 2 \times 2 \times 2 = 8$

Set A is occurance of geeting head and tail alternately = $\{HTH, THT\}$

$P(A) = \frac{2}{8} = \frac{1}{4}$

44. What is the sum of the squares of the intercepts cut off by the circle on the axes?

(a) $\left(\frac{a^2+b^2}{a^2-b^2}\right)^2$

(b) $2\left(\frac{a^2+b^2}{a-b}\right)^2$

(c) $4\left(\frac{a^2+b^2}{a-b}\right)^2$

(d) None of the above

Solution: Equation of circle is

$$x^2+y^2+2gx+2fy+c=0$$

Circle passes through origin, (a, b) and (-b, -a).

$$0^2+0^2+2g\times 0+2f\times 0+c=0$$

$$c=0$$

$a^2+b^2+2ga+2fb=0$ - - - (1)

$$(-b)^2+(-a)^2-2gb-2fa=0$$

$a^2+b^2-2gb-2fa=0$ - - - (2)

Solving equation (1) and (2) we get,

$$g=-\frac{a^2+b^2}{2(a-b)}$$

$$f=\frac{a^2+b^2}{2(a-b)}$$

For x-intercept, substitute y = 0 we get,

$$x^2+2gx=0$$

$$x=0\,,-2g$$

For y-intercept, substitute x = 0 we get,

$$y^2+2fy=0$$

$$y=0,-2f$$

The sum of the squares of the intercepts cut off by the circle on the axes

$$=4g^2+4f^2=\left(\frac{a^2+b^2}{a-b}\right)^2$$

For the next two (2) items that follow:

Let f(x) be the greatest integer function and g(x) be the modulus function.

45. What is $g^\circ f\left(-\frac{5}{3}\right)-(f^\circ g)\left(-\frac{5}{3}\right)$ equal to?

(a) -1 (b) 0

(b) 1 (d) 2

Solution: $\mathrm{f(x)=[x]}$ and $g(x)=|x|$

$$f\left(-\frac{5}{3}\right)=\left[-\frac{5}{3}\right]=-2$$

$$g(-2)=|-2|=2$$

$$g\left(-\frac{5}{3}\right)=\left|-\frac{5}{3}\right|=\frac{5}{3}$$

$$f\left(\frac{5}{3}\right)=\left[\frac{5}{3}\right]=1$$

$$g^\circ f\left(-\frac{5}{3}\right)-(f^\circ g)\left(-\frac{5}{3}\right)=2-1=1$$

46. What is $(f^\circ f)\left(-\frac{9}{5}\right)+(g^\circ g)(-2)$ equal to?

(a) -1 (b) 0

(c) 1 (d) 2

Solution: $f\left(-\frac{9}{5}\right)=\left[-\frac{9}{5}\right]=-1$

$$f(-1)=[-1]=-1$$

$$g(-2)=|-2|=2$$

$$g(2)=|2|=2$$

$$(f^\circ f)\left(-\frac{9}{5}\right)+(g^\circ g)(-2)=-1+2=1$$

47. What is the binary equivalent of the decimal number 0.3125?

(a) 0.0111 (b) 0.1010

(c) 0.0101 (d) 0.1101

Solution:

0.0111

$$=0\times 2^{-1}+1\times 2^{-2}+1\times 2^{-3}+1\times 2^{-4}$$

$$=0+0.25+0.125+0.0625=0.4375$$

0.1010

$$=1\times 2^{-1}+0\times 2^{-2}+1\times 2^{-3}+0\times 2^{-4}$$

$=0.5+0.125=0.625$

0.0101

$$=0\times 2^{-1}+1\times 2^{-2}+0\times 2^{-3}+1\times 2^{-4}$$

$=0.25+0.0625=0.3125$

48. If A $=(\cos 12^0-\cos 36^0)(\sin 96^0+\sin 24^0)$

B $=(\sin 60^0-\sin 12^0)(\cos 48^0-\cos 72^0)$

then what is $\frac{A}{B}$ equal to ?

(a) -1 (b) 0

(c) 1 (d) 2

Solution:

$\cos \mathrm{A}-\cos \mathrm{B}=2\sin\frac{\mathrm{A+B}}{2}\sin\frac{\mathrm{B-A}}{2}$

$\sin \mathrm{A}+\sin \mathrm{B}=2\sin\frac{\mathrm{A+B}}{2}\cos\frac{\mathrm{A-B}}{2}$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$\cos 12^0 - \cos 36^0$

$= 2\ \sin\frac{12^0+36^0}{2}\sin\frac{36^0-12^0}{2}$

$= 2\sin 24^0 \sin 12^0$

$\sin 96^0 + \sin 24^0$

$= 2\sin\frac{96^0+24^0}{2}\cos\frac{96^0-24^0}{2}$

$= 2\sin 60^0 \cos 36^0$

$\sin 60^0 - \sin 12^0$

$= 2\cos\frac{60^0+12^0}{2}\sin\frac{60^0-12^0}{2}$

$= 2\cos 36^0 \sin 24^0$

$\cos 48^0 - \cos 72^0$

$= 2\sin\frac{48^0+72^0}{2}\sin\frac{72^0-48^0}{2}$

$= 2\sin 60^0 \sin 12^0$

$$\frac{A}{B} = \frac{(2\sin 24^0 \sin 12^0)\times(2\sin 60^0\cos 36^0)}{(2\cos 36^0\sin 24^0)\times(2\sin 60^0 \sin 12^0)} = 1$$

49. Consider the following statements

1. If ABC is an equilateral triangle, then $3\tan(A+B)\tan C = 1$

2. If ABC is a triangle in which $= 78^0$, $B\ = 66^0$, then

$$\tan\left(\frac{A}{2}+C\right) < \tan A$$

3. If ABC is any triangle, then

$$\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$$

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) 1 and 2

(d) 2 and 3

Solution: If ABC is an equilateral triangle then $\angle A = \angle B = \angle C = 60^0$

$3\tan(A+B)\tan C = 3\tan(120^0)\tan 60^0$

$= -3\tan 60^0 \tan 60^0 = -9$

$\tan 120^0 = \tan(180^0 - 60^0) = -\tan 60^0$

If $A = 78^0$, $B = 66^0$ then $C = 180^0 - (78^0 + 66^0) = 36^0$

$$\tan\left(\frac{A}{2}+C\right) = \tan\left(\frac{78^0}{2}+36^0\right) = \tan 72^0$$

$$\tan A = \tan 78^0$$

Since $72^0 < 78^0$

$\tan 72^0 < \tan 78^0$

$\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right)$

$= \tan\left(\frac{78^0+66^0}{2}\right)\sin\frac{36^0}{2}$

$= \tan\frac{144^0}{2}\sin 18^0$

$= \tan 72^0 \sin 18^0$

$\sin 18^0 = \sin 90^0 - 72^0 = \cos 72^0$

$\tan 72^0 \sin 18^0 = \sin 72^0$

$$\cos\left(\frac{C}{2}\right) = \cos 18^0 = \sin 72^0$$

$$\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right) = \cos\left(\frac{C}{2}\right)$$

For the next three (3) items that follow:

Consider a parallelogram whose vertices are $A(1,2), B(4,y), C(x,6)$ and $D(3,5)$ taken in order.

50. What is the value of $AC^2 - BD^2$?

(a) 25 (b) 30

(c) 36 (d) 40

Solution: Diagonal of parallelogram bisect each other.

Midpoint of AC is point P.

X-coordinate of point P

$x_p = \frac{1+x}{2}$

Midpoint of BD is point P.

$x_p = \frac{4+3}{2}$

Midpoint of BD = Midpoint of AC

$7 = 1 + x$

$x = 6$

Y-coordinate of point P

$y_p = \frac{y+5}{2}$

$y_p = \frac{2+6}{2}$

Y- Coordinate of midpoint of AC = Y-coordinate of midpoint of BD.

$y + 5 = 8$

$y = 3$

$\mathrm{A}(1,2), \mathrm{B}(4,3), \mathrm{C}(6,6)$ and $\mathrm{D}(3,5)$

$\mathrm{AC} = \sqrt{(1-6)^2 + (2-6)^2} = \sqrt{25+16}$

$= \sqrt{41}$

$BD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$

$\mathrm{AC}^2 - \mathrm{BD}^2 = 41 - 5 = 36$

51. What is the point of intersection of the diagonals?

(a) $\left(\frac{7}{2}, 4\right)$ (b) $(3,4)$

(c) $\left(\frac{7}{2}, 5\right)$ (d) $(3,5)$

Solution: The point of intersection of the diagonal.

X-coordinate of point P = $\frac{7}{2}$

Y- Coordinate of point P = 4

52. What is the area of the parallelogram?

(a) $\frac{7}{2}$ square units

(b) 4 square units

(c) $\frac{11}{2}$ square units

(d) 7 square units

Solution:

Area of the parallelogram

$= |\vec{r}_{AB} \times \vec{r}_{AD}|$

$\vec{r}_{AB} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath}$

$= (4-1)\hat{\imath} + (y-2)\hat{\jmath} = 3\hat{\imath} + \hat{\jmath}$

$\vec{r}_{AD} = (x_D - x_A)\hat{\imath} + (y_D - y_A)\hat{\jmath}$

$= (3-1)\hat{\imath} + (5-2)\hat{\jmath}$

$= 2\hat{\imath} + 3\hat{\jmath}$

$|\vec{r}_{AB} \times \vec{r}_{AD}| = |9\hat{\imath} \times \hat{\jmath} + 2\hat{\jmath} \times \hat{\imath}| = |7\hat{\imath} \times \hat{\jmath}| = 7$

For the next three (2) items that follow:

A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1).

53. What are the direction ratios of the line of intersection of the given planes?

(a) $(2, -5, -3)$

(b) $(1, -5, -3)$

(c) $(2, 5, 3)$

(d) $(1, 3, 5)$

Solution: Direction ratios of the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$.

Substitute z = 0

$$2x - y = 2$$

$$x + y = 1$$

$$\mathrm{x} = 1 \text{ and } \mathrm{y} = 0$$

Point P (1, 0, 0)

Substitute y = 0

$$2x + 3z = 2$$

$$x - z = 1$$

$$\mathrm{x} = 1 \text{ and } \mathrm{z} = 0$$

Point Q (1, 0 , 0)

Direction ratio of line PQ

54. What is the equation of the plane P?

(a) $2x + 5y - 2 = 0$

(b) $5x + 2y - 5 = 0$

(c) $x + z - 2 = 0$

(d) $2x - y - 2z = 0$

Solution: Equation of plane passing through Planes P_1 and P_2

P = $P_1 + \lambda P_2$

$$2x - y + 3z - 2 + \lambda(x + y - z - 1) = 0$$

$$(2+\lambda)x + (\lambda - 1)y + (3-\lambda)z - 2 - \lambda = 0$$

Plane P passes through (1, 0, 1)

$$(2+\lambda) + (3-\lambda) - 2 - \lambda = 0$$

$$5 - 2 - \lambda = 0$$

$$\lambda = 3$$

$$5x + 2y = 5$$

55. If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to?

(a) $\frac{2}{\sqrt{29}}$

(b) $\frac{4}{\sqrt{29}}$

(c) $\frac{5}{\sqrt{29}}$

(d) 1

Solution: Centre of sphere $x^2 + y^2 + z^2 = r^2$

C(0, 0, 0)

Perpendicular from C on the plane P.

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{|5x_1 + 2y_1 - 5|}{\sqrt{5^2 + 2^2}}$$

$$d = \frac{5}{\sqrt{29}}$$

For the next two (2) items that follow:

Consider the function $f(x) = |x^2 - 5x + 6|$

56. What is $f'(4)$ equal to ?

(a) -4 (b) -3

(c) 3 (d) 2

Solution: $f(x) = |x^2 - 5x + 6|$

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$f(x) > 0 \; if \; x > 3 \; and \; x < 2$$

$$f(x) < 0 \; if \; 2 < x < 3$$

$$f(x) = x^2 - 5x + 6 \quad x < 2$$

$$= -(x^2 - 5x + 6 \;) \; 2 < x < 3$$

$$= x^2 - 5x + 6 \quad x > 3$$

$$f'(x) = 2x - 5 \, , x > 3$$

$$f'(4) = 3$$

57. What is $f'(2.5)$ equal to ?

(a) -3 (b) -2

(c) 0 (d) 2

Solution:

$$f(x) = -(x^2 - 5x + 6 \;) \; 2 < x < 3$$

$$f'(x) = -2x + 5$$

$$f'(2.5) = 0$$

58. If

$$\int_{-2}^{5} f(x)\, dx = 4 \; and$$

$\int_0^5 \{1 + f(x)\}\, dx =$

$7 \; then \; what \; is \int_{-2}^{0} f(x)\, dx$ equal to?

(a) -3 (b) 2

(c) 3 (d) 5

Solution:

$$\int_{-2}^{5} f(x)\, dx = 4$$

$$\int_{-2}^{0} f(x)\, dx + \int_{0}^{5} f(x) = 4$$

$$\int_{0}^{5} \{1 + f(x)\}\, dx = 7$$

$$\int_{0}^{5} dx + \int_{0}^{5} f(x)\, dx = 7$$

$$\int_{0}^{5} f(x)\, dx = 7 - 5 = 2$$

$$\int_{-2}^{0} f(x)\, dx + 2 = 4$$

$$\int_{-2}^{0} f(x)\, dx = 2$$

For the next two (2) items that follow:

Let z be a complex number satisfying

$$\left|\frac{z - 4}{z - 8}\right| = 1$$

and

$$\left|\frac{z}{z - 2}\right| = \frac{3}{2}$$

59. What is $|z|$ equal to?

(a) 6 (b) 12

(c) 18 (d) 36

Solution:

$$\left|\frac{z - 4}{z - 8}\right| = 1$$

$$(x - 4)^2 + y^2 = (x - 8)^2 + y^2$$

$$(x - 4 + x - 8)(x - 4 - x + 8) = 0$$

$$(2x - 12)4 = 0$$

$$x = 6$$

$$\left|\frac{z}{z - 2}\right| = \frac{3}{2}$$

$$\frac{\sqrt{x^2+y^2}}{\sqrt{(x-2)^2+y^2}}=\frac{3}{2}$$

$$\frac{x^2+y^2}{(x-2)^2+y^2}=\frac{9}{4}$$

$$\frac{6^2+y^2}{(6-2)^2+y^2}=\frac{9}{4}$$

$$y=0$$

$$z=x+iy=6$$

$$|z|=6$$

60. What is $\left|\frac{z-6}{z+6}\right|$ equal to?

(a) 3 (b) 2

(c) 1 (d) 0

Solution:

$$\left|\frac{z-6}{z+6}\right|=\left|\frac{6-6}{6+6}\right|=0$$

For the next two (2) items that follow:

Given that $\log_x y, \log_z x, \log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.

61. Which one of the following is correct?

x, y and z are

(a) in AP only

(b) in GP only

(c) in both AP and GP

(d) neither in AP nor in GP

Solution: $\log_x y, \log_z x, \log_y z$ are in GP

$$\frac{\log_z x}{\log_x y}=\frac{\log_y z}{\log_z x}$$

$$(\log_z x)^2=\log_x y \log_y z=\log_x z$$

$$\log_z x=\frac{1}{\log_x z}$$

$$(\log_z x)^2=\frac{1}{\log_z x}$$

$$(\log_z x)^3=1$$

$$\log_z x=1=\log_z z$$

$$z=x$$

62. Which one of the following is correct?

xy, yz and zx are

(a) in AP only

(b) in GP only

(c) in both AP and GP

(d) neither in AP nor in GP

For the next two (2) items that follow:

Given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2+bx+c=0$ with $b\neq 0$.

63. What is $\tan(\alpha+\beta)$ equal to?

(a) $b(c-1)$ (b) $c(b-1)$

(c) $c(b-1)^{-1}$ (d) $b(c-1)^{-1}$

Solution: If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2+bx+c=0$ then

$$\tan\alpha+\tan\beta=-b$$

$$\tan\alpha\tan\beta=c$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=-b(1-c)^{-1}$$

$$=b(c-1)^{-1}$$

64. What is $\sin(\alpha+\beta)\sec\alpha\sec\beta$ equal to?

(a) b (b) –b

(c) c (d) –c

Solution:

$$\sin(\alpha+\beta)\sec\alpha\sec\beta$$

$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$=\tan\alpha+\tan\beta$$

$$=-b$$

For the next two (2) items that follow:

Consider the curves

$$y=|x-1| and\ |x|=2$$

65. What is/are the point(s) of intersection of the curves?

(a) $(-2,3)$ only

(b) $(2,1)$ only

(c) $(-2,3)$ and $(2,1)$

(d) Neither $(-2,3)$ nor $(2,1)$

Solution: $y=|x-1|$ and

$$|x|=2$$

$$x=\pm 2$$

$$y=|2-1|=1$$

$$y=|-2-1|=3$$

Point of intersection are (2,1) and (-2, 3)

66. What is the area of the region bounded by the curves and x-axis?

(a) 3 square units

(b) 3 square units

(c) 5 square units

(d) 6 square units

Solution:

Area of the region bounded by the curves and x-axis

Area = $\left|\frac{1}{2} \times (-2-1) \times 3\right| + \left|\frac{1}{2} \times (2-1) \times 1\right|$

$= \frac{9+1}{2} = 5$

For the next two (2) items that follow:

Consider the function

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

where p is a constant.

67. What is the value of $f'(0)$?

(a) p^3 (b) $3p^3$

(c) $6p^3$ (d) $-6p^3$

Solution:

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f(x) = x^3 \begin{vmatrix} -1 & 0 \\ p^2 & p^3 \end{vmatrix} - \sin x \begin{vmatrix} 6 & 0 \\ p & p^3 \end{vmatrix} + \cos x \begin{vmatrix} 6 & -1 \\ p & p^2 \end{vmatrix}$$

$$f'(x) = 3x^2 \begin{vmatrix} -1 & 0 \\ p^2 & p^3 \end{vmatrix} - \cos x \begin{vmatrix} 6 & 0 \\ p & p^3 \end{vmatrix} - \sin x \begin{vmatrix} 6 & -1 \\ p & p^2 \end{vmatrix}$$

$f'(0) = -6p^3$

68. What is the value of p for which $f''(0) = 0$?

(a) $-\frac{1}{6}$ or 0

(b) – 1 or 0

(c) $-\frac{1}{6}$ or 1

(d) – 1 or 1

Solution:

$$f''(x) = 6x \begin{vmatrix} -1 & 0 \\ p^2 & p^3 \end{vmatrix} + \sin x \begin{vmatrix} 6 & 0 \\ p & p^3 \end{vmatrix} - \cos x \begin{vmatrix} 6 & -1 \\ p & p^2 \end{vmatrix}$$

$f''(0) = -(6p^2 + p) = -p(1+6p) = 0$

$p = 0, -\frac{1}{6}$

For the next two (2) items that follow:

Consider the function

$$f(x) = \frac{a^{[x]+x} - 1}{[x]+x}$$

Where [.] denotes the greatest integer function.

69. What is $\lim_{x \to 0^+} f(x)$ equal to ?

(a) 1

(b) $\ln a$

(c) $1 - a^{-1}$

(d) Limit does not exist

Solution: $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{a^x - 1}{x} = \ln a$

70. What is $\lim_{x \to 0^-} f(x)$ equal to?

(a) 1

(b) $\ln a$

(c) $1 - a^{-1}$

(d) Limit does not exist

Solution: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{a^{[x]+x} - 1}{[x]+x} =$

$\lim_{x \to 0^-} \frac{a^{x-1} - 1}{x-1} = \frac{a^{-1} - 1}{0-1} = 1 - a^{-1}$