1. What is the principal argument of $(-1-i)$ where $= \sqrt{-1}$?

(a) $\pi/4$

(b) $-\pi/4$

(c) $-3\pi/4$

(d) $3\pi/4$

Solution: Principal argument lies between $[-\pi, \pi]$.

$$\arg(-1-i) = \tan^{-1}\frac{-1}{-1} = -\frac{3\pi}{4}$$

Answer: (c)

2. How many numbers between 100 and 1000 can be formed with the digits 5, 6, 7, 8, 9 if the repetition of digits is not allowed?

(a) 3^5 (b) 5^3

(c) 120 (d) 60

Solution: Total number lies between 100 and 1000 = $5 \times 4 \times 3 = 60$

Answer: (d)

3. The number of non-zero integral solutions of the equation $|1-2i|^x = 5^x$ is

(a) Zero (No solution)

(b) One

(c) Two

(d) Three

Solution:

$$|1-2i| = \sqrt{5}$$

$$|1-2i|^x = 5^x$$

$$5^{\frac{x}{2}} = 5^x$$

$$\frac{x}{2} = x$$

$$x = 0$$

So number of non-zero integral solution is **ZERO**.

Answer: (a)

4. If the ratio of AM to GM of two positive numbers a and b is 5:3, then a : b is equal to

(a) $3:5$ (b) $2:9$

(c) $9:1$ (d) $5:3$

Solution: Arithmetic Mean of two positive number a and b = $\frac{a+b}{2}$

Geometric Mean of two positive number a and b = $\sqrt{ab}$

$$\frac{AM}{GM} = \frac{5}{3}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{5}{3}$$

$$Let\ \frac{a}{b} = x$$

$$\frac{bx+b}{2\sqrt{b \times bx}} = \frac{5}{3}$$

$$\frac{1+x}{2\sqrt{x}} = \frac{5}{3}$$

$$3 + 3(\sqrt{x})^2 = 10\sqrt{x}$$

$$3(\sqrt{x})^2 - 10\sqrt{x} + 3 = 0$$

$$3(\sqrt{x})^2 - 9\sqrt{x} - \sqrt{x} + 3 = 0$$

$$(3\sqrt{x}-1)(\sqrt{x}-3) = 0$$

$$\sqrt{x} = 3$$

$$x = 9$$

Answer: (c)

5. If the coefficients of a^m and a^n in the expansion of $(1+a)^{m+n}$ are α and β, then which one of the following is correct?

(a) $\alpha = 2\beta$ (b) $\alpha = \beta$

(c) $2\alpha = \beta$ (d) $\alpha = (m+n)\beta$

Solution:

$$(1+x)^n = \sum_{r=0}^{r=n} C(n,r)x^{n-r}$$

$(1+a)^{m+n} = \sum_{r=0}^{r=m+n} C(m+n,r)(a)^{m+n-r}$

Coefficient of $a^m = C(m+n,n)$

Coefficient of $a^n = C(m+n,m)$

$$C(m+n,n) = \frac{(m+n)!}{n!\,m!}$$

$$C(m+n,m) = \frac{(m+n)!}{n!\,m!}$$

Coefficient of $a^m =$ Coefficient of a^n

6. How many four-digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits?

(a) 24 (b) 36

(c) 44 (d) 64

Solution: Number which is divisible by 10 then last digit should be equal to 0.

Total number of four digit = $4 \times 3 \times 2 = 24$

Answer: (a)

7. The equation $|1-x| + x^2 = 5$ has

(a) a rational root and an irrational root

(b) two rational roots

(c) two irrational roots

(d) no real roots

Solution:

If $1 - x > 0$

$1 - x + x^2 = 5$

$x^2 - x - 4 = 0$

$$x = \frac{1 \pm \sqrt{17}}{2}$$

If $1 - x < 0$

$x - 1 + x^2 = 5$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

Answer: option (a)

8. The binary number expression of the decimal number 31 is

(a) 1111 (b) 10111

(c) 11011 (d) 11111

Solution:

		Remainder
2	31	
2	15	1
2	7	1
2	3	1
	1	1

$$(31)_{10} = (11111)_2$$

9. What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal (where i = $\sqrt{-1}$)?

(a) 0 (b) i

(c) –i (d) 1

Solution:

$$i^{1000} = 1$$
$$i^{1001} = i$$
$$i^{1002} = -1$$
$$i^{1003} = -i$$

$i^{1000} + i^{1001} + i^{1002} + i^{1003} = 0$

10 The modulus-amplitude form of $\sqrt{3} + i$, where $i = \sqrt{-1}$ is

(a) $2\left(\cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3}\right)$

(b) $2\left(\cos\frac{\pi}{6}+i\ \sin\frac{\pi}{6}\right)$

(c) $4\left(\cos\frac{\pi}{3}+i\ \sin\frac{\pi}{3}\right)$

(d) $4\left(\cos\frac{\pi}{6}+i\ \sin\frac{\pi}{6}\right)$

Solution:

$z = \sqrt{3}+i$

Magnitude of z = $\sqrt{x^2+y^2} = \sqrt{\left(\sqrt{3}\right)^2+1^2} = 2$

Argument of z = $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$$z = re^{i\theta} = 2e^{i\frac{\pi}{6}} = 2\left(\cos\frac{\pi}{6}+i\ \sin\frac{\pi}{6}\right)$$

Answer: (b)

11.If A is a 2×3 matrix and AB is a 2×5 matrix, then B must be a

(a) 3 × 5 matrix (b) 5 × 3 matrix

(c) 3 × 2 matrix (d) 5 × 2 matrix

Solution:

If AB exist then number of column of A is equal to number of rows of B. Dimension of AB is [no of row of A x no of column of B]

Dimension of B = 3 x 5

Answer: (a)

12. Let [x] denote the greatest integer function . What is the number of solutions of the equation $x^2-4x+[x]=0$ in the interval [0, 2]?

(a) Zero (No solution) (b) One

(c) Two (d) Three

Solution: $[x]=0 \quad 0\le x<1$

$[x]=1 \quad 1\le x<2$

$[x]=2 \quad x=2$

if $0\le x<1$

$x^2-4x=0$

$x=0,4$

Roots lies between 0 to 1 is 0.

if $1\le x<2$

$x^2-4x+1=0$

$$x=\frac{4}{2}\pm\frac{\sqrt{16-4}}{2}=2\pm\sqrt{3}$$

Root lies between 1 and 2 is $2-\sqrt{3}$

if x= 2

$x^2-4x+2=0$

$$x=\frac{4}{2}\pm\frac{\sqrt{16-8}}{2}=2\pm\sqrt{2}$$

Roots are 0, $2-\sqrt{3}$.

13. What is the sum of all two-digit numbers which when divided by 3 leave 2 as the remainder?

(a) 1565 (b) 15 85

(c) 1635 (d) 1655

Solution: For of number when divided by 3 we get remainder is equal to 2.

N = 3m +2

m= { 3, 4, . . . , 32}

Number of element in set m = 30

Total number of two digit number when divided by 3 get 2 remainder is equal to 30

$$\sum_{m=3}^{m=32} 3m + 2 = 3\sum m + \sum 2$$

$$= 3 \times \frac{30}{2} \times (3 + 32) + 2 \times 30$$

$$= 1635$$

14. If $\sin x = \frac{1}{\sqrt{5}}$, $\sin y = \frac{1}{\sqrt{10}}$, where $0 < x < \frac{\pi}{2}$ $0 < y < \frac{\pi}{2}$, then what is $(x + y)$ equal to?

(a) π (b) $\pi/2$

(c) $\pi/4$ (d) 0

Solution: $\sin x = \frac{1}{\sqrt{5}}$

$$\cos x = \frac{2}{\sqrt{5}}$$

$$\sin y = \frac{1}{\sqrt{10}}$$

$$\cos y = \frac{3}{\sqrt{10}}$$

$$\sin(x + y) = \sin x \cos y + \sin y \cos x$$

$$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{2}}$$

$$x + y = \frac{\pi}{4}$$

15. What is $\sin 105^0 + \cos 105^0$ equal to ?

(a) $\sin 50^0$ (b) $\cos 50^0$

(c) $1/\sqrt{2}$ (d) 0

Solution:

$$\sin 105^0 + \cos 105^0$$

$$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 105^0 + \frac{1}{\sqrt{2}}\cos 105^0\right)$$

$$\sqrt{2}(\cos 45^0 \sin 105^0 + \sin 45^0 \cos 105^0)$$

$$\sqrt{2}\sin(45^0 + 105^0)$$

$$\sqrt{2}\sin 150^0 = \sqrt{2}\sin(180^0 - 30^0)$$

$$\sqrt{2}\sin 30^0 = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

Answer: (c)

16. In a triangle ABC if $a = 2$, $b = 3$ and $Sin\ A = 2/3$, then what is angle B equal to?

(a) $\pi/4$ (b) $\pi/2$

(c) $\pi/3$ (d) $\pi/6$

Solution:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{2}{\frac{2}{3}} = \frac{3}{\sin B}$$

$$\sin B = 1$$

$$B = \frac{\pi}{2}$$

Answer: (b)

17. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

(a) $\pi/4$ (b) $\pi/2$

(c) $\pi/3$ (d) $2\pi/3$

Solution:

$$\sin\frac{2\pi}{3} = \sin 120^0 = \frac{\sqrt{3}}{2}$$

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

Answer: (c)

18. If x , x – y and x + y are the angles of a triangle (not an equilateral triangle) such that $\tan(x-y), \tan x \; and \tan(x+y)$ are in GP, then what is x equal to?

(a) $\pi/4$ (b) $\pi/3$

(c) $\pi/6$ (d) $\pi/2$

Solution:

$x + x - y + x + y = 180^0$

$x = 60^0$

Answer: (b)

19. ABC is a triangle inscribed in a circle with centre O. Let α = $\angle BAC$, where $45^0 < \alpha < 90^0$. Let $\beta = \angle BOC$. Which one of the following is correct?

(a) $\cos\beta = \frac{1-ta\ ^2\alpha}{1+tan^2\alpha}$

(b) $\cos\beta = \frac{1+tan^2\alpha}{1-tan^2\alpha}$

(c) $\cos\beta = \frac{2\tan\alpha}{1+tan^2\alpha}$

(d) $\sin\beta = 2sin^2\alpha$

Solution:

$\angle BOC = 2\angle BAC$

$\beta = 2\alpha$

$\cos\beta = \cos 2\alpha$

$\cos\beta = cos^2\alpha - sin^2\alpha$

$$\cos\beta = \frac{cos^2\alpha - sin^2\alpha}{cos^2\alpha + sin^2\alpha}$$

$$\cos\beta = \frac{\frac{cos^2\alpha - sin^2\alpha}{cos^2\alpha}}{\frac{cos^2\alpha + sin^2\alpha}{cos^2\alpha}}$$

$$\cos\beta = \frac{1 - tan^2\alpha}{1 + tan^2\alpha}$$

Answer: (b)

20. The maximum value of $\sin\left(x+\frac{\pi}{5}\right) + \cos\left(x+\frac{\pi}{5}\right)$, where $x \in \left(0, \frac{\pi}{2}\right)$, is attained at

(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{15}$

(c) $\frac{\pi}{10}$ (d) $\frac{\pi}{2}$

Solution:

$$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\left(x+\frac{\pi}{5}\right) + \frac{1}{\sqrt{2}}\cos\left(x+\frac{\pi}{5}\right)\right)$$

$$\sqrt{2}\sin\left(x+\frac{\pi}{5}+\frac{\pi}{4}\right)$$

When $\sin\left(x+\frac{\pi}{5}+\frac{\pi}{4}\right) = 1$ then f(x) will attain maximum value.

$$x + \frac{\pi}{5} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$x = \frac{(10-4-5)\pi}{20} = \frac{\pi}{20}$$

21 What is the distance between the points which divide the line segment joining (4, 3) and (5, 7) internally and externally in the ratio 2:3?

(a) $\frac{12\sqrt{17}}{5}$ (b) $\frac{13\sqrt{17}}{5}$

(c) $\frac{\sqrt{17}}{5}$ (d) $\frac{6\sqrt{17}}{5}$

Solution:

Co-ordinate of point P which divides A(4, 3) and B(5, 7) internally in the ratio 2:3.

$$x = \frac{mx_B + nx_A}{m+n} = \frac{2\times 5 + 3\times 4}{2+3} = \frac{22}{5}$$

$$y = \frac{my_B + ny_A}{m+n} = \frac{2\times 7 + 3\times 3}{2+3} = \frac{23}{5}$$

Co-ordinate of point Q which divides A(4, 3) and B(5,7) externally in the ratio 2:3

$$x = \frac{mx_B - nx_A}{m-n} = \frac{2\times 5 - 3\times 4}{2-3} = 2$$

$$y = \frac{my_B - ny_A}{m-n} = \frac{2\times 7 - 3\times 3}{2-3} = -5$$

Distance between point P($\frac{22}{5}, \frac{23}{5}$) and Q $(2, -5)$.

$$PQ = \sqrt{\left(\frac{22}{5} - 2\right)^2 + \left(\frac{23}{5} + 5\right)^2} = \frac{12\sqrt{17}}{5}$$

Answer: (a)

22. What is the equation of the line passing through the point of intersection of the lines through the point of intersection of the lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ and parallel to the line $y - x + 10 = 0$?

(a) $7x - 7y + 18 = 0$

(b) $5x - 7y + 18 = 0$

(c) $5x - 7y + 18 = 0$

(d) $x - y + 5 = 0$

Solution: Equation of line passing through line L_1: $x + 2y - 3 = 0$ and Line L_2: $2x - y + 5 = 0$ is

$$(x + 2y - 3) + \boldsymbol{\lambda}(2x - y + 5) = \mathbf{0}$$

$$(1 + 2\lambda)x + (2 - \lambda)y + 5\lambda - 3 = 0$$

Slope of line: $m = \frac{1+2\lambda}{\lambda-2}$

Slope of line: $y - x + 10 = 0$ is m = 1

$$\frac{1 + 2\lambda}{\lambda - 2} = 1$$

$$1 + 2\lambda = \lambda - 2$$

$$\lambda = -3$$

Substitute $\lambda = -3$ in equation of line $(1 + 2\lambda)x + (2 - \lambda)y + 5\lambda - 3 = 0$ we get

$$5x - 7y + 18 = 0$$

23. **C**onsider the following statements:

1. The length p of the perpendicular from the origin to the origin to the line $ax + by = c$ satisfies the relation $p^2 = \frac{c^2}{a^2+b^2}$.

2. The length p of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$ satisfies the relation $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

3. The length p of the perpendicular from the origin to the line $y = mx + c$ satisfies the relation

$$\frac{1}{p^2} = \frac{1 + m^2 + c^2}{c^2}.$$

Which of the above is/are correct?

(a) 1, 2 and 3

(b) 1 only

(c) 1 and 2 only

(d) 2 only

Solution: Perpendicular distance from point $P(x_1, y_1)$ on line $ax + by + c = 0$ is d.

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

If point P is origin $x_1 = 0\ and\ y_1 = 0$

$$p = \frac{|c|}{\sqrt{a^2 + b^2}}$$

Equation of line $\frac{x}{a} + \frac{y}{b} = 1$

$$p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Equation of line $y = mx + c$

$$p = \frac{c}{\sqrt{1 + m^2}}$$

$$\frac{1}{p^2} = \frac{1 + m^2}{c^2}$$

24. What is the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are at $(\pm 4, 0)$?

(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(b) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(c) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

(d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Solution: Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Co-ordinate of vertices are $(\pm a\ ,\ 0)$.

Co-ordinate of foci are $(\pm ae, 0)$.

$a = 5$ and $ae = 4$

$b^2 = a^2(1 - e^2) = 25 - 16 = 9$

Equation of ellipse is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

25. What is the equation of the straight line passing through the point (2, 3) and making an intercept on the positive y-axis equal to twice its intercept on the positive x-axis?

(a) $2x + y = 5$

(b) $2x + y = 7$

(c) $x + 2y = 7$

(d) $2x - y = 1$

Solution: General equation of line in intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\mathrm{a = x - intercept}$

$\mathrm{b = y - intercept}$

Given $\mathrm{b} = 2\mathrm{a}$ and line passing through (2, 3)

$$\frac{2}{a} + \frac{3}{b} = 1$$

$$\frac{2}{a} + \frac{3}{2a} = 1$$

$$\frac{7}{2a} = 1$$

$$a = \frac{7}{2}$$

$$\frac{2x}{7} + \frac{y}{7} = 1$$

$$2x + y = 7$$

26. What is the equation of the plane passing through the points (-2, 6, -6), (-3, 10, -9) and (-5, 0, -6) ?

(a) $2x - y - 2z = 2$

(b) $2x + y + 3z = 3$

(c) $x + y + z = 6$

(d) $x - y - z = 3$

Solution:

Plane P_1: $2x - y - 2z = 2$

If point P(-2, 6, -6) lies on plane P_1 then it should satisfy equation of plane.

$$(2 \times -2) - 6 - (2 \times -6) - 2$$
$$= -4 - 6 + 12 - 2 = 0$$

If point Q(-3, 10, -9) lies on plane P_1 then it should satisfy equation of plane.

$$(2 \times -3) - 10 - (2 \times -9) - 2$$
$$= -6 - 10 + 18 - 2 = 0$$

If point R(-5, 0, -6) lies on plane P_1 then it should satisfy equation of plane.

$$(2 \times -5) - 0 - (2 \times -6) - 2$$
$$= -10 + 12 - 2 = 0$$

Since all three point lies on plane P_1.

27. What is the equation to the sphere whose centre is at (-2, 3, 4) and radius is 6 units?

(a) $x^2 + y^2 + z^2 + 4x - 8y - 8z = 7$

(b) $x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$

(c) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 4$

(d) $x^2 + y^2 + z^2 + 4x + 6y + 8z = 4$

Solution: Equation of sphere whose centre is (x_0 , y_0, z_0) and radius R is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

Centre C (-2, 3, 4) and Radius R = 6

$$(x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$
$$x^2 + y^2 + z^2 + 4x - 8y - 8z = 7$$

28. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ and $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \lambda(\vec{b} \times \vec{c})$ then what is the value of λ?

(a) 2 (b) 3

(c) 4 (d) 6

Solution:

$$\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$$
$$\vec{a} = -2\vec{b} - 3\vec{c}$$
$$\vec{a} \times \vec{b} = -2\vec{b} \times \vec{b} - 3\vec{c} \times \vec{b}$$
$$\vec{a} \times \vec{b} = 3(\vec{b} \times \vec{c})$$
$$\vec{c} \times \vec{a} = \vec{c} \times (-2\vec{b}) + \vec{c} \times (-3\vec{c})$$
$$\vec{c} \times \vec{a} = 2(\vec{b} \times \vec{c})$$
$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \lambda(\vec{b} \times \vec{c})$$
$$3(\vec{b} \times \vec{c}) + \vec{b} \times \vec{c} + 2(\vec{b} \times \vec{c}) = \lambda(\vec{b} \times \vec{c})$$
$$\lambda = 6$$

29. if the vector $\vec{k}$ and $\vec{A}$ are parallel to each other, then what is $k\vec{k} \times \vec{A}$ equal to?

(a) $k^2\vec{A}$ (b) $\vec{0}$

(c) $-k^2\vec{A}$ (d) $\vec{A}$

Solution:

If $\vec{k}$ *and* $\vec{A}$ are parallel to each other therefore angle between them is equal to zero . Cross product of these two vector is equal to zero vector.

$k\vec{k}$ is also parallel to $\vec{k}$ vector . So $k\vec{k} \times \vec{A} = \vec{0}$

30. Suppose $f: R \to R$ is defined by $f(x) = \frac{x^2}{1+x^2}$. What is the range of the function?

(a) [0, 1) (b) [0, 1]

(c) (0, 1] (d) (0, 1)

Solution: $f(x) = \frac{x^2}{1+x^2}$. Denominator is always greater than numerator. So f(x) should be less than 1.

31. What is the area of the region bounded by the parabola $y^2 = 6(x - 1)$ and $y^2 = 3x$?

(a) $\frac{\sqrt{6}}{3}$

(b) $\frac{2\sqrt{6}}{3}$

(c) $\frac{4\sqrt{6}}{3}$

(d) $\frac{5\sqrt{6}}{3}$

Solution: Find the point of intersection of curves $y^2 = 6(x - 1)$ and $y^2 = 3x$

$$6(x - 1) = 3x$$

$$x = 2$$

Area bounded by the curves = $2\left[\int_0^2 \sqrt{3x}\, dx - \int_1^2 \sqrt{6(x - 1)}dx\right] = \frac{4\sqrt{6}}{3}$

32. If $(x) = \frac{x^2-3}{x^2-2x-3}$, $x \neq 3$ is continuous x =3, then which one of the following is correct?

(a) $f(3) = 0$ (b) $f(3) = 1.5$

(c) $f(3) = 3$ (d) $f(3) = -1.5$

Solution: $f(x) = \frac{x^2-3}{x^2-2x-3}$

$$\lim_{x\to 3}\frac{x^2-3}{x^2-2x-3} = \lim_{x\to 3}\frac{2x}{2x-2} = \frac{2\times 3}{2\times 3-2} = \frac{6}{4} = 1.5$$

Answer: (b)

33. What is $\int_1^e x \ln x \, dx$ equal to?

(a) $\frac{e+1}{4}$ (b) $\frac{e^2+1}{4}$

(c) $\frac{e-1}{4}$ (d) $\frac{e^2-1}{4}$

Solution: $\int_1^e x \ln x \, dx$

$$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\times\frac{1}{x}dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4}$$

$$= \frac{e^2}{2}\ln e - \frac{e^2}{4} - \frac{1}{2}\ln 1 + \frac{1}{4}$$

$$= \frac{e^2+1}{4}$$

34. What is $\int_0^{\sqrt{2}}[x^2]\,dx$ equal to (where [.] is the greatest integer function) ?

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$

(c) $2(\sqrt{2}-1)$ (d) $\sqrt{3}-1$

Solution: $\int_0^{\sqrt{2}}[x^2]\,dx = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx = \sqrt{2}-1$

35. What is the maximum value of $16\sin\theta - 12sin^2\theta$?

(a) $3/4$ (b) $4/3$

(c) $16/3$ (d) 4

Solution:

$f(\theta) = 16\sin\theta - 12sin^2\theta$

$f'(\theta) = 16\cos\theta - 24\sin\theta\cos\theta$

$f'(\theta) = \cos\theta\,(16 - 24\sin\theta)$

$f'(\theta) = 0$

$\cos\theta\,(16 - 24\sin\theta) = 0$

$\cos\theta = 0$

$16 - 24\sin\theta = 0$

$\sin\theta = \frac{16}{24} = \frac{2}{3}$

$f(\theta) = 16\sin\theta - 12sin^2\theta$

$$f(\theta) = 16\times\frac{2}{3} - 12\times\frac{4}{9} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

$f(\theta) = 16 - 12 = 4$

Answer: (c)

36. For f to be a function, what is the domain of if $(x) = \frac{1}{\sqrt{|x|-x}}$?

(a) $(-\infty,\ 0)$ (b) $(0,\ \infty)$

(c) $(-\infty,\ \infty)$ (d) $(-\infty, 0]$

Solution: $f(x) = \frac{1}{\sqrt{|x|-x}}$

$$f(x) = \frac{1}{\sqrt{-2x}} \quad x < 0$$

$$for\ x > 0\ f(x) = \frac{1}{\sqrt{x-x}} = \frac{1}{\sqrt{0}}$$

Domain of function is $(-\infty,\ 0)$

37. What is the solution of the differential equation $xdy - ydx = 0$?

(a) $xy = c$ (b) $y = cx$

(c) $x + y = c$ (d) $x - y = c$

Solution:

$xdy - ydx = 0$

$$\frac{xdy}{xy}-\frac{ydx}{xy}=0$$

$$\frac{dy}{y}-\frac{dx}{x}=0$$

Integrate this differential equation we get,

$$\ln y-\ln x=lnc$$

$$\ln\frac{y}{x}=\ln c$$

$$\mathrm{y}=\mathrm{cx}$$

Answer: (b)

38. What is the derivative of the function $f(x)=\ e^{\tan}\ \ +\ln(\sec x)-e^{\ln x}$ at $x=\frac{\pi}{4}$?

(a) $e/2$ (b) e

(c) $2e$ (d) $4e$

Solution: $\frac{dy}{dx}=\ e^{\tan x}sec^2x+\frac{1}{\sec x}\times$ $\sec x\tan x-e^{\ln x}\times\ \frac{1}{x}$

$$\frac{dy}{dx}=2e+1-1=2e$$

39. What is the period of the function $f(x)=sin\ x$?

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) 2π

Answer: (d)

40. What is $\lim_{x\to0}\frac{tan\ x}{\sin 2x}$ equal to?

(a) $1/2$ (b) 1

(c) 2 (d) Limit does not exist

Solution:

$$\lim_{x\to0}\frac{tan\ x}{\sin 2x}=\lim_{x\to0}\frac{x}{2x}=\frac{1}{2}$$

Answer: (a)

41. What is $\lim_{h\to0}\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$ equal to?

(a) $\frac{1}{2\sqrt{2x}}$ (b) $\frac{3}{\sqrt{2x}}$

(c) $\frac{3}{2\sqrt{2x}}$ (d) $\frac{3}{4\sqrt{2x}}$

Solution:

$$\lim_{h\to0}\frac{\sqrt{2x+3h}\ -\sqrt{2x}}{2h}$$

$$\lim_{h\to0}\frac{\sqrt{2}\left(\sqrt{x+\frac{3h}{2}}-\sqrt{x}\right)}{\frac{3h}{2}\times\frac{2}{3}\times2}=\frac{3\sqrt{2}}{4}\times\frac{1}{2\sqrt{x}}$$

$$=\frac{3}{4\sqrt{2x}}$$

42. What is the solution of

$(1+2x)dy-(1-2y)dx=0$?

(a) $x-y-2xy=c$

(b) $y-x-2xy=c$

(c) $y+x-2xy=c$

(d) $x+y+2xy=c$

Solution:

$$(1+2\mathrm{x})\mathrm{dy}-(1-2\mathrm{y})\mathrm{dx}=0$$

$$\frac{\mathrm{dy}}{1-2\mathrm{y}}=\frac{\mathrm{dx}}{1+2\mathrm{x}}$$

Integrate both sides we get,

$$\int\frac{\mathrm{dy}}{1-2\mathrm{y}}=\int\frac{\mathrm{dx}}{1+2\mathrm{x}}$$

Le $1-2\mathrm{y}=\mathrm{u}$

$$-2\mathrm{dy}=\mathrm{du}$$

$$\int\frac{\mathrm{dy}}{1-2\mathrm{y}}=\int\frac{\mathrm{du}}{-2\mathrm{u}}=\frac{\ln\mathrm{u}}{-2}=\frac{\ln(1-2\mathrm{y})}{-2}$$

$$\int\frac{\mathrm{dx}}{1+2\mathrm{x}}=\int\frac{\mathrm{dv}}{2\mathrm{v}}=\frac{\ln\mathrm{v}}{2}=\frac{\ln(1+2\mathrm{x})}{2}$$

$$\frac{\ln(1-2\mathrm{y})}{-2}=\frac{\ln(1+2\mathrm{x})}{2}+\ln\mathrm{c}$$

$$\frac{\ln(1+2\mathrm{x})(1-2\mathrm{y})}{2}+\ln\mathrm{c}=0$$

$$\ln(1-2\mathrm{y}+2\mathrm{x}-4\mathrm{xy})=\ln\frac{1}{\mathrm{c}^2}$$

$$1-2\mathrm{y}+2\mathrm{x}-4\mathrm{xy}=\mathrm{constant}$$

$$2(\mathrm{x}-\mathrm{y}-2\mathrm{xy})=\mathrm{constant}$$

$$\mathrm{x}-\mathrm{y}-2\mathrm{xy}=\mathrm{constant}$$

43. What is the median of the numbers 4.6, 0, 9.3 , - 4.8, 7.6, 2.3, 12.7, 3.5, 8.2, 6.1, 3.9, 5.2 ?

(a) 3.8 (b) 4.9

(c) 5.7 (d) 6.0

Solution: Arrange the numbers is ascending order

1	-4.8
2	0
3	2.3
4	3.5
5	3.9
6	4.6
7	5.2
8	6.1
9	7.6
10	8.2
11	9.3
12	12.7

Median = $\frac{4.6+5.2}{2} = 4.9$

44. A train covers the first 5 km of its journey at speed of 30 km/hr and the next 15 km at speed of 45 km/hr. What is the average speed of the train?

(a) 35 km/hr (b) 37.5 km/hr

(c) 39.5 km/hr (d) 40 km/hr

Solution: $Average\ speed = \frac{Total\ distance}{Total\ time} = \frac{5+15}{\frac{5}{30}+\frac{15}{45}} = \frac{20}{\frac{15+3}{90}} = 40\ km/hr$

114. Two fair dice are rolled. What is probability of getting a sum of 7?

(a) $1/36$ (b) $1/6$

(c) $7/12$ (d) $5/12$

Solution: Number of sample space = 36

Set of number whose sum is equal to 7 = $\{(1,6)\ (2,5)(3,4)\ (4,3)\ (5,2)\ (6,1)\}$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

45. One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the chance that it is white?

(a) $3/8$ (b) $49/80$

(c) 8/13 (d) ½

Solution: Let Bag A contains 3 white and 2 black balls and Bag B contains 5 white and 3 black balls.

Probability of selecting Bag A = $\frac{1}{2}$

P(W/A) = Probability of white ball when bag A is selected

P(W/A) = $\frac{1}{2} \times \frac{3}{5}$

Probability of selecting Bag B = $\frac{1}{2}$

P(W/B) = Probability of white ball when bag B is selected

P(W/B) = $\frac{1}{2} \times \frac{5}{8}$

P(W) = P(W/A) + P(W/B) = $\frac{3}{10} + \frac{5}{16} = \frac{49}{80}$

46. The tird term of a GP is 3. What is the product of the first five terms?

(a) 216

(b) 226

(c) 243

(d) Cannot be determined due to insufficient data

Solution:

First term of G.P. is a and common ratio is r.

Third term $= ar^2 = 3$

Product of first five terms

$= a \times ar \times ar^2 \times ar^3 \times ar^4 = a^5 \times r^{10} = (ar^2)^5 = 3^5 = 81 \times 3 = 243$

47. What is the equation of the straight line cutting off an intercept 2 from the negative direction of y-axis and inclined at 30^0 with the positive direction of x-axis?

(a) $x - 2\sqrt{3}y - 3\sqrt{2} = 0$

(b) $x + 2\sqrt{3}y - 3\sqrt{2} = 0$

(c) $x+\sqrt{3}y-2\sqrt{3}=0$

(d) $x-\sqrt{3}y-2\sqrt{3}=0$

Solution: Slope intercept form of equation of line is $\mathrm{y}=\mathrm{mx}+\mathrm{c}$

Slope of line m = $\tan 30^0 = \frac{1}{\sqrt{3}}$

y-intercept of line $c = -2$

$$y=\frac{1}{\sqrt{3}}x-2$$

$$\sqrt{3}y=x-2\sqrt{3}$$

$$x-\sqrt{3}y-2\sqrt{3}=0$$

48. Let the coordinates of the points A, B, C be (1, 8, 4), (0, -11, 4) and (2, -3, 1) respectively. What are the coordinates of the point D which is the foot of the perpendicular from A on BC?

(a) $(3,4,-2)$ (b) (4, -2, 5)

(c) (4, 5, -2) (d) (2, 4, 5)

Solution: Let coordinate of point D (x, y, z)

$$\overrightarrow{AD} = (x-1)\hat{\imath}+(y-8)\hat{\jmath}+(z-4)\hat{k}$$

$$\overrightarrow{BC} = (2-0)\hat{\imath}+(-3+11)\hat{\jmath}+(1-4)\hat{k}$$

$$\overrightarrow{BC} = 2\hat{\imath}+8\hat{\jmath}-3\hat{k}$$

Since AD is perpendicular to BC. So dot product is equal to zero.

$$\overrightarrow{AD}.\overrightarrow{BC} = 2(x-1)+8(y-8)-3(z-4) = 0$$

$$2x-2+8y-64-3z+12=0$$

$$2x+8y-3z=44$$

point (3, 4, -2) satisfy above equation.

49. What is the distance between the straight lines $3x+4y=9$ and $6x+8y=15$?

(a) $3/2$ (b) $3/10$

(c) 6 (d) 5

Solution:

Lines $3x+4y=9$ and $6x+8y=15$ are parallel to each other.

Distance between line

$$d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=\frac{\left|9-\frac{15}{2}\right|}{\sqrt{3^2+4^2}}=\frac{3}{2\times 5}=\frac{3}{10}$$

50. What is the equation of the straight line cutting off an intercept 2 from the negative direction of y-axis and inclined at 30^0 with the positive direction of x-axis?

(a) $x-2\sqrt{3}y-3\sqrt{2}=0$

(b) $x+2\sqrt{3}y-3\sqrt{2}=0$

(c) $x+\sqrt{3}y-2\sqrt{3}=0$

(d) $x-\sqrt{3}y-2\sqrt{3}=0$

Solution: Equation of line in slope intercept form

$$y=mx+c$$

$$m=\tan\theta = \tan 30^0 = \frac{1}{\sqrt{3}}$$

$$y\ intercept\ \ c = -2$$

$$y=\frac{1}{\sqrt{3}}x-2$$

$$x-\sqrt{3}y-2\sqrt{3}=0$$

51. The coordinates of the vertices P, Q and R of a triangle PQR are (1, -1, 1), (3, -2, 2) and (0, 2, 6) respectively. If $\angle RQP=\theta$, then what is $\angle PRQ$ equal to?

(a) $30^0+\theta$ (b) $45^0-\theta$

(c) $60^0-\theta$ (d) $90^0-\theta$

Solution:

$$PQ = \sqrt{(1-3)^2+(-1+2)^2+(1-2)^2} = \sqrt{6}$$

$$QR = \sqrt{(3-0)^2+(-2-2)^2+(2-6)^2} = \sqrt{41}$$

$$PR = \sqrt{(1-0)^2+(-1-2)^2+(1-6)^2} = \sqrt{35}$$

$$PQ^2+PR^2=QR^2$$

Triangle PQR is right angled.

$\angle P = 90^0,\ \angle Q=\theta$ and $\angle R = 90^0-\theta$

52. The equation of the line, when the portion of it intercepted between the axes is divided by the point (2, 3) in the ratio of 3 : 2 , is

(a) Either $x+y=4$or $9x+y=12$

(b) Either $x+y=5$or $4x+9y=30$

(c) Either $x+y=4$or $x+9y=12$

(d) Either $x+y=5$ or $9x+4y=30$

Solution:

Let equation of Line is

$$\frac{x}{a}+\frac{y}{b}=1$$

Let A is intercept of line at x-axis and B is intercept of line at y-axis.

A(a, 0) and B(0, b)

If Point P(2, 3) divide AB in the ratio of 3 : 2.

$$x_p=\frac{mx_A+nx_B}{m+n}$$

$$2=\frac{3\times a+2\times 0}{3+2}$$

$$a=\frac{10}{3}$$

$$y_p=\frac{my_A+ny_B}{m+n}$$

$$3=\frac{3\times 0+2\times b}{3+2}$$

$$b=\frac{15}{2}$$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{3x}{10}+\frac{2y}{15}=1$$

$$\frac{9x+6y}{30}=1$$

$$9x+4y=30$$

53. What is the moment about the point $\hat{\imath}+2\hat{\jmath}-\hat{k}$ of a force represented by $\widehat{3\imath}+\hat{k}$ acting through the point $\widehat{2\imath}-\hat{\jmath}+3\hat{k}$?

(a) $-3\hat{\imath}+11\hat{\jmath}+9\hat{k}$

(b) $3\hat{\imath}+2\hat{\jmath}+9\hat{k}$

(c) $3\hat{\imath}+4\hat{\jmath}+9\hat{k}$

(d) $\hat{\imath}+\hat{\jmath}+\hat{k}$

Solution: $Moment\ \vec{T}=\vec{r}\times\vec{F}$

$$\vec{r}=(2-1)\hat{\imath}+(-1-2)\hat{\jmath}+(3+1)\hat{k}$$

$$\hat{r}=\hat{\imath}-3\hat{\jmath}+4\hat{k}$$

$$\vec{F}=\widehat{3\imath}+\hat{k}$$

$$\vec{T}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -3 & 4\\ 3 & 0 & 1\end{vmatrix}$$

$$\vec{T}=-3\hat{\imath}+11\hat{\jmath}+9\hat{k}$$

54. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}|=2, |\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{\imath}+2\hat{\jmath}+6\hat{k},$ then what is the acute angle between $\vec{a}$ and $\vec{b}$?

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

Solution: $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

$$|\vec{a}\times\vec{b}|=\sqrt{3^2+2^2+6^2}=\sqrt{9+4+36}=7$$

$$|\vec{a}||\vec{b}|\sin\theta=7$$

$$\sin\theta=\frac{7}{2\times 7}=\frac{1}{2}$$

Acute angle between $\vec{a}$ and $\vec{b}$ are vectors is 30^0.

55. Which one of the following differential equations has a periodic solution?

(a) $\frac{d^2x}{dt^2}+\mu x=0$

(b) $\frac{d^2x}{dt^2}-\mu x=0$

(c) $x\frac{dx}{dt}+\mu t=0$

(d) $\frac{dx}{dt}+\mu xt=0$

where $\mu>0$

Solution: $\vec{a}\ +\ \omega^2x=0$

This is equation of simple harmonic motion.

$\vec{a}=\frac{d^2x}{dt^2}$ and $\omega^2=\mu$

56. What is the value of $\int_{-\pi/4}^{\pi/4}(sin\ x-tanx)\ dx$?

(a) $-\frac{1}{\sqrt{2}}+\ln\left(\frac{1}{\sqrt{2}}\right)$ (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) $\sqrt{2}$

Solution:

$$\int_{-\pi/4}^{\pi/4}(sin\ x-tanx)\ dx$$

$$\int\sin x\,dx=-\cos x$$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = -\ln \cos x$$

$$\int_{-\pi/4}^{\pi/4} (\sin x - \tan x)\, dx$$

$$= -\cos x + \ln \sec x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 0$$

57. What is the angle between the straight lines $(m^2 - mn)y = (mn + n^2)x + n^3$ and $(mn + m^2)y = (mn - n^2)x + m^3$, where $m > n$?

(a) $\tan^{-1}\left(\frac{2mn}{m^2+n^2}\right)$

(b) $\tan^{-1}\left(\frac{4m^2n^2}{m^4-n^4}\right)$

(c) $\tan^{-1}\left(\frac{4m^2n^2}{m^4+n^4}\right)$

(d) 45^0

Solution: Angle between two line

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Slope of line L_1 : $(m^2 - mn)y = (mn + n^2)x + n^3$

$$m_1 = \frac{mn + n^2}{m^2 - mn} = \frac{n}{m} \times \frac{m+n}{m-n}$$

Slope of line L_2: $(mn + m^2)y = (mn - n^2)x + m^3$

$$m_2 = \frac{mn - n^2}{mn + m^2} = \frac{n}{m} \times \frac{m-n}{m+n}$$

$$m_1 m_2 = \frac{n^2}{m^2}$$

$$m_1 - m_2 = \frac{n}{m}\left(\frac{(m+n)^2 - (m-n)^2}{m^2 - n^2}\right)$$

$$= \frac{n}{m}\left(\frac{4mn}{m^2 - n^2}\right)$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{n}{m}\left(\frac{4mn}{m^2 - n^2}\right)}{1 + \frac{n^2}{m^2}}$$

$$= \frac{n}{m} \times \frac{4mn}{m^4 - n^4} \times m^2$$

$$= \frac{4m^2n^2}{m^4 - n^4}$$

$$\theta = \tan^{-1}\left(\frac{4m^2n^2}{m^4 - n^4}\right)$$

58. The third term of a GP is 3. What is the product of the first five terms?

(a) 216 (b) 226

(c) 243 (d) Cannot be determined due to insufficient data

Solution:

Let first term of GP = a

Let common ratio of GP = r

Third term of GP = ar^2

Product of first five terms = $a \times ar \times ar^2 \times ar^3 \times ar^4 = a^5 r^{10} = (ar^2)^5 = 3^5 = 243$

59. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by which one of the following?

(a) 1904 (b) 2000

(c) 2002 (d) 2006

Solution: Take n =1

$$121^n - 25^n + 1900^n - (-4)^n$$

$$= 121 - 25 + 1900 + 4$$

$$= 96 + 1904 = 2000$$

Number is divisible by 2000.

60. If $n = (2007)!$, then what is

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{2017} n}$$

equal to?

(a) 0 (b) 1

(c) $\frac{n}{2}$ (d) n

Solution: Logarithmic properties

$$\log_a b = \frac{1}{\log_b a}$$

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots+\frac{1}{\log_{2017} n}$$
$$=\log_n 2+\log_n 3+\log_n 4+\ldots+\log_n 2017$$
$$=\log_n(2\times 3\times 4\times..\times 2017)=\log_n 2017!$$
$$=\log_n n=1$$

61. In the expansion of $(1+x)^{43}$, if the coefficients of $(2r+1)^{th}$ and $(r+2)^{th}$ terms are equal, then what is the value of r $(r\neq 1)$?

(a) 5 (b) 14
(c) 21 (d) 22

Solution:

Coefficient of $(2r+1)^{th}=C(43,2r)$
Coefficient of $(r+2)^{th}=C(43,r+1)$
$C(n,r)=C(n,n-r)$
$C(43,2r)=C(43,r+1)$
$2r+r+1=43$
$r=14$

62. Let α and β be real numbers and z be complex number. If $z^2+\alpha z+\beta=0$ has two distinct non-real roots with $Re(z)=1$,then it is necessary that

(a) $\beta\in(-1,0)$ (b) $|\beta|=1$
(c) $\beta\in(1,\infty)$ (d) $\beta\in(0,1)$

Solution: Roots of $z^2+\alpha z+\beta=0$ has two distinct non-real roots with $Re(z)=1$.

$z_1=1+iy$
$z_2=1-iy$
$z_1+z_2=2$
$z_1z_2=1+y^2=\beta$
$\beta\geq 1$

63. What is
$$\frac{1}{\log_2 N}+\frac{1}{\log_3 N}+\frac{1}{\log_4 N}+\ldots+\frac{1}{\log_{100} N}$$
equal to $(N\neq 1)$?

(a) $\frac{1}{\log_{100!} N}$ (b) $\frac{1}{\log_{99!} N}$
(c) $\frac{99}{\log_{100!} N}$ (d) $\frac{99}{\log_{99!} N}$

Solution:

$$\frac{1}{\log_2 N}+\frac{1}{\log_3 N}+\frac{1}{\log_4 N}+\ldots+\frac{1}{\log_{100} N}$$
$$=\log_N 2+\log_N 3+\log_N 4+\ldots+\log_N 100$$
$$=\log_N(2\times 3\times 4\times..\times 100)=\log_N 100!$$
$$=\frac{1}{\log_{100!} N}$$

64. If $x=1-y+y^2-y^3+\ldots$ up to infinite terms, where $|y|<1$, then which one of the following is correct?

(a) $x=\frac{1}{1+y}$ (b) $x=\frac{1}{1-y}$
(c) $x=\frac{y}{1+y}$ (d) $x=\frac{y}{1-y}$

Solution: Sum of infinite G.P series
$$x=\frac{a}{1-r}=\frac{1}{1+y}$$

65. What is the inverse of the matrix

$$A=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}?$$

(a) $\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

(b) $\begin{pmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{pmatrix}$

(c) $\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{pmatrix}$

(d) $\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

Solution: Cofactor matrix of A

$$C_{11}=(-1)^{1+1}\begin{vmatrix}\cos\theta & 0\\ 0 & 1\end{vmatrix}=\cos\theta$$

$$C_{12}=(-1)^{1+2}\begin{vmatrix}-\sin\theta & 0\\ 0 & 1\end{vmatrix}=\sin\theta$$

$$C_{13}=(-1)^{1+3}\begin{vmatrix}-\sin\theta & \cos\theta\\ 0 & 0\end{vmatrix}=0$$

$$C_{21}=(-1)^{2+1}\begin{vmatrix}\sin\theta & 0\\ 0 & 1\end{vmatrix}=-\sin\theta$$

$C_{22} = (-1)^{2+2}\begin{vmatrix}\cos\theta & 0\\ 0 & 1\end{vmatrix} = \cos\theta$

$C_{23} = (-1)^{2+3}\begin{vmatrix}\cos\theta & -\sin\theta\\ 0 & 0\end{vmatrix} = 0$

$C_{31} = (-1)^{3+1}\begin{vmatrix}\sin\theta & 0\\ \cos\theta & 0\end{vmatrix} = 0$

$C_{32} = (-1)^{3+2}\begin{vmatrix}\cos\theta & 0\\ -\sin\theta & 0\end{vmatrix} = 0$

$C_{33} = (-1)^{3+3}\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}$

$= cos^2\theta + sin^2\theta = 1$

Cofactor matrix C

$$= \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Ad joint of matrix A = transpose of matrix C

$$Adj(A) = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\text{Det (A)} = \begin{vmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{vmatrix} = 1$$

$$A^{-1} = \frac{Adjoint\ of\ matrix\ A}{determinat\ of\ matrix\ A}$$

$$= \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

66. What is the number of triangles that can be formed by choosing the vertices from a set of 12 points in a plane, seven of which lie on the same straight line?

(a) 185 (b) 175

(c) 115 (d) 105

Solution: Number of triangle formed by choosing the vertices from a set of 12 points in a plane,seven of which lie on the same straight line is

(a) Select three point form 5 points which are non-collinear.

Number of ways =

$$C(5,3) = \frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = 10$$

(b) Select two points from 5 non-collinear points and one point from 7 collinear points.

Number of ways

$= C(5,2) \times C(7,1) = 10 \times 7 = 70$

(c) Select one point from 5 non-collinear points and two points from 7 collinear points.

Number of ways

$= C(5,1) \times C(7,2) = 5 \times 21 = 105$

Total number of triangle

= 10 +70 +105 =185

67. A survey of 850 students in a University that 680 students like music and 215 like dance. What is the least number of students who like both music and dance?

(a) 40 (b) 45

(c) 50 (d) 55

Solution:

Let Set A is number of students like music.

Set B is number of students like dance.

$n(A) = 680$

$n(B) = 215$

$n(A \cup B) \leq Total\ number\ of\ students.$

$n(A) + n(B) - n(A \cap B) \leq 850$

$680 + 215 - 850 \leq n(A \cap B)$

$45 \leq n(A \cap B)$

Minimum number of students who like both music and dance is 45.

68. if $0 < a < 1$, the value of $\log_{10} a$ is negative. This is justified by

(a) Negative power of 10 is less than 1

(b) Negative power of 10 is between 0 and 1

(c) Negative power of 10 is positive

(d) Negative power of 10 is neagtive

Answer: (b)

69. If x, 3/2, z are in AP; x, 3, z are in GP; then which one of the following will be in HP?

(a) x, 6, z (b) x, 4, z

(c) x, 2, z (d) x, 1, z

Solution:

If x, 3/2, z are in AP

$$2 \times \frac{3}{2} = x + z$$

$$x + z = 3$$

If x, 3, z are in GP

$$3^2 = xz$$

$$zx = 9$$

Let x , b, z are in HP.

$$\frac{2}{b} = \frac{1}{x} + \frac{1}{z} = \frac{x+z}{xz} = \frac{3}{9} = \frac{1}{3}$$

$$b = 6$$

70. What is the value of the sum

$$\sum_{n=2}^{11} (i^n + i^{n+1}),$$

where $i = \sqrt{-1}$?

(a) i (b) $2i$

(c) $-2i$ (d) $1 + i$

Solution:

$$\sum_{n=2}^{11} (i^n + i^{n+1}) = \sum_{n=2}^{11} i^n + \sum_{n=2}^{11} i^{n+1}$$

$$\sum_{n=2}^{11} i^n = \frac{a(r^n - 1)}{r - 1} = \frac{i^2(i^{10} - 1)}{i - 1}$$

$$= \frac{-1((-1)^5 - 1)}{i - 1} = \frac{2}{i - 1}$$

$$\sum_{n=2}^{11} i^{n+1} = \frac{i^3(i^{10} - 1)}{i - 1} = \frac{-i(-2)}{i - 1} = \frac{2i}{i - 1}$$

$$\sum_{n=2}^{11} i^n + \sum_{n=2}^{11} i^{n+1} = \frac{2(1+i)}{(1-i)}$$

$$= \frac{2(1+i)(1+i)}{1 - i^2}$$

$$= 1 + i^2 + 2i = 2i$$

71. If a flag-staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then what is the angle that the sun makes with the ground?

(a) 60^0 (b) 45^0

(c) 30^0 (d) 15^0

Solution:

$$\tan\theta = \frac{height\ of\ tower}{Shado\quad leng} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\theta = 60^0$$