1. The value of x, satisfying the equation $\log_{\cos x}\sin x = 1$, where $0 < x < \frac{\pi}{2}$, is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Solution: $\log_{\cos x}\sin x = 1$

$$\log_{cosx}\sin x = \log_{\cos x}\cos x$$

$$\sin x = \cos x$$

$$x = \frac{\pi}{4}$$

2. If $C_0, C_1, C_2, \dots, C_n$ are the coefficients in the expansion of $(1+x)^n$, then what is the value of $C_1 + C_2 + C_3 \dots + C_n$?

(a) 2^n (b) $2^n - 1$

(c) 2^{n-1} (d) $2^n - 2$

Solution:

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots + C_n x^n$$

Put $x = 1$

$$2^n = C_0 + C_1 + C_2 + C_3 + \dots + C_n$$

$$2^n - C_0 = C_1 + C_2 + C_3 + \dots + C_n$$

$$C_0 = \frac{n!}{0!\,n!} = 1$$

$$2^n - 1 = C_1 + C_2 + C_3 + \dots + C_n$$

3. If the roots of the quadratic equation $x^2 + 2x + k = 0$ are real, then

(a) $k < 0$ (b) $k \le 0$

(c) $k < 1$ (d) $k \le 1$

Solution:

$$x^2 + 2x + k = 0$$

$$D = b^2 - 4ac = 4 - 4k > 0$$

$$1 > k$$

4. If $Z = 1 + i$, where $i = \sqrt{-1}$, then what is the modulus of $Z + \frac{2}{Z}$?

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

$$z = 1 + i$$

$$\frac{1}{z} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1-i^2}$$

$$\frac{1}{z} = \frac{1-i}{2}$$

$$\frac{2}{z} = 1 - i$$

$$z + \frac{2}{z} = 1 + i + 1 - i = 2$$

5. If $\tan x = -\frac{3}{4}$ and x is in the second quadrant, then what is the value of $sinx.cosx$?

(a) $\frac{6}{25}$

(b) $\frac{12}{25}$

(c) $-\frac{6}{25}$

(d) $-\frac{12}{25}$

Solution:

$$\tan x = -\frac{3}{4}$$

$$\sin x = \frac{3}{5}$$

$$\cos x = -\frac{4}{5}$$

$$\sin x \cos x = -\frac{12}{25}$$

6. What is the value of the following?

$$cosec\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right)$$

(a) $\frac{4}{3}$

(b) 4

(c) -4

(d) $-\frac{4}{\sqrt{3}}$

Solution:

$$\operatorname{cosec}\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right)$$

$$\sin\left(\frac{7\pi}{6}\right) = \sin(\pi + \frac{\pi}{6}) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(2\pi - \frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\operatorname{cosec}\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right) = \frac{1}{\sin\left(\frac{7\pi}{6}\right)\times\cos\left(\frac{5\pi}{3}\right)}$$

$$= -4$$

7. If the determinant $\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$ then what is x equal to?

(a) -2 or 2

(b) -3 or 3

(c) -1 or 1

(d) 3 or 4

Solution:

$$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$$

$$-\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 0$$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

8. A chord subtends an angle 120^0 at the centre of a unit circle. What is the length of the chord?

(a) $\sqrt{2} - 1$ units

(b) $\sqrt{3} - 1$ units

(c) $\sqrt{2}$ units

(d) $\sqrt{3}$ units

Solution: Let O is the centre of the circle and AB is the chord of the circle.

$$\cos\angle AOB = \frac{OA^2 + OB^2 - AB^2}{2\times OA\times OB}$$

$$\cos 120^0 = \frac{1^1 + 1^2 - x^2}{2\times1\times1}$$

$$-\frac{1}{2} = \frac{2 - x^2}{2}$$

$$x = \sqrt{3}$$

9. What is $(1 + cot\theta - cosec\theta)(1 + tan\theta + sec\theta)$ equal to ?

(a) 1

(b) 2

(c) 3

(d) 4

Solution:

$$(1 + cot\theta - cosec\theta)(1 + tan\theta + sec\theta)$$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta - 1)(\cos\theta + \sin\theta + 1)}{\sin\theta\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta} = 2$$

10. What is $\frac{1+tan^2\theta}{1+cot^2\theta} - \left(\frac{1-tan\theta}{1-cot}\right)^2$ equal to ?

(a) 0

(b) 1

(c) $2tan\theta$

(d) $2cot\theta$

Solution:

$$\frac{B^2 + P^2}{\frac{P^2 + B^2}{P^2}} \times \frac{1}{B^2} - \frac{(B-P)^2}{(P-B)^2} \times \frac{P^2}{B^2} = \frac{P^2}{B^2} - \frac{P^2}{B^2}$$

$$= 0$$

11. What is the interior angle of a regular octagon of side length 2 cm?

(a) $\frac{\pi}{2}$

(b) $\frac{3\pi}{4}$

(c) $\frac{3\pi}{5}$

(d) $\frac{3\pi}{8}$

Solution:

$$8\theta = 2\pi$$

$$\theta = \frac{\pi}{4}$$

$$\pi = 2\alpha + \frac{\pi}{4}$$

$$2\alpha = \frac{3\pi}{4}$$

12. If $7\sin\theta + 24cos\theta = 25$, then what is the value of $(sin\theta + cos\theta)$?

(a) 1

(b) $\frac{26}{25}$

(c) $\frac{6}{5}$

(d) $\frac{31}{25}$

Solution:

$$7\sin\theta + 24\cos\theta = 25$$

$$\frac{7}{25}\sin\theta + \frac{24}{25}\cos\theta = 1$$

$$\cos\varphi = \frac{7}{25}$$

$$\sin\varphi = \frac{24}{25}$$

$$\cos\varphi\sin\theta + \sin\varphi\cos\theta = 1$$

$$\sin(\theta + \varphi) = 1$$

$$\theta + \varphi = \frac{\pi}{2}$$

$$\sin\theta + \cos\theta = \sin\left(\frac{\pi}{2} - \varphi\right) + \cos\left(\frac{\pi}{2} - \varphi\right)$$

$$= \cos\varphi + \sin\varphi = \frac{7}{25} + \frac{24}{25}$$

$$= \frac{31}{25}$$

13. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 75^0. What is the height of the flagstaff?

(a) 12 m

(b) 9 m

(c) $(6 + \sqrt{3})$ m

(d) $(6 + 3\sqrt{3})$ m

Solution:

$$\tan 75^0 = \frac{H}{B}$$

$$(H - 6)^2 + B^2 = 36$$

$$(H - 6)^2 + \frac{H^2}{tan^2 75^0} = 36$$

$$H^2 - 12H + \frac{H^2}{tan^2 75^0} = 0$$

$$H^2(1 + cot^2 75^0) - 12H = 0$$

$$H = \frac{12}{1 + cot^2 75^0} = 12sin^2 75^0$$

$$= 12\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2$$

$$= 3(2 + \sqrt{3})$$

$$\sin 75^0 = \sin(45^0 + 30^0)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

14. The shadow of a tower is found to be x meter longer, when the angle of elevation of the sun changes from 60^0 to 45^0. If the height of the tower is $5(3 + \sqrt{3})$ m, then what is x equal to?

(a) 8 m

(b) 10 m

(c) 12 m

(d) 15 m

Solution:

$$\tan 60^0 = \frac{5(3+\sqrt{3})}{B}$$

$$B = \frac{5(3+\sqrt{3})}{\sqrt{3}}$$

$$5(3+\sqrt{3}) - \frac{5(3+\sqrt{3})}{\sqrt{3}} = x$$

$$5(3+\sqrt{3}) - 5(\sqrt{3}+1) = x$$

$x = 10$

$f(x) = x^2 - 5x + 6$

15. If $3\cos\theta = 4\sin\theta$, then what is the value of $\tan(45^0 + \theta)$?

(a) 10

(b) 7

(c) $\frac{7}{2}$

(d) $\frac{7}{4}$

Solution:

$$3\cos\theta = 4\sin\theta$$

$$\tan\theta = \frac{3}{4}$$

$$\tan(45^0 + \theta) = \frac{\tan 45^0 + \tan\theta}{1 - \tan 45^0 \tan\theta}$$

$$= \frac{1 + \tan\theta}{1 - \tan\theta} = \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = 7$$

16. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ holds, when

(a) $x \in R$

(b) $x \in R - \{-1,1\}$ only

(c) $x \in R - \{0\}$ only

(d) $x \in R - \{-1,1\}$ only

17. If $f(x+1) = x^2 - 3x + 2$, then what is f(x) equal to?

(a) $x^2 - 5x + 4$

(b) $x^2 - 5x + 6$

(c) $x^2 + 3x + 3$

(d) $x^2 - 3x + 1$

Solution:

$$f(x+1) = x^2 - 3x + 2$$

Let $x + 1 = t$

$$f(x+1) = x^2 - 3x + 2 = x^2 - 2x - x + 2$$
$$= (x-1)(x-2)$$

$$f(t) = (t-1-1)(t-1-2)$$
$$= (t-2)(t-3)$$
$$= t^2 - 5t + 6$$

18. If $x^2, x, -8$ are in AP, then which one of the following is correct?

(a) $x \in \{-2\}$

(b) $x \in \{4\}$

(c) $x \in \{-2,4\}$

(d) $x \in \{-4, 2\}$

Solution:

$x^2, x, -8$ are in AP

$$x - x^2 = -8 - x$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = -2, 4$$

19. The third term of a GP is 3. What is the product of its first five terms?

(a) 81

(b) 243

(c) 729

(d) Cannot be determined due to insufficient data

Solution:

The third term of a GP is 3.

Let first term is a and common difference is r

$t_1 = a,\ t_2 = ar, t_3 = ar^2$, $t_4 = ar^3$ and $t_5 = ar^4$

$$t_1t_2t_3t_4t_5 = a^5r^{1+2+3+4} = a^5r^{10} = (t_3)^5$$
$$= 3^5 = 243$$

20. The element in the $i\,th$ row and the $j\,th$ column of a determinant of third order is equal to 2(I + j). What is the value of the determinant?

(a) 0

(b) 2

(c) 4

(d) 6

Solution: $a_{ij} = 2(i + j)$

$$A = \begin{bmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{vmatrix} = 8\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= 8\left\{2\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 3\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} + 4\begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}\right\}$$

$$= 8\big(2 \times (24 - 25) - 3 \times (18 - 20) + 4(15 - 16)\big)$$

$=8(-2 + 6 - 4) = 0$

21. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?

(a) 14 units

(b) 10.5 units

(c) 7 units

(d) 3.5 units

Solution:

Equation of circle

$$4x^2 + 4y^2 - 20x + 12y - 15 = 0$$

$$x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$$

$$x^2 - 2 \times \frac{5}{2}x + \left(\frac{5}{2}\right)^2 + y^2 + 2 \times \frac{3}{2}y + \left(\frac{3}{2}\right)^2 - \frac{15}{4} - \frac{9}{4} - \frac{25}{4} = 0$$

$$\left(x - \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{49}{4}$$

Radius of circle is $\frac{7}{2}$

22. A parallelogram has three consecutive vertices $(-3, 4), (0, -4)$ and $(5, 2)$. The fourth vertex is

(a) (2, 10)

(b) (2, 9)

(c) (3, 9)

(d) (4, 10)

Solution:

Vertex of parallelogram are A(-3, 4), B(0, -4), C(5, 2) and D(x, y)

Diagonal of parallelogram are bisect each other.

x- coordinate of AC = $\frac{-3+5}{2} = 1$

x- coordinate of BD = $\frac{0+x}{2} = \frac{x}{2}$

$$\frac{x}{2} = 1$$

$$x = 2$$

y-cordinate of AC = $\frac{4+2}{2} = 3$

y-coordinate of BD = $\frac{-4+y}{2}$

$$\frac{y - 4}{2} = 3$$

$$y = 10$$

D(2, 10)

23. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct?

(a) $pq + 1 = 0$

(b) $p + q + 1 = 0$

(c) $pq - 1 = 0$

(d) $p - q + 1 = 0$

Solution: Line $y + px = 1$ and $y - qx = 2$ are perpendicular to each other.

$$m_1 m_2 = -1$$

$-pq = -1$

$pq - 1 = 0$

24. If A, B and C are in AP, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is

(a) (0, 0)
(b) (-1, 1)
(c) (1, -2)
(d) (1, -1)

Solution:

if A , B and C are in AP then $2B = A + C$

$$A - 2B + C = 0$$

Straight line $Ax + 2By + C = 0$ passes through fixed point (1, -1)

25. If the image of the point (-4, 2) by a line mirror is (4, -2), then what is the equation of the line mirror?

(a) y =x
(b) $y = 2x$
(c) $4y = x$
(d) $y = 4x$

Solution:

Image of point P(-4, 2) by a line mirror is Q(4, -2). Mid point of PQ lies on straight line.

$x = 0$

$y = 0$

Slope of PQ = $\frac{2+2}{-4-4} = -\frac{1}{2}$

Slope of line = 2

Equation of line y = 2x

26. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$?

(a) 0^0
(b) 30^0
(c) 45^0
(d) 60^0

Solution: $x - 2 = 0$ and $\sqrt{3}x - y - z = 0$

$$\theta = 30^0$$

27. The point of intersection of diagonals of a square ABCD is at the origin and one of its vertices is at A(4,2). What is the equation of the diagonal BD?

(a) $2x + y = 0$
(b) $2x - y = 0$
(c) $x + 2y = 0$
(d) $x - 2y = 0$

Solution: Slope of AC = $\frac{2}{4} = \frac{1}{2}$

Slope of BD = -2

$y = -2x$

28. If any point on a hyperbola is (3 tan Θ , 2 secΘ), then what is the eccentricity of the hyperbola?

(a) $\frac{3}{2}$
(b) $\frac{5}{2}$
(c) $\frac{\sqrt{11}}{2}$
(d) $\frac{\sqrt{13}}{2}$

Solution: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{9tan^2\theta}{a^2} - \frac{4sec^2\theta}{b^2} = 1$$

$a^2 = 9$

$b^2 = 4 = 9(e^2 - 1)$

$\frac{4}{9} = e^2 - 1$

$$\frac{13}{9} = e^2$$

$$e = \frac{\sqrt{13}}{3}$$

29. Consider the following with regard to eccentricity (e) of a conic section :

1. $e = 0$ for circle
2. $e = 1$ for parabola
3. $e < 1$ for ellipse

Which of the above are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Solution: For parabola , eccentricity e =1

For ellipse, eccentricity e < 1

For circle, e =0

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

For circle $a^2 = b^2$

$$a^2 = a^2(1 - e^2)$$

$$e = 0$$

30. What is the angle between the two lines having direction ratios (6, 3, 6) and (3, 3, 0)?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

Solution: Direction ratios are (6, 3, 6) and (3, 3, 0)

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{6 \times 3 + 3 \times 3 + 6 \times 0}{\sqrt{6^2 + 3^2 + 6^2}\sqrt{3^2 + 3^2}} = \frac{27}{\sqrt{81}\sqrt{18}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

31. If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to?

(a) 1

(b) 11

(c) $\frac{13}{27}$

(d) 4

Solution: $x - 1 = 2(y + 3) = 1 - z$

$$\frac{x-1}{1} = \frac{y+3}{\frac{1}{2}} = \frac{z-1}{-1}$$

$$a = 1,\ b = \frac{1}{2} and\ c = -1$$

$$k^2 + \frac{k^2}{4} + k^2 = 1$$

$$\frac{9k^2}{4} = 1$$

$$k = \pm\frac{2}{3}$$

$$l = \frac{2}{3}, m = bk = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} and\ n = -\frac{2}{3}$$

$$l^4 + m^4 + n^4 = \left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)^4 + \left(-\frac{2}{3}\right)^4 = \frac{11}{27}$$

32. What is the projection of the line segment joining A(1, 7, -5) and B(-3, 4, -2) on y-axis?

(a) 5

(b) 4

(c) 3

(d) 2

Solution: $A(1,7,-5)\ B(-3,4,-2)$

$$\overrightarrow{AB} = (-3-1)\hat{\imath} + (4-7)\hat{\jmath} + (-2+5)\hat{k}$$

$$= -4\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$$

Projection $\overrightarrow{AB}$ on y-axis

$$= |\vec{r}_{AB}.\hat{\jmath}| = |-3| = 3$$

33. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is

(a) $(0, 1, 2)$

(b) (0, 0, 3)

(c) (1, 1, 1)

(d) (-1, 1, 3)

Solution: $x + y + z = 3$

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

$$x = y = z = k$$

$$3k = 3$$

$$k = 1$$

(1, 1, 1)

34. The curve $y = -x^3 + 3x^2 + 2x - 27$ has the maximum slope at

(a) $x = -1$

(b) $x = 0$

(c) $x = 1$

(d) $x = 2$

Solution: $y = -x^3 + 3x^2 + 2x - 27$

$$y' = -3x^2 + 6x + 2$$

maximum value of quadratic equation occurs at $x = -\frac{b}{2a}$ if a <0

$$x = -\frac{6}{-6} = 1$$

maximum slope $y'_{max} = -3 + 6 + 2 = 5$

35. If $f(x) = e^{|x|}$, then which one of the following is correct?

(a) $f'(0) = 1$

(b) $f'(0) = -1$

(c) $f'(0) = 0$

(d) $f'(0)$ does not exist

Solution: $f(x) = e^{|x|}$

$$y = e^x \quad x > 0$$

$$= e^{-x}, \quad x < 0$$

$$y' = e^x \quad x > 0$$

$$= -e^{-x}, \quad x < 0$$

$$y'(0^+) = 1$$

$$y'(0^-) = -1$$

36. What is $\int \frac{dx}{sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$

(b) $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$

(d) $\cos^{-1} x + c$

Solution:

$$\int \frac{dx}{sec^2(\tan^{-1} x)}$$

$$\sec^2\theta - tan^2\theta = 1$$

$$sec^2\theta = 1 + tan^2\theta$$

$$\tan\tan^{-1} x = x$$

$$\int \frac{dx}{1 + x^2} = \tan^{-1} x + c$$

37. If $x + y = 20$ and $P = xy$, then what is the maximum value of P?

(a) 100 (b) 96

(c) 84 (d) 50

Solution: $x + y = 20$

$$P = x(20 - x)$$

$$\frac{dP}{dx} = 20 - 2x$$

$$0 = 20 - 2x$$

$$x = 10$$

$$P = 100$$

38. If $x = e^t cost$ and $= e^t sint$, then what is $\frac{dx}{dy}$ at t =0 equal to?

(a) 0

(b) 1

(c) 2e

(d) -1

Solution: $x = e^t \cos t$

$y = e^t \sin t$

$$\frac{dx}{dt} = e^t \cos t - \sin t \, e^t$$

$$\frac{dy}{dt} = e^t \sin t + \cos t \, e^t$$

$$\frac{dx}{dy} = \frac{\cos t - \sin t}{\cos t + \sin t}$$

at t =0,

$$\frac{dx}{dy} = 1$$

39. What is the maximum value of $sin2x.cos2x$?

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 4

Solution: $y = \sin 2x \,.\cos 2x$

$$y = \frac{1}{2}\sin 4x$$

$$y_{max} = \frac{1}{2}$$

40. If the function $f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$ is continuous, then what is the value of (a+b)?

(a) 5 (b) 10

(c) 15 (d) 20

Solution: $a + b = 5$

41. What is the domain of the function $f(x) = 3^x$?

(a) $(-\infty, \infty)$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(-\infty, \infty) - \{0\}$

Solution: $y = 3^x$

Domain $(-\infty, \infty)$

42. What is the degree of the following differential equation?

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

(a) 1

(b) 2

(c) 3

(d) Degree is not defined

Solution: $x^2 = 1 + \frac{d^2y}{dx^2}$

Order = 2

Degree = 1

43. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + y = 1$

(d) $\frac{dy}{dx} - y = 0$

44. What is the solution of the following differential equation? $ln\left(\frac{dy}{dx}\right) + y = x$

(a) $e^x + e^y = c$

(b) $e^{x+y} = c$

(c) $e^x + e^y = c$

(d) $e^{x-y} = c$

Solution:

$$\ln\left(\frac{dy}{dx}\right) + y = x$$

$$\frac{dy}{dx} = e^{x-y}$$

$$e^y dy = e^x dx$$

$$e^x - e^y = c$$

45. What is $\int e^{(2lnx+\ln x^2)}dx$ equal to?

(a) $\frac{x^4}{4} + c$

(b) $\frac{x^4}{3} + c$

(c) $\frac{2x^5}{5} + c$

(d) $\frac{x^5}{5} + c$

Solution: $\int e^{2lnx}.e^{\ln x^2}dx = \int x^2x^2dx = \frac{x^5}{5} + c$

46. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is

(a) 810 (b) 900

(c) 30000 (d) 81000

Solution: $GM = \sqrt[10]{x_1x_2....x_n}$

$$GM' = \sqrt[10]{3^{10}(x_1x_2....x_n)^4} = 3 \times (GM)^4$$

$$= 3 \times 10^4 = 30000$$

47. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, then which of the following is/are correct?

1. A and B are independent events.
2. A and B are mutually exclusive events.

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

$P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$

$P(B) = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$P(A).P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} = P(A \cap B)$

if $P(A \cap B) = P(A).P(B)$ then A and B are called as independent events.

For mutually exclusive event $P(A \cup B) = P(A) + P(B)$

But $P(A \cap B) \neq 0$ therefore A and B are not mutually exclusive events.

48. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

(a) reduced by $\frac{1}{3}$

(b) increased by $\frac{10}{3}$

(c) reduced by $\frac{10}{3}$

(d) reduced by 50

Solution: $\bar{x} = \frac{\sum_{i=1}^{i=15} x_i}{15}$;

$$\bar{x}' = \frac{\sum x_i}{15} = \bar{x} - \frac{50}{15} = \bar{x} - \frac{10}{3}$$

49. A coin is tossed twice. If E and F demote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{1}{3}$

Solution: If a coin is tossed twice set of sample space S = {HH, HT, TH, TT}

Event E of occurrence of Head in first toss and Tail in second toss E = {HT}

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

50. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10 (b)12

(c) 13 (d) 15

Solution:

Mode is equal maximum frequency of observation.

frequency of 10, 12, 13 are 2. So frequency of 15 should be equal to three to become mode of observations.

x =15

51. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements:

1. The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.
2. The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution: $P(A) = \frac{3}{4} = 0.75$

$$P(B) = \frac{5}{8} = 0.625$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since $P(A) > P(B)$ therefore for minimum value of $P(A \cup B)$, maximum value of $P(A \cap B)$ should occur. It happens when B is subset of set A. $P(A \cap B) = P(B)$

$[P(A \cup B)]_{min} = P(A) = \frac{3}{4}$

$[P(A \cap B)]_{max} = P(B) = \frac{5}{8}$

52. What is the area of the triangle ABC with sides a = 10 cm, c =4 cm and angle B =30^0?

(a) 16 cm^2

(b) 12 cm^2

(c) 10 cm^2

(d) 8 cm^2

Solution:

Area of the triangle ABC

$= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 10 \times 4 \times \sin 30^0$

= 10 cm^2

53. Consider the following statements:

1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Solution: If a set has n elements, then number of power set is equal to 2^n.

Number of power sets = $2^n = 2^{10} = 1024$

54. Let R be a relation defined as xRy if and only if and only if $2x + 3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R?

(a) 2

(b) 3

(c) 4

(d) 6

Solution:

Relation R = $\{((1,6), (4,4), (7,2))\}$

55. Consider the following in respect of a complex number Z :

1. $\overline{(Z^{-1})} = (\bar{Z})^{-1}$
2. $ZZ^{-1} = |Z|^2$

Which of the above is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution:

$Z^{-1} = \frac{1}{Z}$

$\overline{(Z^{-1})} = \frac{1}{\bar{Z}} = (\bar{Z})^{-1}$

$|Z|^2 = Z\bar{Z}$

56. Consider the following statements in respect of an arbitrary complex number Z:

1. The difference of Z and its conjugate is an imaginary number.
2. The sum of Z and its conjugate is a real number.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution: $Z = x + iy$

$\bar{Z} = x - iy$

$Z - \bar{Z} = 2iy = an\ imaginary\ number$

$Z + \bar{Z} = 2x = a\ real\ number$

57. The sides of a triangle are m, n and $\sqrt{m^2 + n^2 + mn}$. What is the sum of the acute angles of the triangle?

(a) 45^0
(b) 60^0
(c) 75^0
(d) 90^0

Solution: Largest side of the triangle is $\sqrt{m^2 + n^2 + mn}$. Angle opposite to largest side is largest angle. Let γ is the largest angle of the triangle.

$$\cos\gamma = \frac{m^2 + n^2 - \left(\sqrt{m^2 + n^2 + mn}\right)^2}{2mn} = -\frac{1}{2}$$

$$\gamma = 120^0$$

Sum of the acute angle is = $180^0 - 120^0 = 60^0$

58. If $\tan A = \frac{1}{7}$, then what is $cos2A$ equal to?

(a) $\frac{24}{25}$
(b) $\frac{18}{25}$
(c) $\frac{12}{25}$
(d) $\frac{6}{25}$

Solution: $\tan A = \frac{1}{7} = \frac{p}{b}$

$h = \sqrt{p^2 + b^2} = \sqrt{50}$

$\sin A = \frac{1}{\sqrt{50}}$

$\cos A = \frac{7}{\sqrt{50}}$

$cos2A = cos^2A - sin^2A = \frac{49}{50} - \frac{1}{50} = \frac{48}{50}$

$= \frac{24}{25}$

59. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$?

(a) -1
(b) 1
(c) $\sqrt{2}$
(d) 2

Solution: $z = i^{2n+1}(-i)^{2n-1}$,

$|z| = |i|^{2n+1}|-i|^{2n-1} = 1 \times 1 = 1$

60. A vector $\vec{r} = a\hat{\imath} + b\hat{\jmath}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively?

(a) $\frac{1}{2}, \frac{1}{2}$
(b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(c) $\sqrt{2}, \sqrt{2}$
(d) 2, 2

Solution: If vector $\vec{r} = a\hat{i} + b\hat{j}$ are equally inclined then $|a| = |b|$.

$$|\vec{r}| = \sqrt{|a|^2 + |b|^2} = \sqrt{2}|a| = 2$$

$$|a| = \sqrt{2}$$

$$|b| = \sqrt{2}$$

61. The equation $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ has

(a) no solution

(b) unique solution

(c) two solutions

(d) infinite number of solutions

Solution:

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$2\sin^{-1} x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\sin^{-1} x = \frac{\pi}{3}$$

$$x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

62. What is the value of the following?

$(\sin 24^0 + \cos 66^0)(\sin 24^0 - \cos 66^0)$

(a) -1 (b) 0

(c) 1 (d) 2

Solution:

$\cos 66^0 = \cos(90^0 - 24^0) = \sin 24^0$

$(\sin 24^0 + \sin 24^0)(\sin 24^0 - \sin 24^0) = 0$

63. What is the value of the following?

$\tan 31^0 \tan 33^0 \tan 35^0 \ldots \tan 57^0 \tan 59^0$

(a) -1 (b) 0

(c) 1 (d) 2

Solution:

$\tan 31^0 \tan 33^0 \tan 35^0 \ldots \tan 57^0 \tan 59^0$

$\tan 59^0 = \tan(99^0 - 31^0) = \cot 31^0$

$\tan 57^0 = \tan(99^0 - 33^0) = \cot 33^0$

$\tan 31^0 \tan 33^0 \tan 35^0 \ldots \tan 57^0 \tan 59^0 = 1$

64. If

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

the what is $f(-1) + f(0) + f(1)$ equal to?

(a) 0 (b) 1

(c) 100 (d) -100

Solution:

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

$$f(x) = (x+1)x(x-1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3(x) & 2(x-2) & x1 \end{vmatrix}$$

$f(-1) = f(0) = f(1) = 0$

65. What is the sum of the coefficients of first and last terms in the expansion of $(1 + x)^{2n}$, where n is a natural number?

(a) 1 (b) 2

(c) n (d) 2n

Solution:

Coefficient of first term = $C(2n, 0) = 1$

Coefficient of last term = C(2n, 2n) = 1

Sum of first and last term = 2

66. Consider the following statements :

1. If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.

2. If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

Let first term of GP is a

Common ratio of GP is r.

GP series a, ar, ar^2, ar^3 , . .

Let k is multiply by each term.

ak, ark, ar^2k, ar^3k

Given series is GP.

67. What is the number of possible values of k for which the line joining the points (k, 1, 3) and (1, -2, k+1) also passes through the points (15, 2, -4) ?

(a) Zero

(b) One

(c) Two

(d) Infinite

Solution: Equation of line

$$\frac{x-k}{k-1}=\frac{y-1}{1+2}=\frac{z-3}{3-k-1}$$

Equation of line passes through (15, 2, -4)

$$\frac{15-k}{k-1}=\frac{2-1}{1+2}=\frac{-4-3}{3-k-1}$$

$$\frac{15-k}{k-1}=\frac{1}{3}$$

$$45-3k=k-1$$

$$k=\frac{46}{4}=\frac{23}{2}$$

$$\frac{2-1}{1+2}=\frac{-4-3}{3-k-1}$$

$$\frac{1}{3}=\frac{-7}{2-k}$$

$$2-k=-21$$

$$k=23$$

$$\frac{15-k}{k-1}=\frac{-4-3}{3-k-1}$$

$$(15-k)(2-k)=-7(k-1)$$

$$30-15k-2k+k^2=-7k+7$$

$$k^2-10k+23=0$$

$$k=\frac{10}{2}\pm\frac{\sqrt{100-92}}{2}=5\pm\frac{\sqrt{8}}{2}=5\pm\sqrt{2}$$

No solution.

68. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ holds, when

(a) $x\in R$

(b) $x\in R-\{-1,1\}$ only

(c) $x\in R-\{0\}$ only

(d) $x\in R-\{-1,1\}$ only

Solution:

Let $\tan^{-1}x=\theta$

$\tan\theta=x$

Range of $\tan^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$$\tan\theta=\cot\left(\frac{\pi}{2}-\theta\right)=x$$

$$\cot^{-1}x=\frac{\pi}{2}-\theta$$

$$\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$$

$$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$$

Domain of x is $[-\infty,+\infty]$

69. Consider the following statements:

1. A = {1, 3, 5} and B = {2, 4, 7} are equivalent sets.

2. A ={ 1, 5, 9} and B ={1, 5, 5, 9, 9} are equal sets.

Which of the above statements is /are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Answer: (d)

70. Consider the following statements:

1. A function $f:Z\to Z$, defined by $f(x)=x+1$, is one-one as well as onto.

2. A function $f:N\to N$, defined by $f(x)=x+1$, is one-one but not onto

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Answer: (c)

71. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player?

(a) 42 (b) 35

(c) 21 (d) 20

Solution:

Number of ways =

$$C(7,5) = \frac{7!}{5!\,2!} = \frac{7\times 6}{2} = 21$$

72. What is $C(n,1)+C(n,2)+\ldots+C(n,n)$ equal to?

(a) $2+2^2+2^3+\ldots+2^n$

(b) $1+2+2^2+2^3+\ldots+2^n$

(c) $1+2+2^2+2^3+\ldots+2^{n-1}$

(d) $2+2^2+2^3+\ldots+2^{n-1}$

Solution:

$2^n - 1 = C(n,1)+C(n,2)+\ldots+C(n,n)$

$1+2+2^2+2^3+\ldots+2^{n-1}$

$= \frac{r^n-1}{r-1}$

$= \frac{2^n-1}{2-1}$

$= 2^n - 1$

73. If the first term of an AP is 2 and sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms?

(a) -500 (b) -250

(c) 500 (d) 250

Solution:

First term of an AP = a = 2

$S_5 = \frac{5(t_1+t_5)}{2}$

$S_{10} = \frac{10(t_1+t_{10})}{2}$

$S_5 = \frac{1}{4}(S_{10}-S_5)$

$5S_5 = S_{10}$

$5\times\frac{5}{2}\times(2a+4d) = 5(2a+9d)$

$10a+20d = 4a+18d$

$2d = -6a = -12$

$d = -6$

$$S_{10} = \frac{10(2a+9d)}{2} = 5\big(4+(9\times -6)\big)$$

$$= -250$$

74. if $n = 100!$, then what is the value of the following?

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots+\frac{1}{\log_{100} n}$$

(a) 0 (b)1

(c) 2 (d) 3

Solution:

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots+\frac{1}{\log_{100} n}$$

$$\log_n 2+\log_n 3+\log_n 4+\ldots+\log_n 100$$

$$= \log_n 100! = \log_n n = 1$$

75. If A and B are two matrices such that AB is of order n x n, then which one of the following is correct?

(a) A and B should be square matrices of same order.

(b) Either A or B should be a square matrix.

(c) Both A and B should be same order.

(d) Orders of A and B need not be the same.

Solution: (d)

76. If A and B are square matrices of order 2 such that $\det(AB) = \det(BA)$, then which one of the following is correct?

(a) A must be a unit matrix

(b) B must be a unit matrix.

(c) Both A and B must be unit matrices.

(d) A and B need not be unit matrices.

Solution: (d)

77. What is $cot2xcot4x - cot4xcot6x - cot6xcot2x$ equal to?

(a) -1 (b) 0

(c) 1 (d) 2

Solution:

$$cot2xcot4x - cot4xcot6x - cot6xcot2x$$

$$\cot 4x\,(\cot 2x - \cot 6x) - \cot 6x \cot 2x$$

$$\cot 4x \times \frac{\cos 2x \sin 6x - \cos 6x \sin 2x}{\sin 2x\ \sin 6x} - \frac{\cos 6x \cos 2x}{\sin 6x \sin 2x}$$

$$\frac{cos4x}{sin4x} \times \frac{\sin 4x}{\sin 6x \sin 2x} - \frac{\cos 6x \cos 2x}{\sin 6x \sin 2x}$$

$$\frac{\cos(6x-2x) - \cos 6x \cos 2x}{\sin 6x \sin 2x}$$

$$= \frac{\cos 6x \cos 2x + \sin 6x \sin 2x - \cos 6x \cos 2x}{\sin 6x \sin 2x}$$

$$= 1$$

78. Consider the following statements in respect of a vector $\vec{c} = \vec{a} + \vec{b}$, where $|\vec{a}| = |\vec{b}| \neq 0$:

1. $\vec{c}$ is perpendicular to $(\vec{a} - \vec{b})$.
2. $\vec{c}$ is perpendicular to $(\vec{a} \times \vec{b})$.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution: $\vec{c}$ is perpendicular to $(\vec{a} \times \vec{b})$

$$\vec{c}.(\vec{a} \times \vec{b}) = (\vec{a} + \vec{b}).(\vec{a} \times \vec{b})$$

$$= \vec{a}.(\vec{a} \times \vec{b}) + \vec{b}.(\vec{a} \times \vec{b})$$

$$\vec{a}.(\vec{a} \times \vec{b}) = 0$$

$$\vec{b}.(\vec{a} \times \vec{b}) = 0$$

Since $\vec{a} \times \vec{b}$ is perpendicular to vectors $\vec{a}$ and $\vec{b}$. S0 dot product should equal to zero.

$\vec{c}$ is perpendicular to $(\vec{a} - \vec{b})$

$\vec{c}.(\vec{a} - \vec{b}) = (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 0$

79. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| = 4$, then which one of the following is correct?

(a) $\vec{a}$ and $\vec{b}$ must be unit vectors.
(b) $\vec{a}$ must be parallel to $\vec{b}$.
(c) $\vec{a}$ must be perpendicular to $\vec{b}$.
(d) $\vec{a}$ must be equal to $\vec{b}$.

Solution: $|\vec{a} + \vec{b}| = 4$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 16$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} = 16$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$4\vec{a}.\vec{b} = 0$$

80. if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, then what is $(2\vec{a} \times 3\vec{b}).4\vec{c} + (5\vec{b} \times 3\vec{c}).6\vec{a}$ equal to?

(a) 114
(b) 66
(c) 0
(d) -66

Solution:

$$(2\vec{a} \times 3\vec{b}).4\vec{c} + (5\vec{b} \times 3\vec{c}).6\vec{a}$$

if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar then scalar triple product is equal to zero.

$$(\lambda_1\vec{a} \times \lambda_2\vec{b}).\lambda_3\vec{c} = 0$$

Similarly

$$(5\vec{b} \times 3\vec{c}).6\vec{a} = 0$$

81. Consider the following statements:

1. The cross product of two unit vector is always a unit vector.
2. The dot product of two unit vectors is always unity.
3. The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are not correct?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

Solution:

Let two unit vector $\vec{a}$ and $\vec{b}$. Cross product of unit vector

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\,\hat{n} = \sin\theta\,\hat{n}$

θ *is angle between vector a and b.*

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \cos\theta$

If vector a and vector b is consider two side of the triangle then sum of side is always greater than difference of their side.

82. A particle starts from origin with a velocity (in m/s) given by the equation $\frac{dx}{dt} = x + 1$. The time (in second) taken by the particle to traverse a distance of 24 m is

(a) ln24

(b) $\ln 5$

(c) $2\ln 5$

(d) $2\ln 4$

Solution: $\frac{dx}{dt} = x + 1.$

$$\frac{dx}{x+1} = dt$$

$$\int_0^{24} \frac{dx}{x+1} = \int_0^t dt$$

$$\ln(x+1)|_0^{24} = t$$

$$t = 2\ln 5$$

83. What is $\int_0^a \frac{f(a-x)}{f(x0+f(a-x)} dx$ equal to ?

(a) a

(b) 2a

(c) 0

(d) $\frac{a}{2}$

Solution:

$$I = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)} dx$$

$$I = \int_0^a \frac{f(x)}{f(a-x)+f(x)} dx$$

$$2I = \int_0^a \frac{f(a-x)+f(x)}{f(x)+f(a-x)} dx = a$$

$$I = \frac{a}{2}$$

84. What is

$$\lim_{x\to -1} \frac{x^3+x^2}{x^2+3x+2}$$

equal to ?

(a) 0

(b) 1

(c) 2

(d) 3

Solution:

$$\lim_{x\to -1} \frac{x^3+x^2}{x^2+3x+2}$$

$$\lim_{x\to -1} \frac{3x^2+2x}{2x+3} = \frac{3-2}{-2+3} = 1$$

85. What is the derivative of $\sin(lnx) + \cos(lnx)$ with respect to x at x = e?

(a) $\frac{cos1-sin1}{e}$

(b) $\frac{sin1-cos1}{e}$

(c) $\frac{cos1+sin1}{e}$

(d) 0

Solution:

$$y = \sin(lnx) + \cos(lnx)$$

$$\frac{dy}{dx} = \frac{cos(\ln x)}{x} - \frac{sin(\ln x)}{x}$$

$$\frac{dy}{dx} = \frac{\cos 1 - \sin 1}{e}$$

86. Consider the following statements in respect of the function $f(x) = \sin x$:

1. f(x) increases in the interval $(0, \pi)$.
2. f(x) decreases in the interval $\left(\frac{5\pi}{2}, 3\pi\right)$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution: $f(x) = \sin x$

Function f(x) is increasing in interval $(0,\ \pi/2)$

Function f(x) is decreasing in interval $(\pi/2\ .\pi)$

f(x) decreases in the interval $\left(\frac{5\pi}{2}, 3\pi\right)$

87. What is the degree of the following differential equation?

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

(a) 1

(b) 2

(c) 3

(d) Degree is not defined

Solution:

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

$$1 + \frac{d^2y}{dx^2} = x^2$$

Order is maximum derivative in differential equation. Degree is power on highest derivative.

Order = 2 and degree = 1

88. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + y = 1$

(d) $\frac{dy}{dx} - y = 0$

Solution: (b)

89. The numbers of Science, Arts and Commerce graduates working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates?

(a) 36^0

(b) 72^0

(c) 120^0

(d) 168^0