NDA-1 2023 Math Solution

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1. If ω is a non-real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) 1 (d) $\frac{4}{\sqrt{3}}$

Solution: $x^3 = 1$

Cubic roots of the unity are 1, ω and ω^2

Sum of roots: $1 + \omega + \omega^2 = 0$

$$\omega + \omega^2 = -1$$

Let $z = \left|\frac{1-\omega}{\omega+\omega^2}\right| = \left|\frac{1-\omega}{-1}\right| = |1-\omega|$

$$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$1 - \omega = \frac{3}{2} - i\frac{\sqrt{3}}{2}$$

$$|1-\omega| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

Answer: (a)

2. What is the number of 6-digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6?

(a) 96
(b) 120
(c) 192
(d) 312

Solution:

A number is divisible by 6 if it is divisible by 2 and 3 both.

A number divisible by 2 then at unit digit, we have to choose 0, 2, 4.

A number divisible by 3 then sum of digit should be divisible by 3.

Sum of number = 0 + 1 + 2 + 3 + 4 + 5 = 15.

15 is divisible by 3.

Total number of number in which unit place is 0 = 5!

Total number of numbers in which unit place is 2 = $4 \times 4!$

Total number of numbers in which unit place is 4 = $4 \times 4!$

Total number = $5! + 4 \times 4! + 4 \times 4!$

$$= 312$$

Answer: (d)

3. What is the binary number equivalent to decimal number 1011?

(a) 1011
(b) 111011
(c) 11111001
(d) 111110011

Solution:

2	1011	Remainder
2	505	1
2	252	1
2	126	0
2	63	0
2	31	1
2	15	1
2	7	1
2	3	1
	1	1

$$(1011)_{10} = (111110011)_2$$

4. The system of linear equations

$x + 2y + z = 4,$

$2x + 4y + 2z = 8$

$3x + 6y + 3z = 10$ has

(a) A unique solution
(b) Infinite many solutions
(c) No solution
(d) Exactly three solutions

Solution:

Plane-1: $x + 2y + z = 4$

Plane-2: $2x + 4y + 2z = 8$

Plane-1 and Plane-2 is coincident plane.

Plane-3, Plane-1(2) are parallel plane.

So there is no solution.

Answer: (c)

5. What is the sum of the roots of the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0 \ ?$$

(a) $a + b + c$(b) $a - b + c$

(c) $a + b - c$(d) $a - b - c$

Solution:

$$\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0$$

$(x+b)(x-a)(x-c) = 0$

$x_1 = a,\ x_2 = -b\ ,\ x_3 = c$

$x_1 + x_2 + x_3 = a - b + c$

Answer: (b)

6. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a+b)$?

(a) -11 (b) -3

(c) 0 (d) 3

Solution:

complex roots are in conjugate pair.So if $2 - i\sqrt{3}$ is roots of quadratic equation then other roots is $2 + i\sqrt{3}$

Sum of roots = 4

Product of roots = 7

$\text{Sum of roots} = -a = 4$

$a = -4$

$\text{Product of roots} = b = 7$

$a + b = -4 + 7 = 3$

Answer: (c)

7. If $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ where $i = \sqrt{-1}$, then what is the argument of z?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{6}$

Solution:

$1 + i\sqrt{3} = 2e^{i\frac{\pi}{3}}$

$1 - i\sqrt{3} = 2e^{-i\frac{\pi}{3}}$

$$z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \frac{2e^{i\frac{\pi}{3}}}{2e^{-i\frac{\pi}{3}}} = e^{i\frac{2\pi}{3}}$$

Answer: (b)

8 If $\log_x a$, a^x and $\log_b x$ are in GP, then what is x equal to?

(a) $\log_a(\log_b a)$ (b) $\log_b(\log_a b)$

(c) $\frac{\log_a(\log_b a)}{2}$ (d) $\frac{\log_b(\log_a b)}{2}$

Solution:

In G.P $\frac{\text{second term}}{\text{first term}} = \frac{\text{third term}}{\text{second term}} = r$

$\frac{a^x}{\log_x a} = \frac{\log_b x}{a^x}$

$a^{2x} = \log_x a\ \log_b x$

$\log_x a\ \log_b x = \log_b a$

$x = \frac{\log_a \log_b a}{2}$

Answer: (c)

9. If $2^{\frac{1}{c}}, 2^{\frac{b}{ac}}, 2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?

(a) a, b, c are in AP
(b) a, b, c are in GP
(c) a, b, c are in HP
(d) ab, bc, ca are in AP

Solution: $2^{\frac{1}{c}}, 2^{\frac{b}{ac}}, 2^{\frac{1}{a}}$ are in GP

$$\frac{2^{\frac{b}{ac}}}{2^{\frac{1}{c}}} = \frac{2^{\frac{1}{a}}}{2^{\frac{b}{ac}}}$$

$$2^{\frac{2b}{ac}} = 2^{\left(\frac{1}{a}+\frac{1}{c}\right)}$$

$$\frac{2b}{ac} = \frac{a+c}{ac}$$

$$2b = a + c$$

a, b, c are in A.P.

Answer: (a)

10 The first and the second terms of AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If n^{th} term is the largest negative term, what is the value of n?

(a) 5 (b) 5

(c) 7 (d) n cannot be determined

Solution:

First term of an A.P. = $\frac{5}{2}$

Second term of an A.P. = $\frac{23}{12}$

Common difference d

$= \frac{23}{12} - \frac{5}{2} = \frac{23-3}{12} = -\frac{7}{12}$

$t_n = a + (n-1)d$

$t_n = \frac{5}{2} - (n-1) \times \frac{7}{12}$

$t_n < 0$

$\frac{5}{2} - (n-1) \times \frac{7}{12} < 0$

$\frac{5}{2} < (n-1) \times \frac{7}{12}$

$30 < 7n - 4$

$\frac{34}{7} < n$

For greatest negative number n = 5

Answer: (b)

10. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5) ?

(a) 3 (b) 4

(c) 5 (d) 6

Solution:

For k = 4, 3, 2,1,0 both roots are real and lies between 0 and 5.

Answer: (c)

11. In an AP, the first term is x and the sum of the first n terms is zero. What is the sum of next m terms?

(a) $\frac{mx(m+n)}{n-1}$ (b) $\frac{mx(m+n)}{1-n}$

(c) $\frac{nx(m+n)}{m-1}$ (d) $\frac{nx(m+n)}{1-m}$

Solution:

First term = x

$S_n = \frac{n}{2}(2x + (n-1)d)$

$S_n = 0$

$\frac{n}{2}(2x + (n-1)d) = 0$

$2x + (n-1)d = 0$

$d = -\frac{2x}{n-1}$

$S_{n+m} = \frac{n+m}{2}(2x + (m+n-1)d)$

$S_{n+m} = \frac{n+m}{2}\left(2x + (m+n-1) \times \frac{-2x}{n-1}\right)$

$S_{n+m} = \frac{n+m}{2} 2x\left(1 - \frac{1}{n-1} \times (m+n-1)\right)$

$S_{n+m} = \frac{xm(n+m)}{1-n}$

Answer: (b)

12. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) 2

Solution:

Let $z = x + iy$

$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$

$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$= \frac{x^2+y^2-1-i(-xy+y+xy+y}{(x+1)^2+y^2}$

If $\frac{z-1}{z+1}$ is purely imaginary number then real part should equal to zero.

$\text{Re}\left(\frac{z-1}{z+1}\right) = 0$

$\frac{x^2+y^2-1}{(x+1)^2+y^2} = 0$

$x^2 + y^2 - 1 = 0$

$x^2 + y^2 = 1$

$|z| = \sqrt{x^2+y^2}$

$|z| = \sqrt{1} = 1$

Answer: (c)

13. How many real numbers satisfy the equation $|x-4| + |x-7| = 15$?

(a) Only one (b) Only two

(c) Only three (d) Infinitely many

Solution:

Case 1: $x > 7$

$x - 4 + x - 7 = 15$

$2x - 11 = 15$

$2x = 26$

$x = 13$

Case 2: $4 < x < 7$

$x - 4 + 7 - x = 15$

$3 = 15$

No solution.

Case 3: $x < 4$

$4 - x + 7 - x = 15$

$11 - 2x = 15$

$x = -2$

So roots of the equation $x = -2, 13$

Answer: (b)

14. p, q, r and s are in AP such that p +s = 8 and qr =15. What is the difference between largest and smallest numbers?

(a) 6 (b) 5

(c) 4 (d) 3

Solution:

$p + s = 8$

Let common difference of A.P. is d and first term is p.

$s = p + 3d$

$2p + 3d = 8$

$qr = 15$

$q = p + d \text{ and } r = p + 2d$

$(p + d)(p + 2d) = 15$

$2p + 3d = 8$

$p + 2d + p + d = 8$

$p + 2d = 8 - (p + d) = 8 - q$

$(p + d)(p + 2d) = 15$

$q(8 - q) = 15$

$q = 5, 3$

$r = 3, 5$

$p + d = 5$

$p + 2d = 3$

$d = -2$

$p = 7$

7, 5, 3, 1 are number.

Difference between largest and smallest is 6.

Consider the following for the next two (02) items that follow:

Let $x = \frac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \leq \frac{\pi}{2}$

15. What is the minimum value of x?

(a) 1 (b) 2

(c) 3 (d) 44

Solution:

$$x = \frac{\sin^2 A + \sin A + 1}{\sin A}$$

$$x = \sin A + \frac{1}{\sin A} + 1$$

$$\text{A.M} \geq \text{G.M.}$$

$$\frac{\sin A + \frac{1}{\sin A}}{2} \geq \sqrt{\sin A \times \frac{1}{\sin A}}$$

$$\sin A + \frac{1}{\sin A} \geq 2$$

$$x \geq 3$$

Answer: (c)

16 At what value of A does x attain the minimum value?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Solution:

When numbers are equal then arithmetic mean is equal to Geometric mean

$$\sin A = \frac{1}{\sin A}$$

$$\sin^2 A = 1$$

$$A = \frac{\pi}{2}$$

Answer: (d)

Consider the following for the next two (02) items that follow:

Consider the function

$f(x) = |x - 2| + |3 - x| + |4 - x|$, where $x \in R$.

17. At what value of x does the function attain minimum value?

(a) 2
(b) 3
(c) 4

(d) 0

Solution:

$f(x) = |x-2| + |3-x| + |4-x|$

$if\ x < 2\ |x-2| = 2-x$

$|3-x| = 3-x$

$|4-x| = 4-x$

$f(x) = 11-3x$

$if\ 2 < x < 3\ |x-2| = x-2$

$|3-x| = 3-x$

$|4-x| = 4-x$

$f(x) = 5-x$

$if\ 3 < x < 4\ |x-2| = x-2$

$|3-x| = 3-x$

$|4-x| = 4-x$

$f(x) = x-1$

$if\ 4 < x\ |x-2| = x-2$

$|3-x| = x-3$

$|4-x| = x-4$

$f(x) = 3x-9$

$f(2) = 5$

$f(3) = 2$

$f(4) = 3$

Function is minimum at x = 3.

Answer: (b)

18. What is the minimum value of the function?

(a) 2
(b) 3
(c) 4
(d) 0

Answer: (a)

Consider the following for the next two (02) items that follow:

Given that $m(\theta) = cot^2\theta + n^2tan^2\theta + 2n$, where n is a fixed real number.

19. What is the least value of $m(\theta)$?

(a) n
(b) 2n
(c) 3n
(d) 4n

20. Under what condition does m attain the least value?

(a) $n = tan^2\theta$
(b) $n = cot^2\theta$
(c) $n = sin^2\theta$
(d) $n = cos^2\theta$

Solution: If a and b are two positive number then $\mathrm{A.M.} \geq \mathrm{G.M.}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$m(\Theta) = \cot^2\theta + n^2\tan^2\theta + 2n$$

$$y(\Theta) = \frac{1}{\tan^2\theta} + n^2\tan^2\theta$$

$$\frac{\frac{1}{\tan^2\theta} + n^2\tan^2\theta}{2} \geq \sqrt{\frac{1}{\tan^2\theta} \times n^2\tan^2\theta}$$

$$\frac{1}{\tan^2\theta} + n^2\tan^2\theta \geq 2n$$

Minimum value of $m(\theta)$ is equal to $4n$

Equality hold when both a and b are equal.

$$\frac{1}{\tan^2\theta} = n^2\tan^2\theta$$

$$\cot^4\theta = n^2$$

$$\cot^2\theta = n$$

Consider the following for the next two (02) items that follows:

A quadrilateral is formed by the lines $x = 0, y = 0, x + y = 1\ and\ 6x + y = 3$.

21. What is the equation of diagonal through origin?

(a) $3x + y = 0$
(b) $2x + 3y = 0$
(c) $3x - 2y = 0$
(d) $3x + 2y = 0$

22. What is the equation of other diagonal?

(a) $x + 2y - 1 = 0$
(b) $x - 2y - 1 = 0$
(c) $2x + y + 1 = 0$
(d) $2x + y - 1 = 0$

Solution:

Co-ordinate of vertex of quadrilateral are

A(0,0), B(1/2 , 0) , C(x, y) and D(0, 1)

Intersection of lines $x + y = 1$ and $6x + y = 3$ is C (x, y).

x = 2/5 and y = 3/5.

Equation of diagonal AC is

Slope of Line AC = $\frac{y_C - y_A}{x_C - x_A} = \frac{\frac{3}{5} - 0}{\frac{2}{5} - 0} = \frac{3}{2}$

$$y = \frac{3}{2}x$$

Equation of diagonal BD is

Slope of line BD

$$\frac{y_B - y_D}{x_B - x_D} = \frac{0 - 1}{\frac{1}{2} - 0} = -2$$

$$y = -2x + 1$$

Consider the following for the next two (02) items that follow:

P(x, y) is any point on the ellipse $x^2 + 4y^2 = 1$. Let E, F be the foci of the ellipse.

23. What is PE + PF equal to?

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

$x^2 + 4y^2 = 1$

$\frac{x^2}{1} + \frac{y^2}{\frac{1}{4}} = 1$

$a^2 = 1$

$b^2 = \frac{1}{4}$

$b^2 = a^2(1 - e^2)$

$\frac{1}{4} = 1 - e^2$

$e = \frac{\sqrt{3}}{2}$

Focus E ≡ $(-ae, 0)$

E ≡ $\left(-\frac{\sqrt{3}}{2}, 0\right)$

Focus F ≡ $(+ae, 0)$

F ≡ $\left(+\frac{\sqrt{3}}{2}, 0\right)$

An **ellipse** is the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane, is constant.

Choose point P≡ $(0, \frac{1}{2})$

$PE + PF = 1 + 1 = 2$

Answer: (b)

Consider the following for the next two (02) items that follow:

The line $y = x$ partitions the circle $(x - a)^2 + y^2 = a^2$ in two segments.

24. What is the area of minor segment?

(a) $\frac{(\pi - 2)a^2}{4}$ (b) $\frac{(\pi - 1)a^2}{4}$

(c) $\frac{(\pi - 2)a^2}{2}$ (d) $\frac{(\pi - 1)a^2}{2}$

Solution:

The equation of circle: $(x - a)^2 + y^2 = a^2$

Equation of line :$y = x$

Point of intersection of line and circle

$(x - a)^2 + x^2 = a^2$

$2x^2 - 2ax = 0$

$2x(x - a) = 0$

$x = 0, a$

Point of intersection are A≡ $(0, 0)$ and B ≡ (a, a).

AB = $\sqrt{2}a$

Centre C ≡ $(a, 0)$

Radius R = a

Angle made by AB at the centre C

$$\cos\theta = \frac{2R^2 - AB^2}{2R^2} = \frac{2a^2 - 2a^2}{2a^2} = 0$$

$$\theta = \frac{\pi}{2}$$

Area of minor arc = $\frac{\pi a^2}{2\pi} \times \frac{\pi}{2} = \frac{\pi a^2}{4}$

Area of triangle ABC = $\frac{1}{2} \times R \times R = \frac{1}{2}a^2$

Area bounded by line and curve

$= \frac{\pi a^2}{4} - \frac{1}{2}a^2 = \frac{(\pi - 2)a^2}{4}$

Answer: (a)

25. What is the area of major segment?

(a) $\frac{(3\pi-2)a^2}{4}$ (b) $\frac{(3\pi+2)a^2}{4}$

(c) $\frac{(3\pi-2)a^2}{2}$ (d) $\frac{(3\pi+2)a^2}{2}$

Solution:

Area of major segment

$= \pi a^2 - \frac{(\pi-2)a^2}{4} = \frac{(3\pi+2)a^2}{4}$

Answer: (b)

Consider the following for the next two (02) items that follow:

Let A(1, -1, 2) and B(2, 1, -1) be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

26. What is $u + v + w$ equal to?

(a) -2
(b) -1
(c) 1
(d) 2

Solution: If A and B are end points of diameter of sphere then mid point of AB is centre of sphere.

Co-ordinate of mid point of AB.

$$x = \frac{x_A + x_B}{2} = \frac{1+2}{2} = \frac{3}{2}$$

$$y = \frac{y_A + y_B}{2} = \frac{-1+1}{2} = 0$$

$$z = \frac{z_A + z_B}{2} = \frac{2-1}{2} = \frac{1}{2}$$

Centre of sphere of $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$

$$(x+u)^2 + (y+v)^2 + (z+w)^2 = 1 + u^2 + v^2 + w^2$$

Centre C ≡ $(-u, -v, -w)$

$-u = x = \frac{3}{2}$

$-v = y = 0$

$-w = z = \frac{1}{2}$

$u + v + w = -2$

Answer: (a)

27.If P(x, y,z) is any point on the sphere, then what is $PA^2 + PB^2$ equal to ?

(a) 15
(b) 14
(c) 13
(d) 6.5

Solution:

If we join PAB, it will form a right angle triangle at point P.

$PA^2 + PB^2 = AB^2$

$AB^2 = (1-2)^2 + (-1-1)^2 + (-1-2)^2$

$AB^2 = 14$

Answer: (b)

Consider the following for the next two (02) items that follow:

Consider two lines whose direction ratios are (2, -1, 2) and (k, 3, 5). They are inclined at an angle $\frac{\pi}{4}$.

28. What is the value of k?

(a) 4
(b) 2
(c) 1
(d) -1

Solution:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\frac{\pi}{4} = \frac{2k - 3 + 10}{\sqrt{2^2 + (-1)^2 + 2^2}\sqrt{k^2 + 3^2 + 5^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{2k+7}{\sqrt{9}\sqrt{k^2+34}}$$

$$\frac{9}{2} = \frac{(2k+7)^2}{k^2+34}$$

$$9k^2 + 306 = 2(4k^2 + 28k + 49)$$

$$k^2 - 56k + 208 = 0$$

$$k^2 - 2k - 54k + 208 = 0$$

$$(k-4)(k-52) = 0$$

$$k = 4 \ or \ 52$$

29. What are the direction ratios of a line which is perpendicular to both the lines?

(a) (1 ,2 ,10)
(b) (-1, -2, 10)
(c) (11, 12, -10)
(d) (11, 2, -10)

Solution:
Let direction cosine of line perpendicular to both lines is (a, b, c).
Since lines are perpendicular to each other therefore dot product of direction cosine is equal to zero.

$2a - b + 2c = 0$

$4a + 3b + 5c = 0$

$$\frac{a}{\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix}}$$

$$\frac{a}{-11} = \frac{b}{-2} = \frac{c}{10}$$

Direction ratio (a, b, c) = (11, 2, -10)

Answer: (d)

Consider the following for the next two (02) items that follow:

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a}.\vec{b} = 27$ and $\vec{a} \times \vec{b} = 9\vec{c}$

30. What is $\vec{b}$ equal to ?

(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$
(b) $5\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$
(d) $3\hat{i} + 3\hat{j} + 4\hat{k}$

Solution:

Let $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\vec{a}.\vec{b} = 27$

$(3\hat{i} + 3\hat{j} + 3\hat{k}).(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}) = 27$

$3b_x + 3b_y + 3b_z = 27$

$b_x + b_y + b_z = 9$ ---- (1)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 3 & 3 \\ b_y & b_z \end{vmatrix} - j\begin{vmatrix} 3 & 3 \\ b_x & b_z \end{vmatrix} + k\begin{vmatrix} 3 & 3 \\ b_x & b_y \end{vmatrix}$$

$= 3\hat{i}(b_z - b_y) + 3\hat{j}(b_x - b_z) + 3k(b_y - b_x)$

$= 9\vec{c} = 9\hat{j} - 9\hat{k}$

Compare L.H.S and R.H.S vectors we get,

$3(b_z - b_y) = 0$

$b_y = b_z$

$3(b_x - b_z) = 9$

$3(b_y - b_x) = -9$

From equation -1 we get

$b_x + b_y + b_z = 9$

$b_x + 2b_y = 9$

$b_y - b_x = -3$

Add above two equations we get,

$b_x + 2b_y + b_y - b_x = 9 - 3$

$3b_y = 6$

$b_y = 2 = b_z$

$b_x + 2b_y = 9$

$b_x + 4 = 9$

$b_x = 5$

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

Second method: Checking method

Since $\vec{a} \times \vec{b} = 9\vec{c}$ therefore vectors $\vec{b}$ and $\vec{c}$ are perpendicular to each other.

$\vec{b}.\vec{c} = 0$

Check each solution one by one

$(3\hat{i} + 4\hat{j} + 2\hat{k}).(\hat{j} - \hat{k}) = 4 - 2 = 2$

$(5\hat{i} + 2\hat{j} + 2\hat{k}).(\hat{j} - \hat{k}) = 2 - 2 = 0$

$(5\hat{i} - 2\hat{j} + 6\hat{k}).(\hat{j} - \hat{k}) = -2 - 6 = -8$

$(3\hat{i} + 3\hat{j} + 4\hat{k}).(\hat{j} - \hat{k}) = 3 - 4 = -1$

Answer: (b)

31. What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$?

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$

Solution:

Angle between $(\vec{a}+\vec{b})$ and $\vec{c}$ is

$$\cos\theta = \frac{(\vec{a}+\vec{b}).\vec{c}}{|\vec{a}+\vec{b}||\vec{c}|}$$

$$\vec{a}+\vec{b} = (3\hat{\imath}+3\hat{\jmath}+3\hat{k}) + (5\hat{\imath}+2\hat{\jmath}+2\hat{k})$$

$$\vec{a}+\vec{b} = 8\hat{\imath}+5\hat{\jmath}+5\hat{k}$$

$$(\vec{a}+\vec{b}).\vec{c} = (8\hat{\imath}+5\hat{\jmath}+5\hat{k}).(\hat{\jmath}-\hat{k}) = 0$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Answer: (a)

Consider the following for the next two (02) items that follow:

Let a vector $\vec{a} = 4\hat{\imath} - 8\hat{\jmath} + \hat{k}$ make angles α, β, γ with the positive directions of x, y, z axes respectively.

32 What is $\cos\alpha$ equal to ?

(a) $\frac{1}{3}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{2}{3}$

Solution:

$$\cos\alpha = \frac{\vec{a}.\hat{\imath}}{|\vec{a}|}$$

$$\vec{a}.\hat{\imath} = 4$$

$$|\vec{a}| = \sqrt{4^2+8^2+1^2} = \sqrt{81}$$

$$\cos\alpha = \frac{4}{9}$$

Answer: (b)

33 What is $\cos 2\beta + \cos 2\gamma$ equal to?

(a) $-\frac{32}{81}$ (b) $-\frac{16}{81}$

(c) $\frac{16}{81}$ (d) $\frac{32}{81}$

Solution:

$$\cos\beta = \frac{\vec{a}.\hat{\jmath}}{|\vec{a}|}$$

$$\vec{a}.\hat{\jmath} = -8$$

$$\cos\beta = \frac{-8}{9}$$

$$\cos\gamma = \frac{\vec{a}.\hat{k}}{|\vec{a}|}$$

$$\vec{a}.\hat{k} = 1$$

$$\cos\gamma = \frac{1}{9}$$

$$\cos 2\beta + \cos 2\gamma$$

$$= 2(\cos^2\beta + \cos^2\gamma) - 2$$

$$= 2\left(\left(\frac{-8}{9}\right)^2 + \left(\frac{1}{9}\right)^2\right) - 2$$

$$= -\frac{32}{81}$$

Answer: (a)

Consider the following for the next two (02) items that follow:

The position vectors of two points A and B are $\hat{\imath}-\hat{\jmath}$ and $\hat{\jmath}+\hat{k}$ respectively.

34 Consider the following points:

1. (-1, -3,1)
2. (-1, 3,2)
3. (-2,5,3)

Which of the above points lie on the line joining A and B?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Solution:

$$\overrightarrow{AB} = -\hat{\imath}+2\hat{\jmath}+\hat{k}$$

Let point P ≡ $(-1,-3,1)$

$$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$$

$$= (-\hat{\imath}-3\hat{\jmath}+\hat{k}) - (\hat{\imath}-\hat{\jmath})$$

$$= -2\hat{\imath}-2\hat{\jmath}+\hat{k}$$

If P lie on line joining A and B then $\overrightarrow{AP}$ is parallel to $\overrightarrow{AB}$.

$$\overrightarrow{AP} \neq \lambda \overrightarrow{AB}$$

Let point Q $\equiv (-1, 3, 2)$

$$\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA}$$
$$= (-\hat{i} + 3\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j})$$
$$= -2\hat{i} + 4\hat{j} + 2\hat{k}$$

Since $\overrightarrow{AQ} = 2\,\overrightarrow{AB}$ therefore point Q lies on line joining AB.

Let point R $\equiv (-2, 5, 3)$

$$\overrightarrow{AR} = \overrightarrow{OR} - \overrightarrow{OA}$$
$$= (-2\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j})$$
$$= -3\hat{i} + 6\hat{j} + 3\hat{k}$$

Since $\overrightarrow{AR} = 3\,\overrightarrow{AB}$ therefore point R lies on line joining AB.

Answer: (b)

35. What is the magnitude of $\overrightarrow{AB}$?

(a) 2 (b) 3
(c) $\sqrt{6}$ (d) $\sqrt{3}$

Solution:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (\hat{j} + \hat{k}) - (\hat{i} - \hat{j})$$
$$= -\hat{i} + 2\hat{j} + \hat{k}$$
$$|\overrightarrow{AB}| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$$

Answer: (c)

Consider the following for the next three (03) items that follow:

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where P, Q, R are real numbers. Further $f(0) = 6$, $f'(\ln 3) = 282$ and $\int_0^{\ln 2} f(x)\,dx = 11$

36. What is the value of Q?

(a) 1 (b) 2
(c) 3 (d) 4

Solution:

$$f(x) = Pe^x + Qe^{2x} + Re^{3x}$$
$$f(0) = Pe^0 + Qe^{2\times 0} + Re^{3\times 0}$$
$$f(0) = P + Q + R$$
$$P + Q + R = 6$$
$$f'(x) = Pe^x + 2Qe^{2x} + 3Re^{3x}$$
$$f'(\ln 3) = Pe^{\ln 3} + 2Qe^{2\ln 3} + 3Re^{3\ln 3}$$
$$3P + 18Q + 81R = 282$$
$$\int_0^{\ln 2} f(x)\,dx = 11$$
$$\int_0^{\ln 2} Pe^x + Qe^{2x} + Re^{3x}\,dx = 11$$
$$Pe^x + \frac{Qe^{2x}}{2} + \frac{Re^{3x}}{3}\Big|_0^{\ln 2} = 1$$
$$Pe^{\ln 2} + \frac{Qe^{2\ln 2}}{2} + \frac{Re^{3\ln 2}}{3} - Pe^0 - \frac{Qe^{2\times 0}}{2} - \frac{Re^{3\times 0}}{3} = 11$$
$$2P + 2Q + \frac{8}{3}R - P - \frac{Q}{2} - \frac{R}{3} = 11$$
$$P + \frac{3}{2}Q + \frac{7}{3}R = 11$$
$$6P + 9Q + 14R = 66$$

Solve these three linear equations we get,

$$P + Q + R = 6$$
$$3P + 18Q + 81R = 282$$
$$6P + 9Q + 14R = 66$$
$$P = 1, Q = 2 \text{ and } R = 3$$

Answer: (b)

37. What is the value of R?

(a) 1
(b) 2
(c) 3
(d) 4

Answer: (c)

38. What is $f'(0)$ equal to?

(a) 18
(b) 16
(c) 15

(d) 14

Solution:

$f'(x) = Pe^x + 2Qe^{2x} + 3Re^{3x}$

$P = 1, Q = 2$ and $R = 3$

$f'(x) = e^x + 4e^{2x} + 9e^{3x}$

$f'(0) = e^0 + 4e^{2\times 0} + 9e^{3\times 0} = 14$

Answer: (d)

Consider the following for the next three (03) items that follow:

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

39 What is f(0) equal to ?

(a) -1
(b) 0
(c) 1
(d) 2

Solution:

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$f(0) = \begin{vmatrix} \cos 0 & 0 & 1 \\ 2\sin 0 & 0^2 & 2\times 0 \\ \tan 0 & 0 & 1 \end{vmatrix} = 0$$

40. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ What is $\lim_{x\to 0} \frac{f(x)}{x}$ equals to?

(a) -1
(b) 0
(c) 1
(d) 2

Solution: $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \frac{1}{x} \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$= \lim_{x\to 0} \frac{x}{x} \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x & x & 2x \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \lim_{x\to 0} \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x & x & 2x \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \cos 0 & 1 & 1 \\ 2\sin 0 & 0 & 0 \\ \tan x & 1 & 1 \end{vmatrix} = 0$$

Answer: (b)

41. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ What is $\lim_{x\to 0} \frac{f(x)}{x^2}$ equals to?

(a) -1
(b) 0
(c) 1
(d) 2

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0} \frac{1}{x^2} \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$\lim_{x\to 0} \frac{1}{x} \begin{vmatrix} \cos x & \frac{x}{x} & 1 \\ 2\sin x & \frac{x^2}{x} & 2x \\ \tan x & \frac{x}{x} & 1 \end{vmatrix}$$

$$= \lim_{x\to 0} \frac{1}{x} \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x & x & 2x \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \lim_{x\to 0} \begin{vmatrix} \cos x & 1 & 1 \\ \frac{2\sin x}{x} & \frac{x}{x} & \frac{2x}{x} \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \lim_{x\to 0} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -1$$

42. For what value of m with $m < 0$, is the area bounded by the lines $y = x, y = mx$ and $x = 2$ equal to 3?

(a) $-\frac{1}{2}$
(b) -1
(c) $-\frac{3}{2}$
(d) -2

Solution:

Area bounded by line y =x , y =mx and x = 2 .

m < 0

Area = $\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2|m|$

Given Area = 3

$3 = 2 + 2|m|$

$m = -\frac{1}{2}$

43. What is the derivative of $cosec(x^o)$?

(a) $-cosec(x^o)\,cot(x^o)$
(b) $-\frac{\pi}{180}cosec(x^o)\,cot(x^o)$
(c) $\frac{\pi}{180}cosec(x^o)\,cot(x^o)$
(d) $-\frac{\pi}{180}cosec(x)\cot(x)$

Solution:

$$\frac{d(Cosec\ x)}{dx} = -cosec\ x \cot x$$

$$Cosec\ x^0 = \text{cosec}\frac{\pi}{180}x$$

$\cdot\ \frac{d(Cosec\ x^0)}{dx}$

$$= \frac{d(Cosec\ x^0)}{dx^0}\frac{dx^0}{dx}$$

$$= -\frac{\pi}{180}cosec\ x^0 \cot x^0$$

44. A solution of the differential equation
$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is

(a) $y = 2x$
(b) $y = 2x + 4$
(c) $y = x^2 - 1$
(d) $y = \frac{(x^2-2)}{2}$

Solution:

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = x$$

$$\int dy = \int xdx$$

$$y = \frac{x^2}{2} + c$$

45. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to

(a) 3
(b) 1
(c) -2
(d) -3

Solution: $f(x) = x^2 + 2$

$g(x) = 2x - 3$

$g(1) = 2 \times 1 - 3 = -1$

$f(g(1)) = f(-1) = 3$

46. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers?

(a) (0 , ∞)
(b) [0, ∞]
(c) (-∞ , ∞)
(d) [1, ∞)

Solution:

$y = x + |x|$

$$y = \begin{cases} 2x & x > 0 \\ 0 & x \le 0 \end{cases}$$

Range of function y is [0 , ∞]

47. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to ?

(a) $-\sin(3\theta)$
(b) $-\cos(3\theta)$
(c) $\sin(3\theta)$
(d) $-\sin(4\theta)$

Solution:

$f(x) = x(4x^2 - 3)$

$f(\sin\theta) = 4sin^3\theta - 3\sin\theta$

$\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\, cos^2\theta + (1 - 2sin^2\theta)\sin\theta$

$= 2\sin\theta\,(1 - sin^2\theta) + \sin\theta - 2sin^3\theta$

$= 3\sin\theta - 4sin^3\theta$

$$f(\sin\theta) = 4sin^3\theta - 3\sin\theta = -\sin(3\theta)$$

48 What is $\lim_{x\to5}\frac{5-x}{|x-5|}$

(a) -1 (b) 0

(c) 1 (d) Limit does not exist

Solution:

$$\lim_{x\to5^-}\frac{5-x}{|x-5|}=\lim_{x\to5^-}\frac{5-x}{5-x}=1$$

$$\lim_{x\to5^+}\frac{5-x}{|x-5|}=\lim_{x\to5^+}\frac{5-x}{x-5}=-1$$

$$\lim_{x\to5^-}\frac{5-x}{|x-5|}\neq\lim_{x\to5^+}\frac{5-x}{|x-5|}$$

Answer: (d)

100 $\lim_{x\to1}\frac{x^9-1}{x^3-1}$

Apply L Hospital rule

$\lim_{x\to1}\frac{9x^8}{3x^2}=3$

49. Let A and B be two independent events such that $P(\bar{A})=0.7$, $P(\bar{B})=k$, $P(A\ U\ B)=0.8$, what is the value of k?

(a) $\frac{5}{7}$
(b) $\frac{4}{7}$
(c) $\frac{2}{7}$
(d) $\frac{1}{7}$

If A and B are two independent events then $P(A\cap B)=P(A)\times P(B)$.

Given $P(\bar{A})=0.7$

$$P(\bar{B})=k.\text{ and }P(A\cup B)=0.8$$

$$P(A)=1-P(\bar{A})=1-0.7=0.3$$

$$P(B)=1-P(\bar{B})=1-k$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$0.8=0.7+(1-k)-(1-k)\times0.7$$

$$k=\frac{2}{7}$$

50. A baised coin with the probability of getting head equal to $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head?

(a) $\frac{81}{512}$
(b) $\frac{81}{1024}$
(c) $\frac{81}{256}$
(d) $\frac{27}{1024}$

If a coin is tossed five times then occurrence of five tail followed by head

E = {TTTTH}

Probability of Event P(E) = $\left(\frac{3}{4}\right)^4\times\frac{1}{4}=\frac{81}{1024}$

51. A coin is baised so that head comes up thrice as likely as tails. In four independent tosses of the coin, what is probability of getting exactly three heads?

a) $\frac{81}{256}$
b) $\frac{27}{64}$
c) $\frac{27}{256}$

$$\frac{9}{256}$$

$P(Head)+P(Tail)=1$

$$P(Head)=3P(Tail)$$

$$4P(Tail)=1$$

$$P(Tail)=\frac{1}{4}$$

$$P(Head)=\frac{3}{4}$$

If coin is tossed four times then probability of occurrence of three head and one tail

$=\left(\frac{3}{4}\right)^3\times\left(\frac{1}{4}\right)^1\times C(4,3)=\frac{27}{64}$

52. If G is the geometric mean of numbers 1, 2, 2^2, 2^3, ….., 2^{n-1}, then what is the value of 1 +2$\log_2 G$?

(a) 1
(b) 4
(c) n-1
(d) n

$$G = \sqrt[n]{2^{0+1+2+3+\cdots+n-1}}$$

$$0+1+2+3+\cdots+n-1 = \frac{n(0+n-1)}{2}$$

$$G = 2^{\frac{n-1}{2}}$$

$$2\log_2 G = 2\log_2 2^{\frac{n-1}{2}} = n-1$$

$$1 + 2\log_2 G = n$$

53. Three dice are thrown. What is the probability that each face shows only multiples of 3?

(a) 1/9 (b) 1/18

(c) 1/27 (d) 1/3

Solution:

$$\mathrm{n(S)} = 6^3 = 216$$

$$\mathrm{E} = \{(3,3,3),(6,6,6)\}$$

$$\mathrm{p(E)} = \frac{2}{216} = \frac{1}{108}$$

Answer: (*)

54. What is the probability that the month of December has 5 Sundays?

(a) 1 (b) 1/4

(c) 3/7 (d) 2/7

Solution:

Total number of days in December is 31.

So 4 weeks and three days. So last three days are in sequence. List of all cases of last three days.

Sample Space S = { (1,2,3) (2,3,4), (3,4,5), (4,5,6), (5,6,7),(6,7,1),(7,1,2)}

1, 2, 3, 4, 5, 6, 7 correspond to Monday to Sunday.

n(S) =7

n(E) =3

$$\mathrm{P(E)} = \frac{\mathrm{n(E)}}{\mathrm{n(S)}} = \frac{3}{7}$$

55. A natural number n is chosen from the first 50 natural numbers. What is the probability that $+ \frac{50}{n} < 50$?

(a) $\frac{23}{25}$

(b) $\frac{47}{50}$

(c) $\frac{24}{25}$

(d) $\frac{49}{25}$

Solution: Let S is set of natural number which is less than equal to 50.

$S = \{1,2,3,4,\ldots,50\}$

$n(S) = 50$

Let E is set of natural number such that $n + \frac{50}{n} < 50$.

$E = \{2,3,4,\ldots,48\}$

Let E' is a set of natural number such that $n + \frac{50}{n} > 50$.

$$E' = \{1,49,50\}$$

$$P(E') = \frac{n(E')}{n(S)} = \frac{3}{50}$$

$$P(E) = 1 - P(E') = 1 - \frac{3}{50} = \frac{47}{50}$$

56. If H is the harmonic mean of numbers 1, 2, 2^2, 2^3, ….., 2^{n-1}, then what is n/H equal to?

(a) $2 - \frac{1}{2^{n+1}}$

(b) $2 - \frac{1}{2^{n-1}}$

(c) $2 + \frac{1}{2^{n-1}}$

(d) $2 - \frac{1}{2^{n}}$

Solution: Harmonic mean of numbers a, b, c , …

$$\frac{1}{H} = \frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+..}{n}$$

$$\frac{n}{H} = \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\ldots$$

$$\frac{n}{H} = \frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{n-1}}$$

$$\frac{n}{H} = \frac{a(1-r^n)}{1-r} = \frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 2 - \frac{1}{2^{n-1}}$$

57. What is the derivative of sin^2x with respect to cos^2x

(a) -1
(b) 1
(c) $\sin 2x$
(d) $\cos 2x$

Solution:

$$\frac{d(sin^2x)}{dcos^2x} = \frac{d(1 - cos^2x)}{d(cos^2x)} = -1$$

58. Consider the following statements:

1. $f(x) = \ln x$ is increasing in (0, ∞)
2. g(x) = $e^x + e^{\frac{1}{x}}$ is decreasing in (0, ∞)

Which of the statements given above is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Answer: (c)

Consider the following for the next three (03) items that follow:

Let $f(x) = |lnx|, x \neq 1$

59. What is the derivative of f(x) at x =0.5?

(a) -2
(b) -1
(c) 1
(d) 2

Solution: $f(x) = -\ln x \quad 0 < x \leq 1$

$$\frac{df(x)}{dx} = -\frac{1}{x} = -\frac{1}{0.5} = -2$$

60. What is the derivative of f(x) at x =2 ?

(a) $-\frac{1}{2}$
(b) -1
(c) $\frac{1}{2}$
(d) 2

Solution: : $f(x) = \ln x \quad 1 \leq x \leq \infty$

$$\frac{df(x)}{dx} = \frac{1}{x} = \frac{1}{2} = 0.5$$

Consider the following for the next two (02) items that follow:

A quadratic equation is given by

$$(3 + 2\sqrt{2})x^2 - (4 + 2\sqrt{3})x + (8 + 4\sqrt{3}) = 0$$

61. What is the HM of the roots of the equation?

(a) 2
(b) 4
(c) $2\sqrt{2}$
(d) $2\sqrt{3}$

Solution: Roots of the quadratic equation of $(3 + 2\sqrt{2})x^2 - (4 + 2\sqrt{3})x + (8 + 4\sqrt{3}) = 0$

$$\alpha + \beta = -\frac{b}{a} = \frac{4 + 2\sqrt{3}}{3 + 2\sqrt{2}}$$

$$\alpha\beta = \frac{c}{a} = \frac{8 + 4\sqrt{3}}{3 + 2\sqrt{2}}$$

HM of the roots of the equation

$$HM = \frac{2\alpha\beta}{\alpha + \beta} = \frac{2(8 + 4\sqrt{3})}{4 + 2\sqrt{3}} = 4$$

Consider the following for the next two(02) items that follow :

Let $\sin\beta$ be the GM of $\sin\alpha$ and $\cos\alpha$; $\tan\gamma$ be the AM of $\sin\alpha$ and $\cos\alpha$.

62. What is cos2β equal to ?

(a) $(\cos\alpha - sin\alpha)^2$
(b) $(\cos\alpha + sin\alpha)^2$
(c) $(\cos\alpha - sin\alpha)^3$
(d) $\frac{(\cos\alpha - sin\alpha)}{2}$

Solution: $\sin\beta = \sqrt{\sin\alpha\cos\alpha}$

$\cos 2\beta = 1 - 2sin^2\beta$

$= sin^2\alpha + cos^2\alpha - 2\sin\alpha\cos\alpha$

$= (\cos\alpha - \sin\alpha)^2$

63. A die is thrown 10 times and obtained the following outputs:1,2,1,1,2,1,4,6,5,4 What will be the mode of data so obtained ?

(a) 6 (b) 4

(c) 2 (d) 1

Solution:

Data	Frequency
1	4
2	2
4	2
5	1
6	1

Maximum frequency is 4 of data 1.

So mode is equal to 1.

Consider the following for the next two (02) items that follow:

Let $f(x) = \sin[\pi^2]x + cos[-\pi^2]x$ where [.] is a greatest integer function

64. What is $f(\frac{\pi}{2})$ equal to?

(a) -1 (b) 0

(c) 1 (d) 2

Solution:

$[\pi^2] = 9 \quad 9 < \pi^2 < 10$

$[-\pi^2] = -10 \quad -10 < -\pi^2 < -9$

$f(x) = \sin[\pi^2]x + cos[-\pi^2]x$

$= \sin 9x + \cos(-10x)$

$= \sin 9x + \cos 10x$

$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{9\pi}{2}\right) + \cos\left(\frac{10\pi}{2}\right)$

$= \sin\left(4\pi + \frac{\pi}{2}\right) + \cos\left(5\pi\right)$

$= \sin\frac{\pi}{2} - 1 = 1 - 1 = 0$

65. What is $f(\frac{\pi}{4})$ equal to?

(a) $-\frac{1}{\sqrt{2}}$ (b) -1

(c) 1 (d) $\frac{1}{\sqrt{2}}$

Solution:

$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{9\pi}{4}\right) + \cos\left(\frac{10\pi}{4}\right)$

$= \sin\left(2\pi + \frac{\pi}{4}\right) + \cos\left(2\pi + \frac{\pi}{2}\right)$

$= \sin\frac{\pi}{4} + \cos\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}$

Consider the following for the next two (02) items that follow:

Let = $\int_a^b \frac{|x|}{x} dx$, a < b

66. What is I equal to when a < 0 <b?

(a) $a + b$ (b) $a - b$

(c) $b - a$ (d) $\frac{(a+b)}{2}$

Solution:

$I = \int_a^b \frac{|x|}{x} dx$

$= \int_a^0 \frac{-x}{x} dx + \int_0^b \frac{x}{x} dx$

$= \int_0^a dx + \int_0^b dx$

$= a + b$

Property of definite integral

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^0 \frac{-x}{x} dx = \int_a^0 -dx = -\int_0^a -dx = \int_0^a dx$

67. What is I equal to when a<b<0?

(a) $a + b$ (b) $a - b$

(c) $b - a$ (d) $\frac{(a+b)}{2}$

Solution:

$I = \int_a^b \frac{|x|}{x} dx$

$= \int_a^b \frac{-x}{x} dx$

$= \int_a^b - dx$

$= -(b - a)$

$= a - b$

Consider the following for the next three (03) items that follow:

Let $f(x) = |lnx|, x \neq 1$

68. What is the derivative of f(x) at x =0.5?

(e) -2 (b) -1

(c) 1 (d) 2

Solution:

$f(x) = -\ln x \quad 0 < x < 1$

$f'(x) = -\frac{1}{x} \quad 0 < x < 1$

$f'(0.5) = -\frac{1}{0.5} = -2$

69. What is the derivative of f(x) at x = 2 ?

(e) $-\frac{1}{2}$ (b) -1

(c) $\frac{1}{2}$ (d) 2

Solution:

$f(x) = \ln x \quad 1 < x$

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

70. What is the derivative of $fo\, f\,(x)$, where 1 < x <2?

(a) $\frac{1}{lnx}$ (b) $\frac{1}{xlnx}$

(c) - $\frac{1}{lnx}$ (d) $-\frac{1}{xlnx}$

Solution: If 1 < x <2 then $\ln 1 < \ln x < \ln 2$

$$\ln 2 < 1$$

$$0 < \ln x < 1$$

$$fo\, f\,(x) = |\ln(f(x))| \quad 0 < f(x) = \ln x < 1$$

$$Let\ y = fo\, f\,(x) = -\ln(\ln x)$$

$$\frac{dy}{dx} = -\frac{1}{\text{x} \ln x}$$

Consider the following for the next two (02) items that follow:

Let $f(x) = \begin{bmatrix} x+6, & x \le 1 \\ px+q, & 1 < x < 2 \\ 5x, & x \ge 2 \end{bmatrix}$

and f(x) is continuous

71. What is the value of p ?

(a) 2 (b) 3

(c) 4 (d) 5

Solution: $\lim_{x \to 1^-} x + 6 = 7$

$$\lim_{x \to 1^+} px + q = p + q$$

$$f(1) = 7$$

if f(x) is continuous at x =1 then

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$p + q = 7$$

$$\lim_{x \to 2^-} px + q = 2p + q$$

$$\lim_{x \to 2^+} 5x = 10$$

$$f(2) = 10$$

if (x) is continuous at x = 2 then

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$2p + q = 10$$

p = 3 and q =4

72. What is the value of q ?

(a) 2 (b) 3

(c) 4 (d) 5

73. If Δ is the value of the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_2 \end{vmatrix}$$

$$(p \ne 0\ or\ 1, q\ \ne 0\ or\ 1)$$

(a) $p\Delta$ (b) $q\Delta$

(c) $(p+q)\Delta$ (d) $pq\Delta$

Solution: $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_1 = \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_2 \end{vmatrix}$

$\Delta_1 = pq \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_2 \end{vmatrix} = pq\Delta$