1. Every quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in R,\ a \neq 0$ has

(a) Exactly one real root.
(b) At least one real root.
(c) At least two real roots.
(d) At most two real roots.

Answer: (d)

2. If $a \neq b \neq c$ are all positive, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) Non-negative
(b) Non-positive
(c) Negative
(d) positive

Solution:

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= a\begin{vmatrix} c & a \\ a & b \end{vmatrix} - b\begin{vmatrix} b & a \\ c & b \end{vmatrix} + c\begin{vmatrix} b & c \\ c & a \end{vmatrix}$

$= a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= 3abc - a^3 - b^3 - c^3$

$A.M \geq G.M$

$$\frac{a^3 + b^3 + c^3}{3} \geq \sqrt[3]{a^3b^3c^3}$$

$0 \geq 3abc - a^3 - b^3 - c^3$

Answer: (c)

3. Let A and B be two matrices such that $AB = A$ and $BA = B$. Which of the following statements are correct?

1. A^2=A
2. $B^2 = B$
3. $(AB\ )^2 = AB$

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1,2 and 3

Solution: $AB = A$

$ABA = AA$

Sin BA = B and AB = A

$AB = A^2$

$A = A^2$

$BA = B$

$BAB = B^2$

$BA = B^2$

$B = B^2$

$(AB\ )^2 = ABAB = ABB = AB$

4. What is $(1001)_2$ equal to?

(a) $(5)_{10}$
(b) $(9)_{10}$
(c) $(17)_{10}$
(d) $(11)_{10}$

Solution:

$(1001)_2 = 1 \times 2^0 + 1 \times 2^3$
$= 9$

Answer: (b)

5. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6$ equal to , where $i = \sqrt{-1}$?

(a) 1
(b) 1/6
(c) 6
(d) 2

Solution:

$\sqrt{3} + i = 2e^{i\frac{\pi}{6}}$

$\sqrt{3} - i = 2e^{-i\frac{\pi}{6}}$

$$\frac{\sqrt{3} + i}{\sqrt{3} - i} = \frac{2e^{i\frac{\pi}{6}}}{2e^{-i\frac{\pi}{6}}} = e^{i\frac{\pi}{3}}$$

$$\left(\frac{\sqrt{3} + i}{\sqrt{3} - i}\right)^6 = e^{i2\pi} = 1$$

Answer: (a)

6 Let z be a complex number such that $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$. What is z equal to?

(a) $2\sqrt{3} + 2i$
(b) $2\sqrt{3} - 2i$
(c) $-2\sqrt{3} + 2i$
(d) $-\sqrt{3} + i$

Solution:

$z = |z|e^{i(\arg(z))}$

$z = 4\, e^{i\frac{5\pi}{6}} = 4\cos\frac{5\pi}{6} + i4\sin\frac{5\pi}{6}$

$\cos\frac{5\pi}{6} = \cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$z = -2\sqrt{3} + 2i$

Answer: (c)

7. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, where $i = \sqrt{-1}$, then what is x equal to?

(a) 3 (b) 2

(c) 1 (d) 0

Solution:

$\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$= 6i\begin{vmatrix} 3i & -1 \\ 3 & i \end{vmatrix} + 3i\begin{vmatrix} 4 & -1 \\ 20 & i \end{vmatrix} + 1\begin{vmatrix} 4 & 3i \\ 20 & 3 \end{vmatrix}$

$= 6i(3i^2 + 3) + 3i(4i + 20) + (12 - 60i)$

$= 12i^2 + 60i + 12 - 60i = 0$

Answer: (d)

8 .If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then what is h equal to?

(a) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

(b) $\frac{1}{2}\left(-\frac{b}{a} + \frac{q}{p}\right)$

(c) $\frac{1}{2}\left(\frac{b}{a} + \frac{q}{a}\right)$

(d) $\frac{1}{2}\left(-\frac{b}{p} + \frac{q}{a}\right)$

Solution:

If α, β are the roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a}$$

If $\alpha + h, \beta + h$ are the roots of

$px^2 + qx + r = 0$

$\alpha + h + \beta + h = -\frac{q}{p}$

$-\frac{b}{a} + 2h = -\frac{q}{p}$

$h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

Answer: (b)

9. If the matrix A is such that $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, then what is A equal to?

(a) $\begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -4 \\ 0 & -1 \end{pmatrix}$

Solution:

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} a + 3c & b + 3d \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$

$c = 0, \quad d = -1$

$a + 3c = 1$

$b + 3d = 1$

$a = 1, b = 4$

$A = \begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$

Answer: (a)

10. Consider the following statements:

1. Determinant is a square matrix.
2. Determinant is a number associated with a square matrix.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

11. if A is an invertible matrix, then what is $\det(A^{-1})$ equal to?

(a) det A (b) $\frac{1}{\det A}$
(c) 1 (d) None of the above

Solution: $AA^{-1} = I$

$$\det(AA^{-1}) = \det(I)$$

$$\det(A)\det(A^{-1}) = 1$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

12. From the matrix equation $AB = AC$, where A, B, C are the square matrices of same order, we can conclude B=C provided

(a) A is non-singular

(b) A is singular.

(c) A is symmetric

(d) A is skew symmetric

Solution: $AB = AC$

Premultiply by A^{-1}

$A^{-1}AB = A^{-1}AC$

$B = C$

Inverse of matrix A exists if matrix A is non-singular matrix.

13. If $A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric, then what is x equal to?

(a) 2
(b) 3
(c) -1
(d) 5

Solution:

If A is symmetric matrix then $a_{ij} = a_{ji}$

$a_{12} = a_{21}$

$x + 2 = 2x - 3$

$x = 5$

Answer: (d)

14. if $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then which one of the following is correct?

(a) $\frac{a}{b}$ is one of the cube roots of unity.

(b) $\frac{a}{b}$ is one of the cube roots of -1.

(c) a is one of the cube roots of unity.

(d) b is one of the cube roots of unity.

Solution: $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$

$a\begin{vmatrix} a & b \\ 0 & a \end{vmatrix} - b\begin{vmatrix} 0 & b \\ b & a \end{vmatrix} = 0$

$a^3 + b^3 = 0$

$\frac{a^3}{b^3} + 1 = 0$-+

$x^3 + 1 = 0$

$x = \frac{a}{b}$ is the cube root of -1.

15. What is

$$\frac{(1+i)^{4n+}}{(1-i)^{4n+3}}$$

equal to, where n is a natural number and $i = \sqrt{-1}$?

(a) 2 (b) 2i

(c) – 2i (d) i

Solution:

$(1+i) = \sqrt{2}\, e^{i\frac{\pi}{4}}$

$(1+i)^{4n+5} = (\sqrt{2})^{4n+5} e^{i\frac{(4n+5)\pi}{4}}$

$1 - i = \sqrt{2}\, e^{-i\frac{\pi}{4}}$

$(1-i)^{4n+5} = (\sqrt{2})^{4n+3} e^{-i\frac{(4n+3)\pi}{4}}$

$$\frac{(1+i)^{4n+}}{(1-i)^{4n+3}} = \frac{(\sqrt{2})^{4n+5}}{(\sqrt{2})^{4n+3}} e^{i\frac{8n+8}{4}\pi}$$
$$= 2(\cos 2(n+1)\pi + i \sin 2(n+1)\pi)$$

$\cos 2(n+1)\pi = 1$ and $\sin 2(n+1)\pi = 0$

$$\frac{(1+i)^{4n+5}}{(1-i)^{4n+}} = 2$$

Answer: (a)

16. What is the number of ways in which one can post 5 letters in 7 letter boxes?

(a) 7^5 (b) 3^5

(c) 5^7 d) 2520

Solution: Number of ways **= 5^7**

17. What is the number of ways that a cricket team of 11 players can be made out of 15 players?

(a) 364 (b) 1001

(c) 1365 (d) 32760

Solution:

Number of ways:

$$C(15,11) = \frac{15!}{11!4!}$$
$$= 1365$$

Answer: (c)

18. A and B are two sets having 3 elements in common. If n(A) =5, n(B) =4, then what is n(A x B) equal to ?

(a) 0 (b) 9

(c) 15 (d) 20

Solution: Number of element in Cartesian product of A and B = n(A) x n (B) = 5 x 4 = 20 elements

19. IF A and B are square matrices of second order such that $|A| = -1$, $|B| = 3$, then what is $|3AB|$ equal to?

(a) 3 (b) -9

(c) -27 (d) None of the above

Solution: $|3AB| = 3|A||B|$

$$= 3 \times -1 \times 3 = -9$$

Consider the function f(x) = $\frac{x-1}{x+1}$

20. What is $\frac{f(x)+1}{f(x)-1} + x$ equal to?

(a) 0 (b) 1`

(c) 2x (d) 4x

Solution:

$$f(x) = \frac{x-1}{x+1}$$

$$\frac{f(x)+1}{f(x)-1} + x = \frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1} + x$$

$$= \frac{\mathbf{2x}}{\mathbf{-2}} + \mathbf{x} = \mathbf{0}$$

Answer: (a)

21. Consider the function f(x) = $\frac{x-1}{x+1}$

What is $f(2x)$ equal to?

(a) $\frac{f(x)+1}{f(x)+3}$

(b) $\frac{f(x)+1}{3f(x)+1}$

(c) $\frac{3f(x)+1}{f(x)+3}$

(d) $\frac{f(x)+3}{3f(x)+1}$

Solution:

$$f(2x) = \frac{2x-1}{2x+1}$$

At x = 0

$$f(0) = -1$$

$$\frac{3f(0)+1}{f(0)+3} = \frac{-3+1}{-1+3} = -1$$

Answer: (c)

22. What is $f(f(x))$ equal to?

(a) x (b) - x

(c) $-\frac{1}{x}$ (d) None

Solution:

$$f\big(f(x)\big) = \frac{f(x)-1}{f(x)+1}$$

$$= \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$$

$$= -\frac{1}{x}$$

Answer: (c)

Consider the expansion$\left(x^2 + \frac{1}{x}\right)^{15}$.

23. What is the independent term in the given expansion?

(a) 2103

(b) 3003
(c) 4503
(d) None of the above

Solution:

$$\left(x^2+\frac{1}{x}\right)^{15} = \sum_{r=0}^{15} C(15,r)(x^2)^r\left(\frac{1}{x}\right)^{15-r}$$

$$(x^2)^r\left(\frac{1}{x}\right)^{15-r} = x^{3r-15}$$

If r = 5 them term is independent of x

$$C(15,5) = \frac{15!}{5!\,10!} = 3003$$

Answer: (b)

24. What is the ratio of coefficient of x^{15} to the term independent of x in the given expansion?

(a) 1
(b) ½
(c) 2/3
(d) ¾

Solution:

x^{3r-}

$3r - 15 = 15$

$r = 10$

$$C(15,10) = \frac{15!}{10!\,5!}$$

$$\frac{C(15,10)}{C(15,5)} = 1$$

Answer: (a)

25. Consider the following statements:

1. There are 15 terms in the given expansion.

2.The coefficient of x^{12} is equal to that of x^3.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution:

$(x+y)^n$

Total number of term is n+1

$$\left(x^2+\frac{1}{x}\right)^{15}$$

Total number of term is 15 + 1 =16

$x^{3r-15} = x^{12}$

$3r - 15 = 12$

$r = 9$

$x^{3r-15} = x^3$

$3r - 15 = 3$

$r = 6$

Coefficient of $x^{12} = C(15,9)$

Coefficient of x^3 = $C(15,6)$

$C(n,r) = C(n,n-r)$

$C(15,9) = C(15,6)$

Answer: (b)

26. What is $\sqrt{1+\sin 2\theta}$ equal to?

(a) $\cos\theta - \sin\theta$
(b) $\cos\theta + \sin\theta$
(c) $2\cos\theta + \sin\theta$
(d) $\cos\theta + 2\sin\theta$

Solution:

$$\sqrt{1+\sin 2\theta}$$

$$= \sqrt{sin^2\theta + cos^2\theta + 2\sin\theta\cos\theta}$$

$$= \sin\theta + \cos\theta$$

Answer: (b)

27.A lamp post stands on a horizontal plane. From a point situated at a distance 150 m from its foot, the angle of elevation of the top is 30^0, What is the height of the lamp post?

(a) 50 m
(b) $50\sqrt{3}$ m
(c) $\frac{50}{\sqrt{3}}$ m
(d) 100m

Solution:

$$\tan 30^0 = \frac{x}{150}$$

$$x = 50\sqrt{3}$$

28. If $\cot A = 2$ and $\cot B = 3$, then what is the value of A + B?

(a) $\pi/6$

(b) π

(c) $\pi/2$

(d) $\pi/4$

Solution:

$$\cot(A+B) = \frac{\cot A + \cot B}{\cot A . \cot B - 1}$$

$$= \frac{2+3}{2\times 3 - 1}$$

$$= 1$$

$$A + B = \frac{\pi}{4}$$

Answer: (d)

29. What is $sin^2 66\frac{1}{2}^0 - sin^2 23\frac{1}{2}^0$ equal to ?

(a) $\sin 47^0$

(b) $\cos 47^0$

(c) $2\sin 47^0$

(d) $2\cos 47^0$

Solution:

$$sin^2 66\frac{1}{2}^0 - sin^2 23\frac{1}{2}^0$$

$$\sin 66\frac{1}{2}^0 = \sin\left(90^0 - 23\frac{1}{2}^0\right)$$

$$= \cos 23\frac{1}{2}^0$$

$$sin^2 66\frac{1}{2}^0 - sin^2 23\frac{1}{2}^0$$

$$cos^2 23\frac{1}{2}^0 - sin^2 23\frac{1}{2}^0$$

$$\cos 47^0$$

Answer: (b)

30. What is $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}$ equal to?

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$

Solution:

$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}$$

$$\sin^{-1}\frac{3}{5} = \theta$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$\theta = \cos^{-1}\frac{4}{5}$$

$$\cos^{-1}\frac{4}{5} + \sin^{-1}\frac{4}{5}$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$x = \frac{4}{5}$$

Answer: (a)

31. What is $\frac{\cos 7x - \cos 3}{\sin 7x - 2\sin 5x + \sin 3x}$ equal to?

(a) $\tan x$

(b) cot x

(c) tan 2x

(d) cot 2x

Solution:

$$\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{B-A}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos 7x - \cos 3x$$

$$= 2\sin\frac{7x+3x}{2}\sin\frac{3x-7x}{2}$$

$$= 2\sin 5x \sin(-2x)$$

$$\sin 7x - 2\sin 5x + \sin 3x$$

$$= \sin 7x - \sin 5x + \sin 3x - \sin 5x$$

$$\sin 7x - \sin 5x$$

$$= 2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}$$

$$= 2\cos 6x \sin x$$

$$\sin 3x - \sin 5x$$

$$= 2\cos\frac{3x+5x}{2}\sin\frac{3x-5x}{2}$$

$$= 2\cos 4x \ \sin(-x)$$

$$\sin 7x - \sin 5x + \sin 3x - \sin 5x$$
$$= 2\cos 6x \sin x - 2\cos 4x \sin x$$

$$= 2\sin x\,(\cos 6x - \cos 4x)$$

$$= 2\sin x \, 2\sin 5x \sin(-x)$$

$$\frac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$$

$$= \frac{-2\sin 5x \sin 2x}{-2\sin x \sin x \sin 5x} = \cot x$$

32. In a triangle ABC, $c = 2, A = 45^0, a = 2\sqrt{2}$, then what is C equal to?

(a) 30^0
(b) 15^0
(c) 45^0
(d) None of the above

Solution:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{2\sqrt{2}}{\sin 45^0} = \frac{2}{\sin C}$$

$$\sin C = \frac{2\sin 45^0}{2\sqrt{2}}$$

$$\sin C = \frac{1}{2}$$

$$C = 30^0$$

Answer: (a)

33. In a triangle ABC, $\sin A - \cos B = \cos C$, then what is B equal to?

(a) π (b) $\pi/3$

(c) $\pi/2$ (d) $\pi/4$

Solution: $\sin A - \cos B = \cos C$

$$\sin A = \cos B + \cos C$$

$$2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\frac{B+C}{2}\cos\frac{C-B}{2}$$

$$A + B + C = \pi$$

$$B + C = \pi - A$$

$$\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2}$$

$$\cos\frac{B+C}{2} = \cos\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\frac{A}{2}$$

$$\cos\frac{A}{2} = \cos\frac{C-B}{2}$$

$$\frac{A}{2} = \frac{C-B}{2}$$

$$A + B = C$$

$$A + B + C = \pi$$

$$C = \frac{\pi}{2}$$

$$\mathrm{Sin}A = \mathrm{Cos}B$$

A = 30^0 and B = 60^0

34. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then what is $\frac{\tan x}{\tan y}$ equal to?

(a) $\frac{b}{a}$
(b) $\frac{a}{b}$
(c) ab
(d) 1

Solution:

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$a\sin(x+y) - b\sin(x+y)$$
$$= a\sin(x-y) + b\sin(x-y)$$

$a(\sin(x+y)-\sin(x-y))$

$=b(\sin(x+y)-\sin(x-y))$

$2a\cos x\sin y=2b\sin x\cos y$

$\frac{a}{b}=\frac{\tan x}{\tan y}$

Answer: (b)

35. If $\sin A\sin(60^0-A)\sin(60^0+A)=k\sin 3A$, then what is k equal to ?

(a) $1/4$

(b) $1/2$

(c) 1

(d) 4

Solution:

$\sin A\sin(60^0-A)\sin(60^0+A)=k\sin 3A$

Take A =30^0

$\sin 30^0\sin(60^0-30^0)\sin(60^0+30^0)$

$=k\sin 3\times 30^0$

$\frac{1}{2}\times\frac{1}{2}\times 1=k$

$k=\frac{1}{4}$

Answer: (a)

36. The line $y=\sqrt{3}$ meets the graph y = tan x, where $x\in\left(0,\frac{\pi}{2}\right)$, in k points. What is k equal to?

(a) one (b) two

(c) Three (d) Infinity

Solution:

tan x is increasing function for given interval. So it cuts at only one point.

Answer: (a)

37.Which one of the following is one of the solutions of the equation $\tan 2\theta\,.\tan\theta=1$?

(a) $\pi/12$

(b) $\pi/6$

(c) $\pi/4$

(d) $\pi/3$

Solution:

$\tan 2\theta\,.\tan\theta=1$

$\frac{2\tan\theta}{1-tan^2\theta}\times\tan\theta=1$

$2tan^2\theta=1-tan^2\theta$

$3tan^2\theta=1$

$tan^2\theta=\frac{1}{3}$

$\theta=30^0$

Answer: (b)

For the next three (03) items that follow:

Given that $16\sin^5 x=p\sin 5x+q\sin 3x+r\sin x$.

38. What is the value of p?

(a) 1 (b) 2

(c) -1 (d) -2

39. What is the value of q?

(a) 3 (b) 5

(c) 10 (d) -5

40. What is the value of r?

(a) 5 (b) 8

(c) 10 (d) -10

Solution:

$4\sin^3\theta=3\sin\theta-\sin 3\theta$

$2\sin^2\theta=1-\cos 2\theta$

$16\sin^5\theta=2\times 4\sin^3\theta\times 2\sin^2\theta$

$=2(3\sin\theta-\sin 3\theta)(1-\cos 2\theta)$

$=2(3\sin\theta-3\sin\theta\ \cos 2\theta-\sin 3\theta+\sin 3\theta\ \cos 2\theta)$

$=6\sin\theta-2\sin 3\theta-6\sin\theta\cos 2\theta+2\sin 3\theta\cos 2\theta$

$=6\sin\theta-2\sin 3\theta-3(\sin(\theta+2\theta)+\sin(\theta-2\theta))+\sin 5\theta+\sin\theta)$

$=\sin 5\theta-5\sin 3\theta+4\sin\theta$

p = 1, q = -5, and r =4

41. What is the length of the latus rectum of the ellipse $25x^2 + 16y^2 = 400$?

(a) 25/2 (b) 25/4

(c) 16/5 (d) 32/5

Solution: Equation of ellipse $25x^2 + 16y^2 = 400$

$$\frac{25x^2}{400} + \frac{16y^2}{400} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Given ellipse have major axis along y-axis and minor axis along x-axis.

$b^2 = 16$ and $a^2 = 25$. If e is eccentricity of the ellipse then $b^2 = a^2(1 - e^2)$

$$16 = 25(1 - e^2)$$

$$e^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$e = \frac{3}{5}$$

Co-ordinate of Focus of ellipse is $(0, \pm ae) \equiv (0, \pm 3)$

At y = 3 , $\frac{x^2}{16} + \frac{3^2}{25} = 1$

$$x = \frac{16}{5}$$

Length of latus rectum = $2x = \frac{32}{5}$

Answer: (d)

For the next (02) items that follow:

Consider the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$.

42. What is the distance between the centers of the two circles?

(a) $\sqrt{a^2 + b^2}$

(b) $a^2 + b^2$

(c) $a + b$

(d) $2(a + b)$

Solution:

$$x^2 + y^2 + 2ax + c = 0$$

Centre $C_1 \equiv (-a, 0)$

Radius $R_1 = \sqrt{a^2 - c}$

$$x^2 + y^2 + 2by + c = 0$$

Centre $C_2 \equiv (0, -b)$

Radius $R_2 = \sqrt{b^2 - c}$

$$C_1C_2 = \sqrt{(-a)^2 + (-b)^2}$$

Answer: (a)

43. The two circles touch each other if

(a) $c = \sqrt{a^2 + b^2}$

(b) $\frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

(c) $c = \frac{1}{a^2} + \frac{1}{b^2}$

(d) $c = \frac{1}{a^2+b^2}$

Solution:

If two circles touch each other externally, then center distance is equal to sum of radius of the circles.

$C_1C_2 = R_1 + R_2$

$\sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$

$a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2 - c)(b^2 - c)}$

$c = \sqrt{(a^2 - c)(b^2 - c)}$

$c^2 = a^2b^2 - c(a^2 + b^2) + c^2$

$c = \frac{a^2b^2}{a^2 + b^2}$

$\frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

Answer: (c)

44. A(3,4) and B(5, -2) are two points and P is a point such that $PA = PB$. If the area of triangle PAB is 1 square unit, what are the coordinates of P?

(a) (1, 0) only

(b) (7, 2) only

(c) (1, 0) or (7, 2)

(d) Neither (1, 0) nor (7, 2)

Solution:

Slope of line passing through AB

$$\frac{y_B - y_A}{x_B - x_A} = \frac{-2-4}{5-3} = -3$$

Slope of line perpendicular to line AB

$$m = \frac{1}{3}$$

Since PA = PB, therefore locus of P is perpendicular bisector of AB.

Coordinate of mid point of AB

$$\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}$$

$$\frac{3+5}{2}, \frac{4-2}{2} \equiv (4, 1)$$

Coordinate of point P $(4 + r\cos\theta, 1 + r\sin\theta)$

$$\tan\theta = \frac{1}{3}$$

$$\cos\theta = \frac{3}{\sqrt{10}}$$

$$\sin\theta = \frac{1}{\sqrt{10}}$$

$$AB = \sqrt{(5-2)^2 + (4+2)^2} = \sqrt{9+36} = \sqrt{45}$$

Area of triangle APB,

$$= \frac{1}{2} \times AB \times PM = 1$$

$$PM = \frac{2}{3\sqrt{5}}$$

$$4 + r\cos\theta, = 4 + \frac{2}{3\sqrt{5}} \times \frac{3}{\sqrt{10}}$$

45. What is the product of the perpendicular drawn from the points $(\pm\sqrt{a^2 - b^2}, 0)$ upon the line $b\,x\,\cos\alpha + a\,y\,\sin\alpha = ab$?

(a) a^2 (b) b^2

(c) $a^2 + b^2$ (d) $a + b$

Solution:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d_1 = \frac{\left|b \times \sqrt{a^2 - b^2}\cos\alpha + b \times 0 \times \sin\alpha - ab\right|}{\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}}$$

$$d_2 = \frac{\left|-b \times \sqrt{a^2 - b^2}\cos\alpha + b \times 0 \times \sin\alpha - ab\right|}{\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}}$$

$$d_1 d_2 = \frac{\left|b\sqrt{a^2 - b^2}\cos\alpha - ab\right|}{\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}} \times \frac{\left|-b\sqrt{a^2 - b^2}\cos\alpha - ab\right|}{\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}}$$

$$d_1 d_2 = \frac{\left|b^2\left((a^2 - b^2)\cos^2\alpha - a^2\right)\right|}{b^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$d_1 d_2 = \frac{|b^2(-a^2\sin^2\alpha - b^2\cos^2\alpha)|}{b^2\cos^2\alpha + a^2\sin^2\alpha} = b^2$$

Answer: (b)

46. Which one of the following is correct in respect of the equation $\frac{x-1}{2} = \frac{y-2}{3}$ and $2x + 3y = 5$?

(a) They represent two lines which are parallel.

(b) They represent two lines which are perpendicular.

(c) They represent two lines which are neither parallel nor perpendicular

(d) The first equation does not represent a line.

Solution:

Equation of line L_1: $\frac{x-1}{2} = \frac{y-2}{3}$

$$y = \frac{3}{2}x - \frac{3}{2} + 2$$

Equation of line L_2: $2x + 3y = 5$

$$y = -\frac{2}{3}x + 5$$

Slope of line L_1: $m_1 = \frac{3}{2}$

Slope of line L_2: $m_2 = -\frac{2}{3}$

$m_1 m_2 = -1$

Lines are perpendicular to each other.

Answer: (b)

Consider a sphere passing through the origin and the points (2, 1, -1), (1, 5, -4), (-2, 4, -6).

47. What is the radius of the sphere?

(a) $\sqrt{12}$

(b) $\sqrt{14}$

(c) 12

(d) 14

Solution:

Let centre of the circle is (x_0, y_0, z_0) and radius of circle is R.

Equation of sphere

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

Since circle passes through origin (0, 0, 0)

$(x_0)^2 + (y_0)^2 + (z_0)^2 = R^2$

$(2 - x_0)^2 + (1 - y_0)^2 + (-1 - z_0)^2 = R^2$

$(1 - x_0)^2 + (5 - y_0)^2 + (-4 - z_0)^2 = R^2$

$(-2 - x_0)^2 + (4 - y_0)^2 + (-6 - z_0)^2 = R^2$

By solving these four equations we $x_0 = -1, y_0 = 2, z_0 = -3$ and R = $\sqrt{14}$

Answer: (b)

48. What is the centre of the sphere?

(a) (-1, 2, -3)
(b) (1, -2, 3)
(c) (1, 2, -3)
(d) (-1, -2, -3)

Answer: (a)

49. Consider the following statements:

1. The sphere passes through the point (0, 4, 0)
2. The point (1, 1, 1) is at a distance of 5 unit from the centre of the sphere.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Solution:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

$x_0 = -1, y_0 = 2, z_0 = -3$ and R = $\sqrt{14}$

$(x + 1)^2 + (y - 2)^2 + (z \mp 3)^2 = 14$

Let point P is (0, 4, 0)

$(0 + 1)^2 + (4 - 2)^2 + (0 + 3)^2 = 14$

$1 + 2 + 9 \neq 14$

So point P not passes through sphere.

The point (1, 1, 1) is at a distance of 5 unit from the centre of the sphere

$$d = \sqrt{(1 - (-1))^2 + (1 - 2)^2 + (-3 - 1)^2}$$

$$= \sqrt{25} = 5$$

Answer: (b)

The line joining the points (2, 1, 3) and (4, -2, 5) cuts the plane $2x + y - z = 3$.

50. Where does the line cut the plane?

(a) (0, -4, -1)
(b) (0, -4, 1)
(c) (1, 4, 0)
(d) (0, 4, 1)

Solution:

Equation of line

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 2}{4 - 2} = \frac{y - 1}{-2 - 1} = \frac{z - 3}{5 - 3}$$

$$\frac{x - 2}{2} = \frac{y - 1}{-3} = \frac{z - 3}{2} = k$$

$x = 2 + 2k$

$y = 1 - 3k$

$z = 3 + 2k$

Equation of plane

$2x + y - z = 3$

$2(2 + 2k) + (1 - 3k) - (3 + 2k) = 3$

$4 + 4k + 1 - 3k - 3 - 2k = 3$

$2 - k = 3$

$k = -1$

$x = 0, y = 4, z = 1$

Answer: (d)

51. What is the ratio in which the plane divides the line?

(a) 1:1
(b) 2:3
(c) 3:4
(d) None of the above

Solution:

Let A = (2, 1, 3) , B = (4, -2, 5).

P = (0, 4, 1)

$AP = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{17}$

$BP = \sqrt{4^2 + 6^2 + 4^2} = \sqrt{68}$

$$\frac{AP}{BP} = \frac{\sqrt{17}}{\sqrt{68}} = \frac{1}{2}$$

Answer: (d)

Consider the plane passing through the points A(2, 2, 1), B(3, 4, 2) and C(7, 0, 6).

52.Which one of the following points lies on the plane?

(a) (1, 0, 0)

(b) (1, 0, 1)

(c) (0, 0, 1)

(d) None of the above

Solution:

Let Equation of plane P

$ax + by + cz = 1$

$2a + 2b + c = 1$

$3a + 4b + 2c = 1$

$7a + 6c = 1$

$a = -\frac{1}{2}, b = \frac{5}{8}, c = \frac{3}{4}$

$-\frac{1}{2}x + \frac{5}{8}y + \frac{3}{4}z = 1$

Answer: (d)

53. What are the direction ratios of the normal to the plane?

(a) < 1, 0, 1>

(b) < 0, 1, 0>

(c) <1, 0, -1>

(d) None of the above

Solution:

Direction ratio (a, b, c)

(-4, 5, 6)

Answer: (d)

Consider the function $f(x) = \begin{cases} x^2 - 5 & x \le 3 \\ \sqrt{x+13} & x > 3 \end{cases}$

54. What is $\lim_{x\to 3} f(x)$ equal to?

(a) 2 (b) 4

(c) 5 (d) 13

Solution:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 - 5 = 4$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \sqrt{x+13} = 4$$

$$f(3) = 3^2 - 5 = 4$$

Answer: (b)

55. Consider the following statements:

1. The function is discontinuous at $x = 3$.
2. The function is not differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3) = 4$$

Function f(x) is continuous at x = 3.

$\frac{dy}{dx} = \frac{d(x^2-5)}{dx} = 2x$

$\left(\frac{dy}{dx}\right)_{x=3^-} = 2 \times 3 = 6$

$\left(\frac{dy}{dx}\right)_{x=3^+} = \frac{1}{2\sqrt{x+13}} = \frac{1}{8}$

56. Where does the line cut the parabola?

(a) At (-2, 3) only

(b) At (4, 12) only

(c) At both (-2, 3) and (4, 12)

(d) Neither at (-2, 3) nor at (4, 12)

Solution:

Point of intersection of line and parabola.

Equation of line L: $2y = 3x + 12$

Equation of parabola: $4y = 3x^2$

$2(3x + 12) = 3x^2$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = -2, 4$

Point of intersection are A(-2, 3) and (4, 12).

Answer: (c)

57.What is the area enclosed by the parabola and the line?

(a) 27 square unit

(b) 36 square unit

(c) 48 square unit

(d) 54 square unit

Solution:

$$I = \int_{x=-2}^{x=4} \frac{3x+1}{2} - \frac{3}{4}x^2\, dx$$

$$I = \frac{3}{2} \times \frac{x^2}{2} + 6x - \frac{3}{4} \times \frac{x^3}{3}$$

$$I = \frac{3}{4}(4^2 - (-2)^2) + 6(4 - (-2))$$

$$-\frac{1}{4}(4^3 - (-2)^3)$$

$$I = 27$$

Answer: (a)

58.What is the area enclosed by the parabola, the line and the y-axis in the first quadrant ?

(a) 7 square unit

(b) 14 square unit

(c) 20 square unit

(d) 21 square unit

Solution: Equation of parabola $4y = 3x^2$

Equation of Line $2y = 3x + 12$

Point of intersection of parabola and line in first quadrant is x = 4 and y = 12

Area enclosed by the parabola, the line and the y-axis in the first quadrant is equal to area enclosed by parabola and y-axis from y = 0 to y= 12 minus area enclosed by line and y-axis from y = 6 to y = 12.

$$\text{Area enclosed by parabola and } y - \text{axis} = \int_0^{12} \frac{2}{\sqrt{3}}\sqrt{y}\, dy = 32$$

Area enclosed by line and y-axis = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times 4 = 12$

Area = 32 – 12 =20

Answer: (c)

59. Consider the function

$$f(x) = \begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$$

What is the non-zero value of k for which the function is continuous at $x = 0$?

(a) 1/4 (b) 1/2

(c) 1 (d) 2

Solution: $\lim_{x \to 0^-} \frac{k \tan kx}{kx} = k$

$$\lim_{x \to 0^+} 3x + 2k^2 = 2k^2$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$k = 2k^2$$

$$k = 0, \frac{1}{2}$$

Answer: (b)

60. Consider the following statements:

1. The function $f(x) = [x]$, where [.] is the greatest integer function defined on R, is continuous at all points except at x =0.

2. The function $f(x) = \sin|x|$ is continuous for all $x \in R$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Solution: $y = \begin{cases} \sin x & x > 0 \\ -\sin x & x < 0 \end{cases}$

Function sin x is continuous function.

$f(x) = [x]$ is discontinuous function.

For the next two(02) items that follow:

Consider the curve $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

61. What is $\frac{dy}{dx}$ equal to?

(a) $\tan\theta$

(b) $\cot\theta$

(c) $\sin 2\theta$

(d) $\cos 2\theta$

Solution:

$x = a(\cos\theta + \theta\sin\theta)$

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$= a\theta\cos\theta$

$y = a(\sin\theta - \theta\cos\theta)$.

$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$= a\theta\sin\theta$

$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos} = \tan\theta$

Answer: (a)

62. What is $\frac{d^2y}{dx^2}$ equal to?

(a) $\sec^2\theta$

(b) $-\text{cosec}^2\,\theta$

(c) $\frac{\sec^3\theta}{a\theta}$

(d) None of the above

$\frac{dy}{dx} = \tan\theta$

$\frac{d^2y}{dx^2} = \frac{d\tan\theta}{dx} = \frac{d\tan\theta}{d\theta}\frac{d\theta}{dx}$

$= \sec^2\theta\,\frac{1}{a\theta\cos\theta} = \frac{\sec^3\theta}{a\theta}$

Answer: (c)

63. What is the area of the parabola $y^2 = 4bx$ bounded by the latus rectum?

(a) $2b^2/3$ square unit

(b) $4b^2/3$ square unit

(c) b^2 square unit

(d) $8b^2/3$ square unit

Solution: Equation of parabola

$y^2 = 4bx$

Focus of parabola F ≡ $(b, 0)$

Area bounded by the latus rectum

$$= 2\int_0^b \sqrt{4bx}\,dx = 2\sqrt{4b}\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^b = \frac{2\times 2\times 2\sqrt{b}b^{\frac{3}{2}}}{3}$$

$$= \frac{8b^2}{3}$$

64. If $y = x\ln x + xe^x$, then what is the value of $\frac{dy}{dx}$ at x =1 ?

(a) 1+ e

(b) $1 - e$

(c) $1 + 2e$

(d) None of the above

Solution:

$y = x\ln x + xe^x$

$\frac{dy}{dx} = \ln x + 1 + e^x + xe^x$

at x =1

$\frac{dy}{dx} = 0 + 1 + e + e = 1 + 2e$

Answer: (c)

65. What is $\lim_{x\to 0}\frac{\log_5(1+x)}{x}$ equal to?

(a) 1

(b) $\log_5 e$

(c) $\log_e 5$

(d) 5

Solution:

$\lim_{x\to 0}\frac{\log_5(1+x)}{x}$

Apply L'Hospital Rule,

$\lim_{x\to 0}\frac{\log_e 5}{1+x} = \log_e 5$

Answer: (c)

The line $2y = 3x + 12$ cuts the parabola $4y = 3x^2$.

66. What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2$?

(a) 1 (b) 2

(c) 3 (d) 4

Solution:

$$\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left(\frac{d^3y}{dx^3}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^4$$

Degree = 3

Order = 3

Answer: (c)

67. What is the solution of the equation $\ln\left(\frac{dy}{dx}\right) + x = 0$?

(a) $y + e^x = c$

(b) $y - e^{-x} = c$

(c) $y + e^{-x} = c$

(d) $y - e^x = c$

where c is an arbitrary constant

Solution:

$\ln\left(\frac{dy}{dx}\right) + x = 0$

$\frac{dy}{dx} = e^{-x}$

$y = \frac{e^{-x}}{-1} + c$

$y + e^{-x} = c$

Answer: (c)

Let $f(x) = ax^2 + bx + c$ such that $f(1) = f(-1)$ and a, b, c are in Arithmetic Progression.

68. What is the value of b?

(a) -1

(b) 0

(c) 1

(d) Cannot be determined due to insufficient data

Solution:

$f(x) = ax^2 + bx + c$

$f(1) = a \times 1^2 + b \times 1 + c = a + b + c$

$f(-1) = a(-1)^2 + b(-1) + c$

$f(-1) = a - b + c$

$f(1) = f(-1)$

$a + b + c = a - b + c$

$2b = 0$

$b = 0$

Answer: (b)

69. $f'(a), f'(b), f'(c)$ are in

(a) A.P.

(B) G.P.

(c) H.P.

(d) Arithmetico-geometric progression

Solution:

Since a, b, and c are in A.P.

$b - a = c - b = d(\text{common difference})$

$f'(x) = 2ax + b$

$f'(a) = 2a^2 + b$

$f'(b) = 2ab + b$

$f'(c) = 2ac + b$

$f'(b) - f'(a) = 2a(b - a) = 2ad$

$f'(c) - f'(b) = 2a(c - b) = 2ad$

$f'(a), f'(b), f'(c)$ are in A.P.

Answer: (a)

70. $f''(a), f''(b), f''(c)$ are

(a) in A.P. only

(b) in G.P. only

(C) in both A.P. and G.P.

(d) neither in A.P. nor in G.P.

Solution:

$f''(x) = 2a$

$f''(a) = f''(b) = f''(c) = 2a$

$f''(a), f''(b), f''(c)$ are in A.P. and G.P. with common difference is 1 and common ratio is 1.

Answer: (c)

71. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then what is $\vec{a}.\vec{b}$ equal to ?

(a) 6 (b) 7

(c) 8 (d) 9

Solution:

$|\vec{a} \times \vec{b}| = 8$

$|\vec{a}||\vec{b}| \sin\theta = 8$

$2 \times 5 \times \sin\theta = 8$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta = 2 \times 5 \times \frac{3}{5} = 6$

Answer: (a)

72. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one of the following is correct?

(a) $|\vec{a}| = |\vec{b}|$

(b) $\vec{a}$ is parallel to $\vec{b}$

(c) $\vec{a}$ is perpendicular to $\vec{b}$

(d) $\vec{a}$ is a unit vector

Solution:

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$4\vec{a}.\vec{b} = 0$$

If dot product of two vector is equal to zero, it means both vectors are perpendicular to each other.

Answer: (c)

73.What is the area of the triangle OAB where O is the origin, $\overrightarrow{OA} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{OB} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$?

(a) $5\sqrt{6}$ square unit

(b) $\frac{5\sqrt{6}}{2}$ square unit

(c) $\sqrt{6}$ square unit

(d) $\sqrt{30}$ square unit

Solution:

$$\overrightarrow{OA} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$|\overrightarrow{OA}| = \sqrt{11}$$

$$\overrightarrow{OB} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$$

$$|\overrightarrow{OB}| = \sqrt{14}$$

$$\cos\theta = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$$

$$= \frac{6-1-3}{\sqrt{11}\sqrt{14}}$$

$$= \frac{2}{\sqrt{154}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\sin\theta = \sqrt{1-\frac{4}{154}} = \frac{\sqrt{150}}{\sqrt{154}}$$

Area of the triangle

$$= \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$$

$$= \frac{1}{2}|\overrightarrow{OA}||\overrightarrow{OB}|\sin\theta$$

$$= \frac{1}{2} \times \sqrt{11} \times \sqrt{14} \times \frac{\sqrt{150}}{\sqrt{154}}$$

$$= \frac{\sqrt{150}}{2} = \frac{5\sqrt{6}}{2}$$

Answer: (b)

74 .Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$?

(a) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

(b) $\hat{k}$

(c) $\frac{\hat{\jmath}+\hat{k}}{\sqrt{2}}$

(d) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$

Solution:

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 2i + 2j$$

$$|\vec{a} \times \vec{b}| = 2\sqrt{2}$$

$$\hat{\boldsymbol{n}} = \frac{\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}}{|\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}|} = \frac{2i+2j}{\mathbf{2\sqrt{2}}} = \frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$$

Answer: (a)

75. What is the interior acute angle of the parallelogram whose sides are represented by the vectors $\frac{1}{\sqrt{2}}\hat{\imath}+\frac{1}{\sqrt{2}}\hat{\jmath}+\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{\imath}-\frac{1}{\sqrt{2}}\hat{\jmath}+\hat{k}$?

(a) 60^0

(b) 45^0

(c) 30^0

(d) 15^0

Solution:

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a}=\frac{1}{\sqrt{2}}\hat{\imath}+\frac{1}{\sqrt{2}}\hat{\jmath}+\hat{k}$$

$$|\vec{a}|=\sqrt{\frac{1}{2}+\frac{1}{2}+1}=\sqrt{2}$$

$$\vec{b}=\frac{1}{\sqrt{2}}\hat{\imath}-\frac{1}{\sqrt{2}}\hat{\jmath}+\hat{k}$$

$$|\vec{b}|=\sqrt{\frac{1}{2}+\frac{1}{2}+1}=\sqrt{2}$$

$$\vec{a}.\vec{b}=\frac{1}{2}-\frac{1}{2}+1=1$$

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{1}{\sqrt{2}\sqrt{2}}=\frac{1}{2}$$

$$\theta=60^0$$

Answer: (a)

76. For what value of λ are the vectors $\lambda\hat{\imath}+(1+\lambda)\hat{\jmath}+(1+2\lambda)\hat{k}$ and $(1-\lambda)\hat{\imath}+(\lambda)\hat{\jmath}+2\hat{k}$ perpendicular?

(a) -1/3 (b) 1/3

(c) 2/3 (d) 1

Solution:

If two vectors perpendicular to each other then dot product is equal to zero.

$$\lambda(1-\lambda)+\lambda(1+\lambda)+2(1+2\lambda)=0$$

$$2\lambda+2+4\lambda=0$$

$$\lambda=-\frac{1}{3}$$

Answer: (a)

For the next four (04) items that follow:

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$ such that $|\vec{a}|=3$, $|\vec{b}|=5$ and , $|\vec{c}|=7$

77. What is the angle between $\vec{a}$ and $\vec{b}$?

(a) $\pi/6$ (b) $\pi/4$

(c) $\pi/3$ (d) $\pi/2$

Solution:

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$

Vector a, b , c are the sides of the triangle.

$$\cos\theta=\frac{a^2+b^2-c^2}{2ab}$$

$$\cos\theta=-\frac{1}{2}$$

$$\theta=120^0$$

Angle between a and b = 60^0 .

Answer: (c)

78. What is $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ equal to?

(a) – 83 (b) -83/2

(c) 75 (d) -75/2

Solution:

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$(\vec{a}+\vec{b}+\vec{c})^2=0$$

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$3^2+5^2+7^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-\frac{83}{2}$$

Answer: (b)

79. What is $|\vec{a}+\vec{b}|$ equal to?

(a) 7 (b) 8

(c) 10 (d) 11

Solution:

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$|\vec{a}+\vec{b}|=|-\vec{c}|=7$$

Answer: (a)

For the next two (02) items that follow:

Consider the function $f''(x)=\sec^4x+4$ with $f(0)=0$ and $f'(0)=0$.

80 What is $f'(x)$ equal to?

(a) $\tan x - \frac{\tan^3 x}{3} + 4x$

(b) $\tan x + \frac{\tan^3 x}{3} + 4x$

(c) $\tan x + \frac{\sec^3 x}{3} + 4x$

(d) $-\tan x - \frac{\tan^3 x}{3} + 4x$

81. What is $f(x)$ equal to?

(a) $\frac{2 \ln \sec x}{3} + \frac{\tan^2 x}{6} + 2x^2$

(b) $\frac{3 \ln se}{2} + \frac{\cot^2 x}{6} + 2x^2$

(c) $\frac{4 \ln \sec x}{3} + \frac{\sec^2 x}{6} + 2x^2$

(d) $\ln \sec\ x + \frac{\tan^4 x}{12} + 2x^2$

Solution:

$f''(x) = \sec^4 x + 4$

$\frac{df'(x)}{dx} = \sec^4 x + 4$

$\int df'(x) = \int \sec^4 x + 4\ dx$

$f'(x) = \int \sec^4 x\ dx + \int 4dx$

$\int \sec^4 x\ dx = \int \sec^2 x \sec^2 x\ dx$

$= \int (1 + \tan^2 x)\sec^2 x dx$

$= \int (1 + \tan^2 x) d(\tan x)$

$= \tan x + \frac{\tan^3 x}{3}$

$\int df'(x) = \tan x + \frac{\tan^3 x}{3} + 4x + c$

$f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$

Answer: (c)

$f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$

$\frac{df(x)}{dx} = \tan x + \frac{\tan^3 x}{3} + 4x$

$\int \tan x\ dx = \int \frac{\sin x}{\cos x} dx = \int \frac{-d(\cos x)}{\cos x}$

$= -\ln \cos x = \ln \sec x$

$\int \tan^3 x\ dx = \int \tan^2 x\ \tan x\ dx$

$= \int (\sec^2 x - 1)\tan x\ dx$

$= \frac{\tan^2 x}{2} + \ln \cos x$

$f(x) = \ln \sec x + \frac{\tan^2 x}{6} + \frac{\ln \cos x}{3} + 2x^2$

Answer: (a)

82. The probability that in a random arrangement of the letters of the word 'UNIVERSITY' , the two I's do not come together is

(a) $4/5$ (b) $1/5$

(c) $1/10$ (d) $9/10$

Solution:

Total No of letters = 10

Total No of arrangement = $\frac{10!}{2!}$

Total No of arrangement in which 2I's come together = 9!

Total No of arrangement in which 2I's do not come together = $\frac{10!}{2!} - 9! = 4 \times 9!$

Probability of 2I's do not come together = $\frac{4 \times 9!}{\frac{10!}{2!}} = \frac{4}{5}$

Answer: (a)

83. A fair coin is tossed four times. What is the probability that at most three tails occur?

(a) 7/8 (b)15/16

(b) 13/16 (d) 3/4

Solution:

Total number of sample space

n(s) = $2 \times 2 \times 2 \times 2 = 16$

E is event of occurrence of four tail.

n(E) = 1

P(E)= $\frac{n(E)}{n(s)} = \frac{1}{16}$

P(at most three tails) = $1 - \frac{1}{16} = \frac{15}{16}$

Answer: (b)

84. What is the mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17?

(a) 2.5 (b) 3

(c) 3.5 (d) 4

Solution: $Mean\ Deviation = \frac{|\sum|X-\mu||}{N}$

Mean μ = $\frac{4+7+8+9+10+12+13+}{8} = \frac{80}{8} = 10$

$$\sum |X - \mu| =$$

$$Mean\ Deviation = \frac{|\sum|X-\mu||}{N} = \frac{24}{8} = 3$$

85. What is $\int_0^{\pi/2} \frac{dx}{a^2cos^2x+b^2sin^2x}$ equal to?

(a) 2ab (b) $2\pi ab$

(c) $\frac{\pi}{2ab}$ (d) $\frac{\pi}{ab}$

Solution:

$$\int_0^{\pi/2} \frac{dx}{a^2cos^2x + b^2sin^2x}$$

$$= \int_0^{\pi/2} \frac{sec^2x\ dx}{a^2 + b^2tan^2x}$$

$$= \frac{1}{b^2}\int_0^{\pi/2} \frac{sec^2x\ dx}{\frac{a^2}{b^2} + tan^2x}$$

Let $\tan x = t$

$$sec^2xdx = dt$$

$$x = 0 \ \rightarrow t = 0$$

$$x = \pi/2 \rightarrow t = \infty$$

$$I = \frac{1}{b^2}\int_0^{\infty} \frac{dt}{\frac{a^2}{b^2} + t^2} = \frac{1}{b^2} \times \frac{1}{\frac{a}{b}} \tan^{-1} t|_0^{\infty}$$

$$= \frac{1}{ab}(\tan^{-1}\infty - \tan^{-1} 0)$$

$$= \frac{\pi}{2ab}$$