Production Function

Introduction

Production in economic terms is generally understood as the transformation of inputs into outputs. The inputs are what the firm buys, namely productive resources, and outputs are what it sells. Production is not the creation of matter but it is the creation of value. Production is also defined as producing goods which satisfy some human want. Production is a sequence of technical processes requiring either directly or indirectly the mental and physical skill of craftsman and consists of changing requiring either directly or indirectly the mental and physical skill of craftsman and consists of changing the shape, size and properties of materials and ultimately converting them into more useful articles.

Factors of production include resource inputs used to produce goods and services. Economist categorise input factors into four major categories such as land, labour, capital and Entrepreneur

Land: Land is heterogeneous in nature. The supply of land is fixed and it is a permanent factor of production but it is productive only with the application of capital and labour.

Labour: The supply of labour is inelastic in nature but it differs in productivity and efficiency and it can be improved.

Capital: is a man-made factor and is mobile but the supply is elastic.

Entrepreneur: The entrepreneur plans, supervises, organizes and controls the business activity and also takes risks.

Measures Of Productivity

Total production (TP): the maximum level of output that can be produced with a given amount of input.
Average Production (AP): output produced per unit of input AP = Q/L
Marginal Production (MP): the change in total output produced by the last unit of an input

The Production Function

The production function expresses a functional relationship between quantities of inputs and out-puts. It shows how and to what extent output changes with variations in inputs during a specified period of time. Basically, the production function is a technological or engineering concept which can be expressed in the form of a table, graph and equation showing the amount of output obtained from various combinations of inputs used in production, given the state of technology. Algebraically, it may be ex-pressed in the form of an equation as

$$Q = f(L,M,N,K,T)$$

Where *Q* stands for the output of a good per unit of time, *L* for labour, *M* for management (or organization), *N* for land (or natural resources), *K* for capital and *T* for given technology, and f refers to the functional relationship.

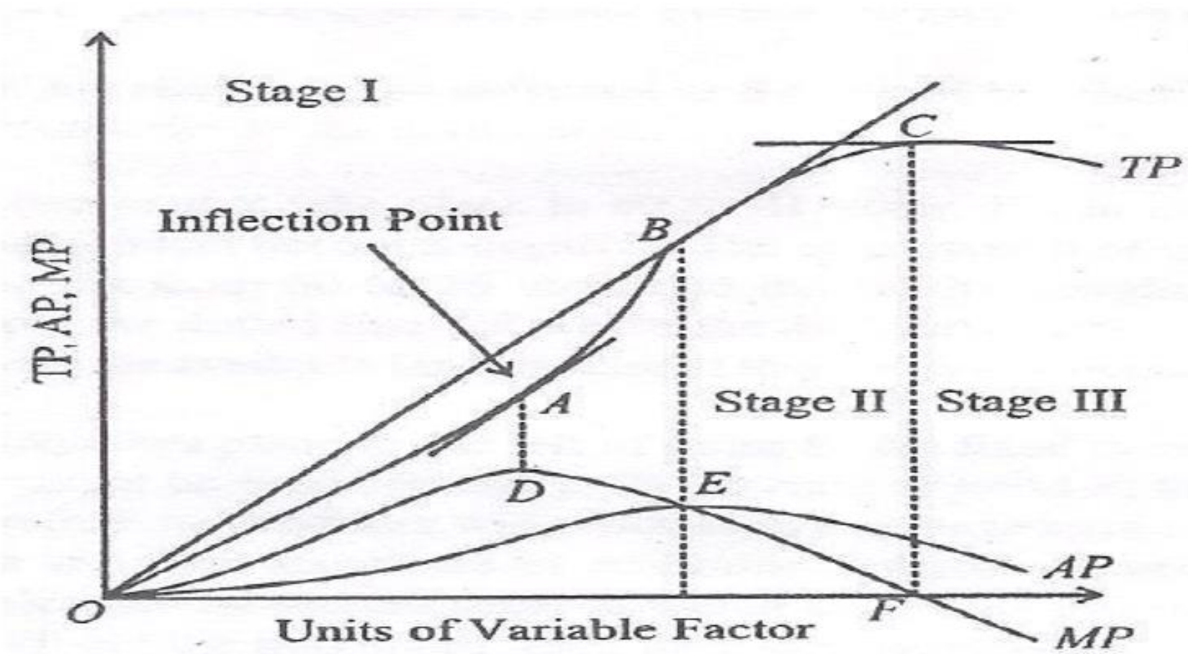
The law of variable proportions

It is referred to as the law which states that when the quantity of one factor of production is increased, while keeping all other factors constant, it will result in the decline of the marginal product of that factor.

It means the marginal production declines up to an extent and it reaches zero and become negative. The point at which the MP becomes zero is the maximum output of the firm with the given set of input factors. This law is applicable in all human activities and business activities.

The law of variable proportions which is a new name given to old classical concept of "Law of diminishing returns. The concept of variable proportions is a short-run phenomenon as in these period fixed factors cannot be changed and all factors cannot be changed

No. of Workers	***Total Product***	***Average Product***	***Marginal Product***
1	8	8	8
2	20	10	12
Stage I			
3	36	12	16
4	48	12	12
5	55	11	7
Stage II			
6	60	10	5
7	60	8.6	0
Stage III			
8	56	7	-4



The rising, the falling and the negative phases of the total, marginal and average products are in fact the different stages of the law of variable proportions which are discussed below.

Stage - I: Increasing Returns: In stage I the average product reaches the maximum and equals the marginal product when 4 workers are employed, as shown in the table. This stage is portrayed in the figure from the origin to point E where the MP curve reaches its maximum and the AP curve is still rising. In this stage, the TP curve also increases rapidly. Thus, this stage relates to increasing returns. Here land is too much in relation to the workers employed. It is, therefore, profitable for a producer to increase workers to produce more and more output. It becomes cheaper to produce the additional output. Consequently, it would be foolish to stop producing more in this stage. Thus, the producer will always expand through this stage I.

Stage-II: Diminishing Returns: It is the most important stage of production. Stage II starts when at point E where the MP curve intersects the AP curve which is at the maximum. Then both continue to decline with AP above MP and the TP curve begins to increase at a decreasing rate till it reaches point C. At this point the MP curve becomes negative when the TP curve begins to decline. Table 1 shows this stage when the workers are increased from 4 to 7 to cultivate the given land. In Figure, it lies between BE and CF. Here land is scarce and is used intensively. More and more workers are employed in order to have larger output. Thus, the total product increases at a diminishing rate and the average and marginal product decline. This is the only stage in which production is feasible and profitable because in this stage the marginal productivity of labour, though positive, is diminishing but is not negative.

Stage - III: Negative Marginal Returns: Production cannot take place in stage II either for in this stage, total product starts declining and the marginal product becomes negative. The employment of the 8th worker actually causes a decrease in total output from 60 to 56 units and makes the marginal product minus 4. In the figure, this stage starts from the dotted line CF where the MP curve is below the X-axis. Here the workers are too many in relation to the available land, making it absolutely impossible to cultivate it.

The Best Stage: In stage I, when production takes place to the left of point E, the fixed factor is excess in relation to the variable factors which cannot be used optimally. To the right of point F, the variable input is used excessively in Stage III. Therefore, no producer will produce in this stage because the marginal production is negative. Thus, the first and third stages are of economic absurdity or economic nonsense. So, production will always take place in the second stage in which total output of the firm increases at a diminishing rate and MP and AP are the maximum, then they start decreasing and production is optimum. This is the optimum and best stage of production.

Law of Return to Scale

In the long run the fixed inputs like machinery, building and other factors will change along with the variable factors like labour, raw material etc. With the equal percentage of increase in input factors various combinations of returns occur in an organization. In the long-term all factors can be changed as made variable. When we study the changes in output when all factors or inputs are changed, we study returns to scale.

Returns to scale describes the relationship between outputs and scale of inputs in the long run when all the inputs are increased in the same proportion. To meet a long run change in demand, the firm increases its scale of production by using more space, more machines and laborers in the factory.

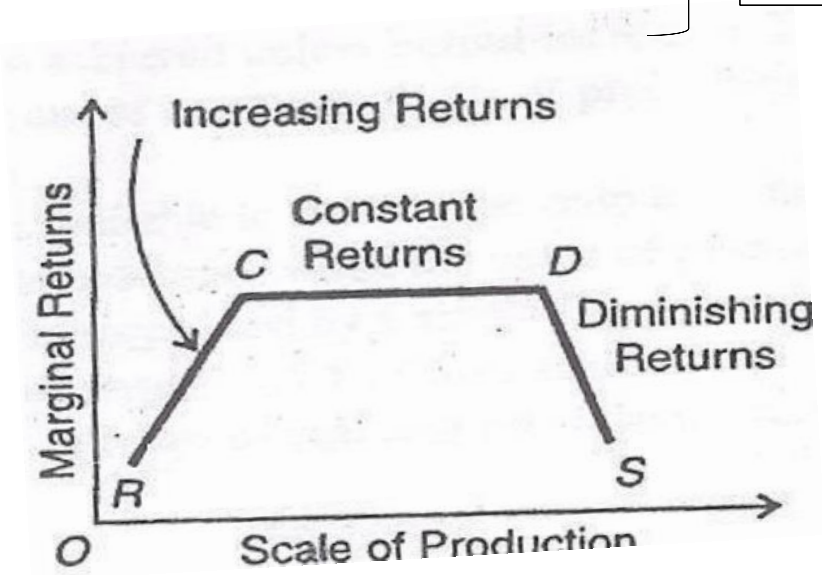
When all inputs are increased in unchanged proportions and the scale of production is expanded, the effect on output three stages: increasing returns to scale, constant returns to scale and diminishing returns to scale. They are explained with the help of Table and Fig.

Unit	Scale of Production	Total Returns	Marginal Returns
1.	1 workers + 2 Acres Land	8	8
2.	2 workers + 4 Acres Land	17	9
3.	3 workers + 6 Acres Land	27	10
4.	4 workers + 8 Acres Land	38	11
5.	5 workers + 10 Acres Land	49	11
6.	6 workers + 12 Acres Land	59	10
7.	7 workers + 14 Acres Land	68	9
8.	8 workers + 16 Acres Land	76	8

Increasing Return

Constant Return

Diminishing Return



1. Increasing Returns to Scale
Returns to scale increase because the increase in total output is more than proportional to the increase in all inputs. The table reveals that in the beginning with the scale of production of (1 worker +2 acres of land), total output is 8. To increase output when the scale of production is doubled (2 workers + 4 acres of land), total returns are more than doubled. They become 17. Now if the scale is trebled (3 workers + 6 acres of land), returns become more than three-fold, i.e., 27. It show increasing returns to scale. In the figure RS is the returns to scale curve where R to C portion indicates increasing returns.

2. Constant Returns to Scale
Returns to scale become constant as the increase in total output is in exact proportion to the increase in inputs. If the scale of production in increased further, total returns will increase in such a way that the marginal returns become constant. In the table, for the 4 and 5 the units of the scale of production, marginal returns are 11, i.e., returns to scale are constant. In the figure, the portion from C to D of the RS curve is horizontal which depicts constant returns to scale. It means that increments of each input are constant at all levels of output.

3. Decreasing Returns to Scale
Returns to scale diminish the increase in output is less than proportional to the increase in inputs. The table shows that when output is increased from the 6th, 7th and 8th units, the total returns increase at a lower rate than before so that the marginal returns start diminishing successively to 10, 9 and 8. In the figure, the portion from D to S of the RS curve shows diminishing returns.

Isoquant Curve

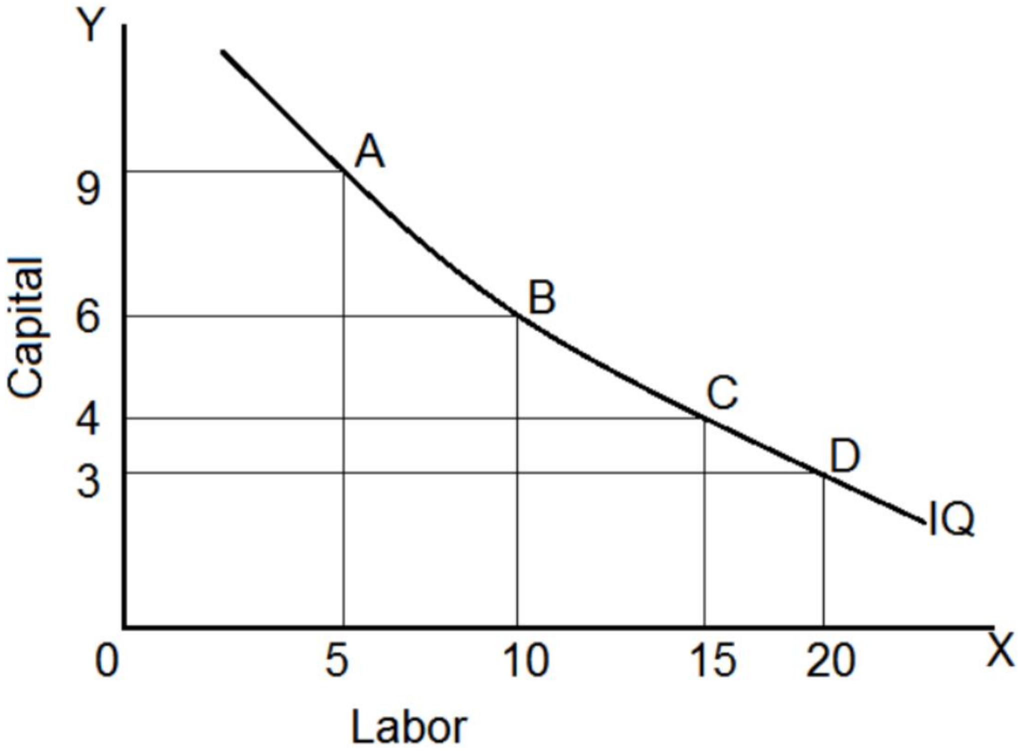
To understand the production function with two variable inputs, iso-quant curve is used. These curves show the various combinations of two variable inputs resulting in the same level of output. The Iso-quant curves show the different combinations of two resources with which a firm can produce equal amount of product.

It Also known as production indifference curve because it is Similar to the indifference curve in the theory of consumer behaviour. The isoquant curve demonstrates the principle of the marginal rate of technical substitution, which shows the rate at which you can substitute one input for another, without changing the level of resulting output.

Combinations of Labour and Capital	Units of Labour (L)	Units of Capital (K)	Output of Cloth (meters)
A	5	9	100
B	10	6	100
C	15	4	100

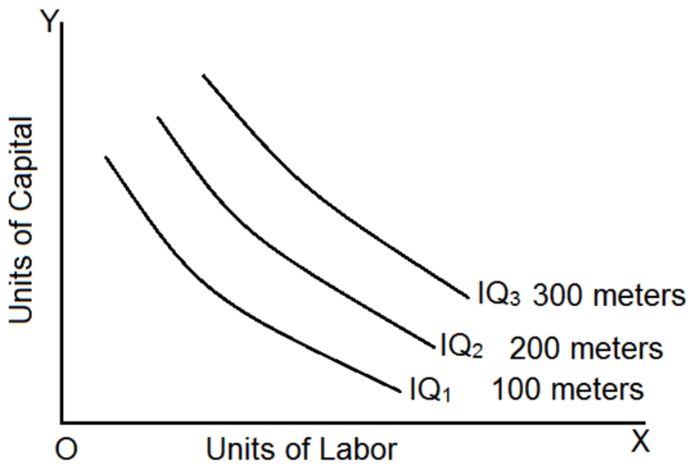
D	20	3	100

The combinations A, B, C and D show the possibility of producing 100 meters of cloth by applying various combinations of labour and capital. Thus, an isoquant schedule is a schedule of different combinations of factors of production yielding the same quantity of output.



Isoquant Map

An isoquant map is a set of isoquants that shows the maximum attainable output from any given combination inputs.



Properties of Isoquants

1. An isoquant must always be convex to the origin. This is because of the operation of the principle of diminishing marginal rate of technical substitution. MRTS is the rate at which marginal unit of an input can be substituted for another input making the level of output remain the same.

2. The iso-quant curve is **negatively sloped**, which means, in order to have a same level of production, the more use of units of one input factor is to be offset with the lesser units of another input factor. This complies with the principle of Marginal Rate of Technical Substitution (MRTP). For example, with more units of capital, the lesser units of labour are to be employed to have a same level of output.

3. Iso-quant curves **cannot intersect or be tangent** to each other. If these intersects, then the results will be incorrect. A common factor combination on both the curves will show the same level of output, which is not feasible.

4. **Upper iso-quant curves yield higher outputs**. This is possible because, at a higher curve, more factors of production are employed either the capital or the labour, which results in more production.

5. **No iso-quant curve touches either of the axis, X or Y**. If it does so, then the rate of technical substitution would be void since it will show that a single factor is producing the given level of output without any units of other factor being employed.

Marginal Rate of Technical Substitution

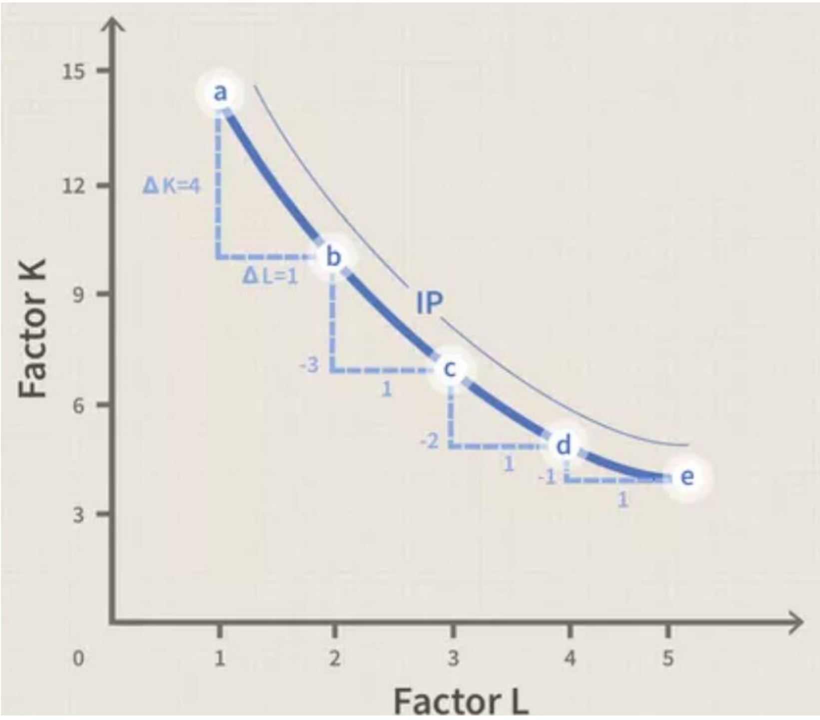
The rate at which one factor has to be decreased in order to retain the same level of productivity if another factor is increased. The marginal rate of technical substitution is different than the marginal rate of substitution (MRS). MRTS focuses on producer equilibrium, while MRS focuses on consumer equilibrium.

$$\textbf{MRTS}(\boldsymbol{L}, \boldsymbol{K}) = -\frac{\Delta K}{\Delta L} = \frac{\text{MP}L}{\text{MP}K}$$

where:
K=Capital
L=Labour
MP=Marginal products of each input
$\frac{\Delta Y}{\Delta L}$ = Amount of capital that can be reduced when labour is increased

The slope of the isoquant indicates the MRTS or at any point along the isoquant how much capital would be required to replace a unit of labour at that production point.

A decline in MRTS along an isoquant for producing the same level of output is called the diminishing marginal rate of substitution. The figure below shows that when a firm moves down from point (a) to point (b) and it uses one additional unit of labour, the firm can give up 4 units of capital (K) and yet remains on the same isoquant at point (b). So, the MRTS is 4. If the firm hires another unit of labour and moves from point (b) to (c), the firm can reduce its use of capital (K) by 3 units but remains on the same isoquant, and the MRTS is 3.



Isocost

In economics an **isocost** line shows all combinations of inputs which cost the same total amount. Although similar to the budget constraint in consumer theory, the use of the isocost line pertains to cost-minimization in production, as opposed to utility-maximization. An isocost line is a curve which shows various combinations of inputs that cost the same total amount. For the two production inputs labour and capital, with fixed unit costs of the inputs, the isocost curve is a straight line. For the two production inputs labour and capital, with fixed unit costs of the inputs, the equation of the isocost line is

$$\boldsymbol{rK + wL = C}$$

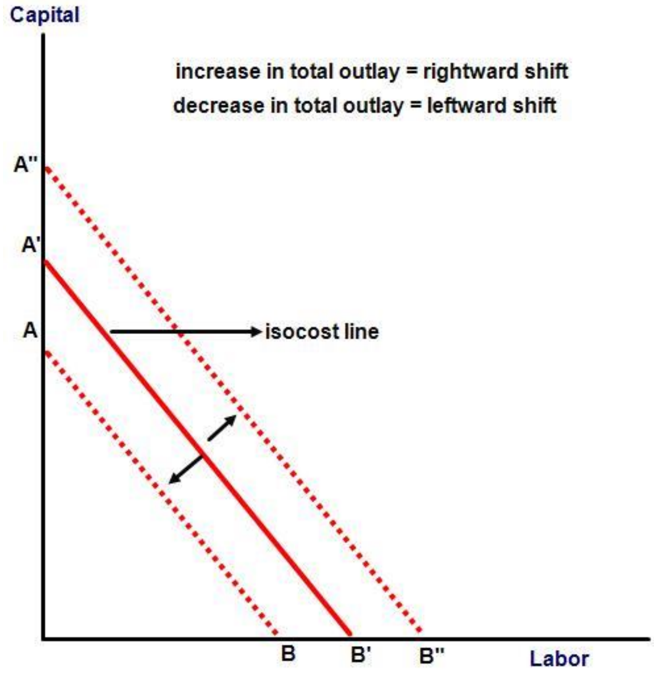
where, w represents the wage rate of labour,

r represents the rental rate of capital,

K is the amount of capital used, L is the amount of labour used, and

C is the total cost of acquiring those quantities of the two inputs.

The absolute value of the slope of the isocost line, with capital plotted vertically and labour plotted horizontally, equals the ratio of unit costs of labour and capital. **The slope is:** $\boldsymbol{\frac{-w}{r}}$



Least Cost Combination

The optimum combination of inputs that is required to produce output at the least possible cost is called the least cost combination.

The main aim of a producer is to continue production activities with least costs. For this purpose, he combines different factors of production in such a way that the costs incurred for hiring their services are minimum. He continues his efforts in this regard until he arrives at the least cost combination of factors.

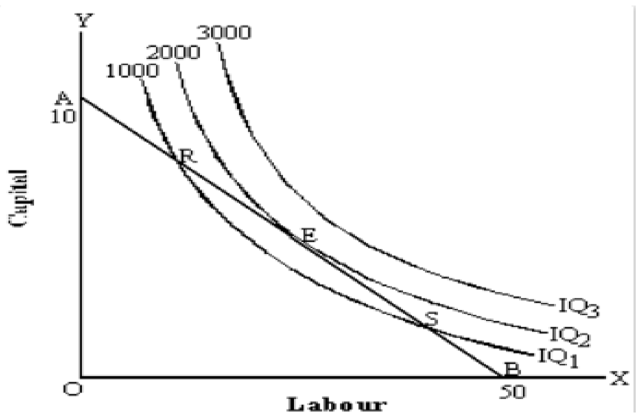
A rational firm/producer seeks maximisation of profit. For this, he tries to minimise its cost of production. The cost is minimum, when input combination is optimal. Optimal input combination indicates the maximum returns to the factors employed. Thus, a rational firm would combine the various factors of production it production function in such a way that with the minimum input and maximum output is obtained at the minimum cost. Such a combination is referred to as the least cost combination.

Assumptions

The assumptions on which this analysis is based are:

1. There are two factors. Capital and labour.
2. All units of capital and labour are homogeneous.
3. The prices of factors of production are given and constant.
4. Money outlay at any time is also given.
5. Perfect competition is prevailing in the factor market.

The firm cannot choose and neither combination beyond line AB nor will it choose any combination below this line. AB is known as iso-cost line. By combining the isoquants and the isocost line, we can find out the optimum combination of factors.



In the Fig. equal product curves IQ_1, IQ_2 and IQ_3 represent outputs of 1,000 units, 2,000 units and 3,000 units respectively. AB is the isocost line. At point E the isocost line is tangent to iso-quant IQ_2 representing 2,000 units of output. Iso-qunat IQ_3 falls outside the isocost line AB and, therefore, cannot be chosen by the firm. On the other hand, iso-quant IQ_1, will not be preferred by the firm even though between R and S it falls within the isocost line. Points R and S are not suitable because output can be increased without increasing additional cost by the selection of a more appropriate input combination. Point E, therefore, is the ideal combination which maximizes output or minimizes cost per units: it is the point at which the firm is in equilibrium.

References:

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