Kinematics and Mechanism (MEE -S203T)
Semester: 2021-22 (Odd Semester)
Year: $\mathbf{2}^{\text {nd }}$ Year (2K21)
End Semester Examination
Time: 3 h
Maximum marks: 50
All questions are compulsory

## Section A

10 marks ( 10 questions of 1 mark each)

1. In a slotted lever quick return mechanism, the distance between fixed centres is 200 mm and driving crank is 100 mm long. Determine the ratio of the time taken on the cutting and return strokes

$\cos \alpha=\frac{O A_{1}}{O B}=\frac{100}{200}=0.5$
$\alpha=60^{0}$
$q r r=\frac{360^{\circ}-2 \alpha}{2 \alpha}=\frac{240}{120}=2$
2. Find the degree of freedom for the mechanism.


Solution:
Number of links $=5$
Number of turning pair $=5$
DOF $=3(\mathrm{~N}-1)-2 \mathrm{j}=3(5-1)-2 * 5=2$

## Section B

20 marks (5 questions of 4 marks each)

1. In the following configuration of a rigid body under certain motion, $\mathrm{V}_{\mathrm{A}}=4 \mathrm{~m} / \mathrm{s}$ and $\theta=30^{\circ}$. The direction of velocity at point B is known to be along the line BC which makes an angle $\varphi=45^{\circ}$ with line AB . The magnitude of velocity at B is


Solution: Velocity component of A along $\mathrm{AB}=V_{A} \cos \theta=4 \cos 30^{\circ}=3.46 \mathrm{~m} / \mathrm{s}$
Velocity component of B along $\mathrm{AB}=V_{B} \cos \varphi=V_{B} \cos 45^{\circ}$
Velocity component of B along $\mathrm{AB}=$ Velocity component of A along AB
$V_{B} \cos 45^{\circ}=3.46 \mathrm{~m} / \mathrm{s}$
$V_{B}=4.9 \mathrm{~m} / \mathrm{s}$
2. In the given configuration of a rigid body in motion, the velocities at points $A$ and $B$ are $V_{A}=4 \mathrm{~m} / \mathrm{s}$ and $\mathrm{V}_{\mathrm{B}}=2 \mathrm{~m} / \mathrm{s}$ with $\theta=45^{\circ}$ and $\varphi=$ $30^{\circ}$, respectively. AC and BC are perpendicular to each other. What is the magnitude of velocity at point C ?


$$
V_{c}=\sqrt{\left(V_{A} \cos \theta\right)^{2}+\left(V_{B} \cos \varphi\right)^{2}}=\sqrt{\left(4 \cos 45^{0}\right)^{2}+\left(2 \cos 30^{0}\right)^{2}}=3.31 \mathrm{~m} / \mathrm{s}
$$

4. The number of teeth on each of the equal spur gears in mesh are 40 . The teeth have $20^{\circ}$ involute profile and the module is 8 mm . If the arc of contact is 1.75 times the circular pitch. Find the addendum

Solution: $\mathrm{Z}_{\mathrm{P}}=40, \mathrm{Z}_{\mathrm{G}}=40, \alpha=20^{\circ}$, module $=8 \mathrm{~mm}$

$$
\mathrm{R}=\mathrm{r}=\frac{m Z}{2}=\frac{2 \times 40}{2}=40 \mathrm{~mm}
$$

Arc of contact $=1.75 * \mathrm{p}_{\mathrm{c}}$
Circular pitch $\mathrm{p}_{\mathrm{c}}=\pi * m=3.14 * 8=25.12 \mathrm{~mm}$
Arc of contact $=1.75 * 25.12 \mathrm{~mm}=43.96 \mathrm{~mm}$
Arc of contact $=\frac{\text { Path of contact }}{\cos \alpha}$
Path of contact $=\operatorname{Arc}$ of contact $* \operatorname{Cos}(\alpha)=43.96 * \operatorname{Cos}\left(20^{\circ}\right)=41.31 \mathrm{~mm}$
Path of contact $=\sqrt{\left(R_{a}\right)^{2}-(R \cos \alpha)^{2}}+\sqrt{\left(r_{a}\right)^{2}-(r \cos \alpha)^{2}}-(R+r) \sin \alpha$
Since gear are equal $\mathrm{R}=\mathrm{r}$

$$
\text { Path of contact }=2 \sqrt{\left(R_{a}\right)^{2}-(R \cos \alpha)^{2}}-2 R \sin \alpha=2 \sqrt{\left(R_{a}\right)^{2}-\left(40 \times \cos 20^{\circ}\right)^{2}}-2 \times 40 \times \sin 20^{0}
$$

$$
41.31=2 \sqrt{\left(R_{a}\right)^{2}-1412.84}-27.36
$$

$$
\sqrt{\left(R_{a}\right)^{2}-1412.84}=34.34
$$

$\left(R_{a}\right)^{2}=1178.9+1412.84$
$\left(R_{a}\right)^{2}=2591.7$
$R_{a}=50.9 \mathrm{~mm}$
Addendum $=\mathrm{R}_{\mathrm{a}}-\mathrm{R}=50.9-40=10.9 \mathrm{~mm}$
5. A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with $20^{\circ}$ pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

## $\underline{\text { Section C }}$

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20 \text { marks (2 questions of } 10 \text { marks each, Each question can two have parts) }
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1. (a) An epicyclic gear train is shown in the figure below. The number of teeth on the gears $A, B$ and $D$ are 20,30 and 20 , respectively. Gear $C$ has 80 teeth on the inner surface and 100 teeth on the outer surface. If the carrier arm is fixed and the sun gear A rotates at 300 rpm in the clockwise direction, then the rpm of D in the clockwise direction is


## Concept:

The speed of the gears in epicyclic gear train can be analyzed from the following table,
If the external toothed gear is meshing with the internal toothed gears then the direction of the velocity remains same, and the velocity ratio between two mating gears is given by
$\frac{N_{2}}{N_{1}}=\frac{T_{1}}{T_{2}}$
Where N is the angular velocity in rpm and T is the number of teeth

| Conditions of <br> motion | Revolutions of elements |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Arm | Gear A | Gear B | Gear C | Gear D |  |
| Arm is fixed, <br> wheel A rotates <br> +1 revolutions | 0 | +1 | $-\frac{T_{A}}{T_{B}}$ | $\frac{T_{A}}{T_{A}} \frac{T_{A}}{T_{B}} \times \frac{T_{B}}{T_{C i}}=-$ | $\frac{T_{A}}{T_{C i}} \times \frac{T_{C_{0}}}{T_{D}}=\frac{T_{A}}{T_{D}}$ |  |
| Arm is fixed, <br> $T_{C i}$ |  |  |  |  |  |  |
| wheel A is <br> rotated trough +x <br> revolution | 0 | +x | $-x \frac{T_{A}}{T_{B}}$ | $-x \frac{T_{A}}{T_{C i}}$ | $x \frac{T_{A}}{T_{D}} \frac{T_{C_{0}}}{T_{C i}}$ |  |
| Add +y <br> revolution to all | y | $\mathrm{Y}+\mathrm{x}$ | $y-x \frac{T_{A}}{T_{B}}$ | $y-x \frac{T_{A}}{T_{C}}$ | $y+x \frac{T_{A}}{T_{D}} \frac{T_{C_{0}}}{T_{C i}}$ |  |

## Calculation:

Given: $T_{A}=20, T_{B}=30, T_{D}=20, T_{C i}=80, T_{C o}=100, N_{\text {Arm }}=0, N_{A}=300 \mathrm{rpm}, N_{D}=$ ?
$\mathrm{N}_{\text {Arm }}=0=\mathrm{y}$
$N_{A}=300 \mathrm{rpm}=\mathrm{y}+\mathrm{x} \Rightarrow \mathrm{x}=300$
$N_{D}=y+x \frac{T_{A}}{T_{D}} \frac{T_{C o}}{T_{c i}}=x \frac{T_{A}}{T_{D}} \frac{T_{C o}}{T_{c i}}=300 \times \frac{20}{20} \times \frac{100}{80}=375 \mathrm{rpm}$
Mistake: Take care of the inner and outer teeth of gear $C$.
(b) Two $20^{\circ}$ full-depth involute gears having 40 and 64 teeth are in mesh. The pinion rotates at 900 rpm . The module is 4 mm . Find the sliding velocities at the engagement and disengagement of a pair of teeth and contact ratio. If the interference is just avoided. Find: (a) The addendum on the wheel and pinion. (b) The path of contact (c) The maximum velocity of sliding at engagement and dis-engagement of a pair of teeth (d) Contact ratio.
2. (a) In the configuration of the mechanism shown in the figure, points $C, A$ and $D$ are collinear. If the $C A=3 \mathrm{~cm}, C D=6 \mathrm{~cm}$ and $\omega_{2}=3 \mathrm{rad} / \mathrm{s}$ CCW, find $\omega_{4} \mathrm{in} \mathrm{rad} / \mathrm{s}$.

(b) In a pin jointed four bar mechanism ABCD , the lengths of the various links are $\mathrm{AB}=30 \mathrm{~mm}, \mathrm{BC}=90 \mathrm{~mm}, \mathrm{CD}=55 \mathrm{~mm}$ and $\mathrm{AD}=85$ mm . The link AD is fixed and angle BAD is $130^{\circ}$. If the velocity of B is $2 \mathrm{~m} / \mathrm{s}$ in clockwise direction, find (a) velocity of mid point of link BC and (b) Angular velocity of CB and CD.

