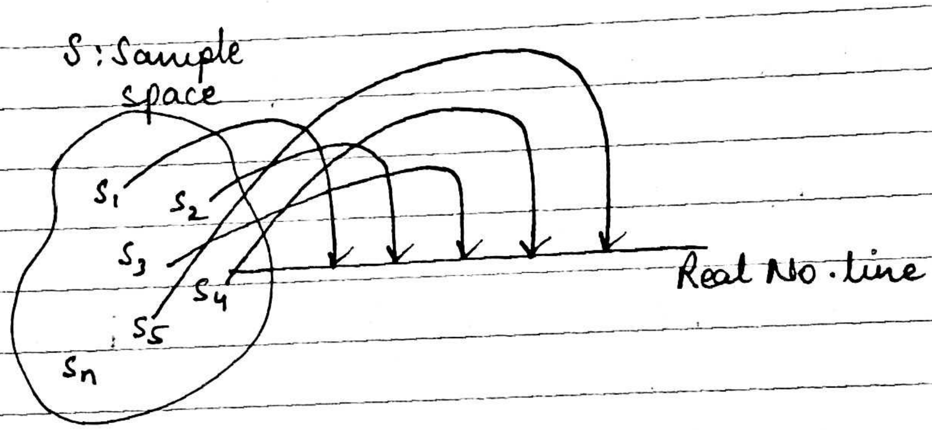
Probability

Experiment → Events

$$P(E) = \frac{N(E)}{N(s)}$$

Random Variable - It is a rule that assigns a numerical value to each possible outcome of an experiment.



The random variable 'x' is a function whose domain is the sample space 'S'.

$$X : S \longrightarrow R$$

$$\text{domain } (X) = S$$

$$\text{Range of } (X) = R$$

Probability Mass Function (pmf)

Any event of sample space Ax is defined as:

$$A_x = \{ s \in S \mid X(s) = x \}$$

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Probability of A_x :

$$P(A_x) = P\{s \in S \mid X(s) = x\}$$

$$P(A_x) = p_X(x)$$

The function $p_X(x)$ is called probability mass function (pmf) or density function. The properties of probability mass function are given as below:

i) $0 \leq P_X(x) \leq 1$

ii) $\sum_{x \in R} p_X(x) = 1$

iii) If the random variable X is discrete random variable then :

$$\sum_i p_X(x_i) = 1$$

Cummulative Distribution Function (CDF)

$X(s) = x$	$p_X(x)$	CDF
$X(s_1) = x_1$	$p_X(x_1)$	$p_X(x_1)$
x_2	$p_X(x_2)$	$p_X(x_1) + p_X(x_2)$
x_3	$p_X(x_3)$	$p_X(x_1) + p_X(x_2) + p_X(x_3)$
$\vdots$	$\vdots$	
x_n	$p_X(x_n)$	$p_X(x_1) + p_X(x_2) + p_X(x_3) + \cdots + p_X(x_n)$

The commulative distribution function is given by:

$$F_X(x) = \sum_{X=x} p_X(x)$$

The function $F_X(x)$ is called commulative distribution function CDF of random variable function. The main properties of CDF are given below:

i) $0 \leq F_X(x) \leq 1$

ii) $F_X(x)$ is a monotonically non decreaing function of x i.e.

if $x_1 \leq x_2$
then $f_X(x_1) \leq F_X(x_2)$

iii) $\lim_{x \to \infty} F_X(x) = 1$ for all sufficiently large x.

$\lim_{x \to -\infty} F_X(x) = 0$ for all sufficiently small x.

iv) The function $F_X(x)$ has a +ve jump to $p_X(x_i)$ at $i = 1, 2, 3, 4 \ldots$

<u>Continuous Random Variables</u>

The commulative distribution function for continuous random variable have no jump but grows continuously. A continuous random variable X is characterised by CDF $F_X(x)$ i.e. a continuous function of x for all $-\infty < x < \infty$.

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$$F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

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Probability Distribution / Density Function (PDF)

The probability distribution function or probability density function of a continuous random variable is defined as below

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$\text{p.d.f} = \frac{d}{dx} \text{C.D.F.}$$

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx$$

The main properties of PDF are given below $f_X(x)$

i) $f_X(x) \ge 0$ for all x

ii) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

Q. The time measured in the X years, X is required to complete a software, have a PDF of following form :

$$f_X(x) = \begin{cases} kx(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find out probability that the project will be