REPLACEMENT MODELS

1.0 MAINTENANCE

"Maintenance is a routine and recurring activity of keeping a particular machine or facility at its normal operating condition so that it can deliver its expected performance or service without causing any loose of time on account of accidental damage or breakdown".

Once equipment is designed, fabricated and installed, the operational availability of the same is looked after by the maintenance requirement. The idea of maintenance is very old and was introduced along with inception of the machine. In the early days, a machine was used as long as it worked. When it stopped working, it was either repaired/serviced or discarded.

The high cost sophisticated machines need to be properly maintained/serviced during their entire life cycle for maximizing their availability. The development of mechanization and automation of production systems and associated equipment, with the accompanying development of ancillary services and safety requirements, has made it mandatory for engineers to think about proper maintenance of equipment.

Maintenance is a function to keep the equipment/machine condition by replacing or repairing some of the components of the machine. The maintenance concept is an outline plan of how the

maintenance function will be performed. Based on the feedback obtained from the users and the history of the equipment, detailed procedures are drawn to concretize the maintenance concept. The procedures developed thus are collectively called the maintenance plan. The development of such a maintenance plan is one of the most important requirements of the maintenance program that requires interaction between the user and the manufacturer. With this information, the manufacturer will be in position to rearrange the design as per user's maintenance requirements.

Maintenance function also involves looking after the safety aspects of certain equipment where the failure of component may cause a major accident. For example, a poorly maintained pressure vessel such as steam boiler may cause a serious accident.

1.1 CHALLENGES IN MAINTENANCE:

The maintenance function of a modern industry faces a number of challenges attributable to:

- Rapid growth of technology resulting in current technology becoming obsolete. Such a challenge is a frequent one in Information and Communications Technology (ICT) industry where computers and computers based system (hardware and Software) is the main component.

- Advent of new advanced diagnostic tools, rapid repair systems, etc.

- Advance store management techniques to incorporate modular technologies.

- Requirements of keeping both outdated and modern machines in service. For example, many industrial organizations have a combination of the old machines working on obsolete technology and new systems utilizing the latest technology and equipment.

The effective management of maintenance aspects under such challenging circumstances is often a difficult job. Besides the rectification of the faults in the equipment, the activities of the maintenance department include:

- Up gradation of the existing plants and equipments and training maintenance personnel to attend the required technical skills.

- Effective maintenance of the old equipment for higher availability

- Cost optimization of all maintenance functions

- Improvement of maintenance activities in the areas of tribology and terrotechnology.

- Reconditioning of used /unserviceable spare parts.

- Development of indigenous sources for parts for import substitution.

- Setting up of an effective maintenance information management systems (MIMS).

- Effective utilization of the maintenance workforce

- Setting up of in house R&D activities for effecting improvements in maintenance practices.

1.2 OBJECTIVIES OF MAINTENANCE:

The objectives of maintenance should be formulated within the framework of the overall organizational setup so that finally the goals of the organization are accomplished. For this, the maintenance division needs to ensure that:

(a) The machinery and/or facilities are always in an optimum working condition at the lowest possible cost

(b) The time schedule of delivering to the customers is not affected because of non-availability of machinery/service in working condition

(c) The performance of the machinery /facility is dependable and reliable.

(d) The performance of the machinery /facility is kept to minimum to the event of the breakdown.

(e) The maintenance cost is properly monitored to control overhead costs.

(f) The life of equipment is prolonged while maintaining the acceptable level of performance to avoid unnecessary replacements.

Maintenance is also related with profitability through equipment output and its running cost. Maintenance work enhances the equipment performance level and its availability in optimum working condition but adds to its running cost.

The objective of maintenance work should be to strike a balance between the availability and the overall running costs. The responsibility of the maintenance function should, therefore, be

ensure that production equipment /facilities are available for use for maximum time at minimum cost over a stipulated time period such that the minimum standard of performance and safety of personal and machines are not sacrificed. These days therefore, separate departments are formed in industrial organizations to look after the maintenance requirements of equipments and machines.

1.3 TYPES OF MAINTENANCE SYSTEMS

Basically, maintenance can be divided into two groups:
(a) Breakdown maintenance
(b) Planned maintenance

1.3.1 Breakdown Maintenance and Its Limitations

The basic concept of breakdown maintenance is not to do anything as long as everything is going on well. Hence, no maintenance or repair work is done until a component or equipment fails or it cannot perform its normal performance. In other words, the maintenance work is called upon when the machine is out of order, and repairs are required to bring back the equipment to its original working condition.

If the system is alone followed, it will lead to poor operational availability of the equipment, as spare parts may not be readily available. Though it appears to be economical proposition, work would greatly suffer if the machine is not restored to operational condition at the earliest. In this type of maintenance, during the repair time, no proper care is taken to know the real cause of the breakdown, which in turn may lead to frequent failures of the same kind.

This type of practice is economical for that machinery whose breakdown time and repair costs are less. But in case of high cost production systems, there are several limitations with breakdown maintenance.

1.3.2 Planned Maintenance:

The planned maintenance is said to be an organized type of maintenance. In this type of maintenance, the maintenance activities are planned well in advance to avoid random failure. It will be pre determined not only the when and what kind of the maintenance work, but also by whom it would be undertaken. The prerequisites for planned maintenance include the conduction of work study that decides the periodicity of maintenance work. Also the conduction of Time Study helps in suggesting ways and means of devising optimal maintenance schedules for the given system.

In planned maintenance, instructions will be in greater detail and specific for each type of equipment. Where safety is of paramount importance, the equipment condition should be checked everyday. Hence, the type of maintenance activity to be carried out will depend upon the nature of equipment and its working conditions.

The planned maintenance can be further classified into:
- Scheduled Maintenance (SM)
- Preventive Maintenance (PM)
- Corrective Maintenance (CM)
- Reliability Centered Maintenance (RCM)

Scheduled Maintenance:

This is a stitch-in-time procedure to avoid break-downs. The actual maintenance program is scheduled in consultation with the production department, so that the relevant equipment is made available for maintenance work. The frequency of such maintenance work is decided well in advance from experience so as to utilize the idle time of the equipment effectively. This also helps the maintenance department to use their manpower effectively. If the schedule of maintenance is known in advance, the specialists for the same can also be made available during the maintenance period. Though scheduled maintenance is costly compared to breakdown maintenance, the availability of equipment is enhanced. This practice is used for overhauling of machines etc.

Preventive Maintenance:

It is said to be preventive maintenance when planned and coordinated inspections, repairs, adjustments, and replacements are carried out to minimize the problems of breakdown maintenance. This is based on the premise that prevention is better than cure. This practice involves planning and scheduling the maintenance work without interruption in production schedule and thus improves the availability of equipment. Under preventive maintenance, a systematic inspection of each item of equipment or at least the critical parts will be carried out at predetermined times to unfold the conditions that lead to production stoppage and harmful depreciation. There is no

readymade preventive maintenance plan that suits for any industry. It should be customized to make it suitable to the requirements of the particular industry.

Planning and implementation of a preventive maintenance practice is a costly affair because it involves the replacement of all deteriorated parts/components during inspection. However, the higher cost of maintenance usually gets compensated by the prolonged operational life of the equipment. To avoid serious breakdowns, the preventive mode of maintenance is usually implemented in complex plants.

Corrective Maintenance:

The practice of preventive maintenance brings out the nature of repetitive failures of a certain part of the equipment. When such repetitive type of failures are observed, corrective maintenance can be applied so that reoccurrence of such failures can be avoided. These types of failures can be reported to the manufacturer to suggest modifications to the equipment.

Corrective maintenance can be defined as the practice carried out to restore the full performance of the equipment that has stopped working to acceptable standards. For example, an IC engine may be in working condition, but does not make its full load because of worn–out piston rings. If the piston rings are replaced, it will bring back the performance of the engine to specified level.

Reliability Centered Maintenance (RCM):

It is used to identify the maintenance requirements of equipment. The RCM establishes the functional requirements and the desired performances standards of equipments and these are then related to design and inherent reliability parameters of the machine. For each function, the associated functional failure is defined, and the failure modes and the consequences of the functional failures are analyzed.

The consequences of each failure are established, which fall in one of the four categories: hidden, safety or environmental, operational, and no operational. Following the RCM logic, preemptive maintenance tasks which will prevent these consequences are selected, provided the applicability and effectiveness criteria for preventive maintenance are satisfied.

The applicability requirements refer to the technical characteristics and effectiveness criteria for preventive maintenance tasks and the frequency at which these should be carried out. Effectiveness criteria depend on the consequences of the failure; probabilities of the multiple failures for hidden failure consequences, acceptable low risk of failure for safety consequences, and non-operational consequences. When the requirements for planned maintenance (PM) are not fulfilled, default tasks include failure finding (for hidden failure, possible redesign of equipment, procedures and training processes) and no-schedule maintenance.

1.4 BENEFITS OF MAINTENANCE:

The high involvement of capital cost in any production system expects proportional returns from the equipment. These expectations will be met only when the equipment keeps working at its normal performance. It is often experienced that the maintenance schedules provided by the manufacturer do not deliver the required results in terms of the production out put and the life of the equipment. In such cases, therefore, it becomes necessary to properly maintain the equipment with extra care in order to obtain the desired levels of production or service.

The following benefits can be derived from a well –organized maintenance system:

(a) The minimization of breakdown time

(b) Improvement in total availability of the system with their optimum capacity

(c) Extended useful life of the equipment

(d) Safety of the personnel.

The consequences of downtime can be very serious when the machine is working in a production line, as its failure will shut down the total system. Following a proper maintenance schedule the normal wear and tear of equipment can be reduced. In certain cases, the safety of the personnel is of prime importance and this also can be assured by proper planned preventive maintenance. For example, all

aircraft systems need to be inspected before and after a flight as safety of the passengers is of prime importance.

1.5 EFFECTS OF MAINTENANCE:

Maintenance, being an important function in any production system, has far reaching effects on the system. If the right practice of maintenance is not established for a particular environment, it may lead to serious problem of either over maintenance or under maintenance. The selection of a particular maintenance policy is also governed by the past history of the equipment. Cost effective maintenance will help in enhancing productivity. It is therefore, is important for the team associated with maintenance work, to know how much to maintain.

The nature of the maintenance function affects the life of equipment. It is known from experience that optimum maintenance will prolong the life of the equipment, and on the other hand, carelessness in maintenance would lead to reduced life of the equipment and in some cases an early failure as well. Further, proper maintenance will help to achieve the production targets. If the availability of the equipment in good working condition is high, the reliability of the production will also be high.

Another important effect of the maintenance function is the working environment. If the equipment is in good working condition, the operator feels comfortable to use it other wise there is a tendency to let the equipment deteriorate further. To get the desired results in

maintenance operations, there should be selective development of skilled, semi skilled, and unskilled labour. And also proper job description is required for the jobs in order to make full use of skilled workforce available.

1.6 EQUIPMENT MAINTENANCE/ REPLACEMENT/ REENGINEERING:

To decide the effective mode of maintenance it is essential to carry out reliability analysis of critical parts of the equipment in all modern automated and semi-automated plants. These critical parts may be individual pieces of equipment or a combination of parts that from systems.

Before considering the purchase of any capital equipment, the evaluation of its reliability is essential, which directly depends upon the probability of failures. It is desirable to obtain a reliability index (numerical value) for each machine which is based on such factors as visual inspection tests and measurements, age, environment duty cycle of the equipment. These numbers, so calculated, represent the reliability of particular equipment. It is also possible to combine these indices and express an aggregate reliability index number for the complete system.

From the evaluation of the above index numbers, schedules can be set for equipment maintenance. Wherever needed, the maintenance efforts can be expanded. From the reliability reports it is possible to determine the actions that are required to maintain the operational availability at the desired level. Cost estimates for such

maintenance for much maintenance functions can also be prepared based on the reliability information.

Similarly, the decision to replace existing equipment will require the consideration of the following questions, economic factors and reliability index numbers calculated for the existing equipment.

(a) Will the maintenance cost come down with the replacement of the old equipment?

(b) Will the cost per unit of production/service come down due to automated test features of the mew equipment?

(c) Is the existing equipment not sufficient to meet the future production/service targets?

(d) Will the new equipment be environment friendly and provide better safety to operators?

(e) Is there any possibility of adding additional accessories to existing equipment in order to make it more versatile for future use, or is the rebuilding of existing equipment possible through minor modifications?

Optimal replacement policy of the equipment can be determined if reliable estimates of revenue (return from equipment), up keep (maintenance cost) cost and replacement costs are available. The equipment in use in industries can be mainly divided into (1) equipment with diminishing efficiency and (2) equipment with constant efficiency. The first category deteriorates with time resulting in increase in operating cost including maintenance cost, and second

category operates at constant efficiency for a certain time period and then deteriorates suddenly.

Several models have been developed using repair vs. time and cost, in order to solve the replacement problem of equipment with diminishing efficiency. Replacement is considered to be the regeneration point of whole life where the operating cost function initially starts. In practice such methods really work well and the life of the equipment/system is enhanced.

On the other hand the concept of reengineering in lieu of replacement is one viable model as the operating cost increases with time. This model maximizes the gain between the operating costs before and after the overhauls. Reengineering can be perceived as the adjustment, alteration, or partial replacement of a process or product in order to make it to meet a new need. Successful implementation of reengineering will improve the equipment or process performance and this reduces the maintenance and operating costs.

1.7 CONVENTIONAL REPLACEMENT PROBLEM:

The replacement problems are concerned with the issues that arises when the performance of an item decreases, failure or breakdown occurs. The decrease in performance or breakdown may be gradual or sometimes sudden. The need for replacement of items is felt when,

1. The existing item or system has become inefficient or require more maintenance.

2. The existing equipment has failed due to accident or otherwise and does not work at all.

3. The existing equipment is expected to fail shortly.

4. The existing equipment has become obsolete due to the availability of equipment with latest technology and better design.

The solution to replacement problem is nothing but arriving at the best policy that determines the time at which the replacement is most economical instead of continuing at an increased maintenance cost. The Main objective of replacement policy is to direct the organization in many situations so that it can take right decision. For Example, few situations are:

(i) Waiting for complete failure of item or to replace earlier at the expense of higher cost of the item.

(ii) Whether to replace the under performing equipment with the similar kind of item or by different kind (latest model) of item.

The problem of replacement occurs in the case of both men and machines. Using probability it is possible to estimate the chance of death (failure) at various ages.

1.8 TYPES OF FAILURES

As the term 'failure' encompasses wider concept, failures can be discussed under the following two categories.

(a) **Gradual Failure**: In this, the failure mechanism is progressive. As the age of an item increases, its performance deteriorates. This results in:

- Increased operating cost

- Decreased productivity of the item

- Decrease in resale value of item

(Ex: Mechanical items like pistons, bearing rings, tyres, etc.,)

(b) **Sudden Failure**: This type of failure can be observed in the items that do not deteriorate gradually with age but which fail suddenly after some period of service. The time period between installation and failure will not be constant for any particular equipment. However the failure pattern will follow certain frequency distribution that may be progressive, retrogressive or random in nature.

- *Progressive failure*: It is said to be progressive failure, when probability of failure increases with the age of an item. Ex: light bulbs, tyres etc.

- *Retrogressive failure*: Certain items will have more probability of failure in the initial years of their life and with the increase in the life of an item the chances of failure become less. That is, the ability of the item to survive in the initial years of life increases its expected life. Aircraft engines exemplify industrial equipments with this type of distribution of life span.

- *Random failure*: It is said to be random failure, when constant probability of failure is associated with equipment that fails because random causes such as physical shocks that are independent of age. In the case of random failure, virtually all items fail before aging has any effect. For example, vacuum

tubes, items made of glass or mirror, fruits, vegetables etc may fail independent of their age.

The replacement situations generally are divided into the following four types:

(1) Replacement of capital equipment whose performance decreases with time, e.g., machine tools, vehicles in a transport organization, airplanes, etc.

(2) Group replacement items that fail completely, e.g., electrical bulbs, etc.

(3) Problem of mortality and staffing.

(4) Miscellaneous problems.

1.9 REPLACEMENT OF ITEMS THAT DETERIORATE

Determining the optimal replacement period for an item can be explained by considering an example of a vehicle owner whose aim is to find the best age at which the old vehicle is to be replaced by a new one. The vehicle owner intends to ship cargo as cheaply as possible. The associated costs are:

   (i)   The running costs and (ii) the capital cost of the vehicle

   These costs can be summarized as average cost per month. It can be observed that the average monthly cost will go on decreasing, with increase in time. However, there will be an age at which the rate of increase in running cost is considerably higher than the savings in average capital costs. Thus, at this age it is justifiable to replace the vehicle.

1.9.1 Case – I: Replacement Policy For Items Whose Maintenance Cost Increases With Time, And Money Value Does Not Change With Time i.e. Constant:

Theorem: The maintenance cost of a machine is given as function increasing with time and machine's scrap value is constant.

(a) When time is a continuous variable, then replacing the machine when the maintenance cost is equal to the average annual cost will minimize the average annual cost.

(b) When time is a discrete variable, then replacing the machine when the maintenance cost in the $(n+1)^{th}$ year becomes greater than the average annual cost in the nth year will minimize the average annual cost.

Proof:

(a)   When time 't' is a continuous variable.

Let R_t =Maintenance cost at time 't'

C= the capital cost of the item

S= the scrap value of the item

Then the annual cost of the item at any time 't' = R_t + C - S

The maintenance cost incurred during 'n' years becomes = $\int_0^n R_t \, dt$

The total cost incurred on the item = P (n) = $\int_0^n R_t \, dt$ + C-S

Hence average total cost is given by

$$F(n)= \frac{P(n)}{n} = \frac{1}{n}\int_0^n R_t \, dt + \frac{C-S}{n} \qquad\qquad ---(1.1)$$

Now, we have to find such time 'n' for which F(n) is minimum. Therefore, differentiating F(n) with respect to 'n',

$$\frac{dF(n)}{dn} = \frac{1}{n} R_n + \left[-\frac{1}{n^2}\right] \int_0^n R_t \, dt - \frac{C-S}{n^2} = 0, \text{ for minimum of F (n),} \qquad ---(1.2)$$

which gives

$$R_n = \frac{1}{n} \int_0^n R_t \, dt + \frac{C-S}{n} = \frac{P(n)}{n}, \text{ by virtue of equation (3.1),} \qquad ---(1.3)$$

Hence, the maintenance cost at time 'n' = average cost in time 'n'

(b) **When time 't' is a discrete variable**

Since the time is measured in discrete units, the cost equation (1.1) can be written as

$$F(n) = \frac{P(n)}{n} = \sum_{t=1}^{n} \frac{R_t}{n} + \frac{C-S}{n} \qquad ---(1.4)$$

By using finite differences, F(n) will be minimum if the following relationship is satisfied :

$$\Delta F(n-1) < 0 < \Delta F(n) \qquad ---(1.5)$$

Now, differencing (3.4) under the summation sign by definition of first difference,

$$\Delta F(n) = F(n+1) - F(n)$$

$$= \left[\sum_{t=1}^{n} \frac{R_t}{n+1} + \frac{C-S}{n+1}\right] - \left[\sum_{t=1}^{n} \frac{R_t}{n} + \frac{C-S}{n}\right]$$

$$= \left[\frac{R_{n+1}}{n+1} + \sum_{t=1}^{n} \frac{R_t}{n+1}\right] - \sum_{t=1}^{n} \frac{R_t}{n} + (C-S)\left[\frac{1}{n+1} - \frac{1}{n}\right]$$

$$= \frac{R_{n+1}}{n+1} - \sum_{t=1}^{n} R_t \left[\frac{1}{n+1} - \frac{1}{n} \right] + (C\text{-}S) \left[\frac{1}{n+1} - \frac{1}{n} \right]$$

$$= \frac{R_{n+1}}{n+1} - \sum_{t=1}^{n} \frac{R_t}{n(n+1)} - \frac{C-S}{n(n+1)}$$

Since $\Delta F(n) > 0$ for minimum of $F(n)$, so

$$\frac{R_{n+1}}{n+1} > \sum_{t=1}^{n} \frac{R_t}{n(n+1)} + \frac{C-S}{n(n+1)} \quad \text{or} \quad R_{n+1} > \sum_{t=1}^{n} \frac{R_t}{n} + \frac{C-S}{n}$$

Or $R_{n+1} > P(n)/n$, by virtue of equation (1.4)

Similarly, it can be shown that $R_n < P(n)/n$, by virtue of $\Delta F(n-1) < 0$

Hence $R_{n+1} > (P(n)/n) > R_n$

This completes the proof.

1.9.2 Case 2: Replacement Policy For Items Whose Maintenance Cost Increases With Time, And Money Value Changes With Constant Rate

Money Value:

When it is said that 'Money is of worth 10% per annum'. it can be interpreted in two ways:

(i) Firstly, investing an amount of Rs.100 today is equivalent to investing Rs.110 after one year i.e. if we plan to invest Rs.110 after one year it is equivalent of investing Rs.100 today.

(ii) Secondly, if Rs.100 is borrowed at the rate of interest 10% per annum and spend this amount today, then it is required to pay Rs. 110 in a year time.

So it can be inferred that Rs.100 in hand today will be equivalent to Rs.110 in hand after one year from now. In other words, Re.1 in hand after one year is of worth Rs. $(1.1)^{-1}$ in hand today.

Present Worth Factor (PWF):

Thus investing Re.1 after one year is equivalent to investing $(1.1)^{-1}$ rupee today with rate of interest 10% per annum. Re.1 to be invested after two years from now is equivalent to investing $(1.1)^{-2}$ Rupees today. Therefore Re.1 to be spent after n years is equivalent to investing $(1.1)^{-n}$ rupees today. The quantity $(1.1)^{-n}$ is considered as **present worth factor (pwf) or present value** of Re.1 to be spent n years from now.

In general, let money carry an interest rate of 'r', then $(1+r)^{-n}$ is called the **present worth factor (pwf) or present value** of Re.1 to be spent after n years from now.

Discount rate (Depreciation value):

Present worth of Re.1 to be spent after 'n' years from now is given by $v=(1+r)^{-n}$, where 'r' is the interest rate. Then, v is called **discount** rate or **depreciation value**.

Theorem.: when the maintenance cost increases with time and money value decreases at constant rate i.e. depreciation value is given, then the replacement policy will be: Replace if the next period's maintenance cost is grater than the weighted average of previous costs.

Proof:

Let C= Capital cost of the item to be replaced

R_i = Maintenance cost incurred at the starting of the ith year

r = interest rate

$v = (1+r)^{-1}$ is the present worth of Re.1 to be spent a year hence.

The proof can be divided into two major steps:

Step1: *To determine the present worth of total cost*

Let the item is replaced at the end of every nth year. The year wise present worth of costs on the item in the successive cycles of 'n' years can be computed as shown in Table 1.1.

Year	1	2	...	n	n+1	n+2	...	2n	2n+1
Present Worth	$C+R_1$	$R_2 v$	...	$R_n v^{n-1}$	$(C+R_1)v^n$	$R_2 v^{n+1}$	...	$R_2 v^{2n-1}$	$(C+R_1)v^{2n}$

Table 1.1: Present worth(Year wise) of future costs

Assuming that the item has no salvage value at the time of replacement, the present value of all future discounted costs associated with the policy of replacing the item at the end of every 'n' year will be given by

$$P(n)=\left[(C+R_1)+R_2v+\ldots+R_nv^{n-1}\right]+$$

$$\left[(C+R_1)v^n+R_2v^{n+1}+\ldots+R_nv^{2n-1}\right]+$$

$$\left[(C+R_1)v^{2n}+R_2v^{2n+1}+\ldots+R_nv^{3n-1}\right]+\ldots \text{ and so on}$$

Summing up the right hand side, we get

$$P(n) = (C+R_1)\left(1+v^n+v^{2n}+\cdots\right)+ R_2v\left(1+v^n+v^{2n}+\cdots\right)+R_nv^{n-1}\left(1+v^n+v^{2n}+\cdots\right)$$

$$= \left(C+R_1+R_2v+\cdots+R_nv^{n-1}\right)\left(1+v^n+v^{2n}+\cdots\right)$$

$$= \left(C+R_1+R_2v+\cdots+R_nv^{n-1}\right)\frac{1}{1-v^n} \qquad\qquad \text{---(1.6)}$$

$$[\text{Since } v<1,\text{the sum of infinite G.P.is } \frac{1}{1-v^n}]$$

$$\therefore\ \ P(n)=\frac{F(n)}{1-v^n}, \qquad P(n+1)=\frac{F(n+1)}{1-v^{n+1}} \qquad\qquad \text{---(1.7)}$$

where, for simplicity, $F(n) = C+ R_1 +\ldots+ R_nv^{n-1}$

Step2: *To determine replacement policy so that P(n) is minimum.*

As 'n' is measured is discrete units, we shall use the method of finite difference method is used in order to minimize the present worth cost P (n).

Therefore, if P(n+1) >P(n)> P(n-1), i.e. $\Delta P(n) > 0 > \Delta P(n-1)$, then P(n) will be minimum. So by the definition of first difference,

$$\Delta P(n) = P(n+1) - P(n) = \frac{F(n+1)}{1-v^{n+1}} - \frac{F(n)}{1-v^n} \text{ from equation (1.7)}$$

$$= \frac{F(n+1)(1-v^n)-F(n)(1-v^{n+1})}{(1-v^{n+1})(1-v^n)} \qquad \left[\frac{N^r}{D^r} form\right]$$

For convenience, we first simplify the N^r of $\Delta P(n)$ only. That is

$N^r = F(n+1)\ (1-v^n) - F(n)(1-v^{n+1})$

$\quad = \left[F(n+1) - F(n)\right] + v^{n+1}\ F(n) - v^n\ F(n+1)$

$\quad = R_{n+1}v^n + v^{n+1}F(n) - v^n[F(n)+v^n R_{n+1}] \quad [\because\ F(n+1) = F(n) + R_{n+1}\ v^n]$

$\quad = v^n(1-v^n)R_{n+1} - v^n(1-v)F(n)$

$\therefore\ \Delta P(n) = \dfrac{v^n(1-v^n)R_{n+1}-v^n(1-v)F(n)}{(1-v^{n+1})(1-v^n)}$

$\qquad\qquad = \dfrac{v^n(1-v)}{(1-v^{n+1})(1-v^n)}\left[\dfrac{(1-v^n)}{(1-v)}R_{n+1} - F(n)\right]$ \hfill ---(1.8)

Simply setting (n-1) for 'n' in equation (1.8)

$\Delta P(n-1) = \dfrac{v^{n-1}(1-v)}{(1-v^n)(1-v^{n-1})}\left[\dfrac{1-v^{n-1}}{(1-v)}R_n - F(n-1)\right]$

After little simplifications of RHS

$\Delta P(n-1) = \dfrac{v^{n-1}(1-v)}{(1-v^n)(1-v^{n-1})}\left[\dfrac{1-v^n}{(1-v)}R_n - F(n)\right]$ \hfill ---(1.9)

The quantity $\dfrac{v^n(1-v)}{(1-v^{n+1})(1-v^n)}$ in eqn. (1.8) is always positive,

$\qquad\qquad\qquad\qquad\qquad\qquad$ since $|v|<1$.

Thus, $\Delta P(n)$ has the same sign as the quantity under [....] in (1.8), with similar explanation for $\Delta P(n-1)$ in (1.9) also.

Hence the condition, $\Delta P(n-1) < 0 < \Delta P(n)$, for minimum present worth expenditure becomes.

$\qquad\qquad \dfrac{1-v^n}{1-v}R_n - F(n) < 0 < \dfrac{1-v^n}{1-v}R_{n+1} - F(n)$ \hfill ---(1.10)

$$\frac{1-v^n}{1-v} R_n < F(n) < \frac{1-v^n}{1-v} R_{n+1} \qquad \text{---(1.11)}$$

$$R_n < \frac{C+R_1+R_2v+\cdots+R_n v^{n-1}}{1+v+v^2+\cdots+v^{n-1}} < R_{n+1} \qquad \text{---(1.12)}$$

$$R_n < \frac{F(n)}{\sum v^{n-1}} < R_{n+1} \qquad \text{---(1.13)}$$

The expression between R_n and R_{n+1} in equations (1.12),and (1.13) is called the 'weighted average cost' of previous 'n' years with weights $1, v, v^2, \cdots, v^{n-1}$ respectively.

The value of 'n' satisfying the relationship (1.10), or (1.12) will be the best replacement age of the item. This proves the theorem.

Annual payment = Weighted average cost for 'n' years.

Selection of best machine:

While making a decision on solution of the best machine, various costs are to be taken into consideration. The costs that are constant with time for each given machine are to be taken into consideration. Some times these costs may differ for each machine. However those costs that are same for the machines under comparison may be excluded.

Considering two machines - M_1 and M_2 - selection of an economically best machine can be done by adopting the following outlined procedure.

Step 1: Find the best replacement age for both machines M_1 and M_2 by making use of

$$R_n < \frac{F(n)}{\sum v^{n-1}} < R_{n+1}$$

Let the optimum replacement age for machines M_1 and M_2 comes out be n_1 and n_2, respectively.

Step 2: calculate the fixed annual cost (or weighted average cost) for each machine by using the formula :

$$x = \frac{C + R_1 + R_2 v + \cdots + R_n v^{n-1}}{1 + v + v^2 + \cdots + v^{n-1}} = \frac{F(n)}{\sum v^{n-1}}$$

And in this formula substitute $n = n_1$ for machine M_1 and $n = n_2$ for machine M_2. Let it be x_1 and x_2 for machines M_1 and M_2 respectively.

Step 3: (i) If $x_1 < x_2$, then select machine M_1

(ii) If $x_1 > x_2$, then select machine M_2

(i)   If $x_1 = x_2$, then both machines are equally good.

1.10 REPLACEMENT OF ITEMS THAT FAIL COMPLETELY

Consider a system usually made up of a large number of low cost items that are prone to failure with age e.g. failure of a resistor in television, radio, computer etc. In some cases the failure of a component may cause the complete failure of the system. In such cases, the cost of overall failure will be quite higher than the cost of

component itself. E.g. the cost of a condenser or tube in an aircraft is little, but its failure may result in total collapse of the airplane.

When dealing with such situations, two types of replacement policies shall be considered.

(i) *Individual replacement*: In this policy, an item is replaced immediately after its failure.

(ii) *Group replacement*: In this policy, decision is about the age when all the items should be replaced, irrespective of whether the items have failed or not. In this policy the items that fail before the optimal time, will be replaced individually.

1.10.1 Case 1: Individual Replacement Policy

Under this policy an item is immediately replaced after its failure. To determine the probability of failure (or life span of any item), mortality tables are used. To discuss such type of replacement policy, we consider the problem of human population.

Assumptions:

(i) All deaths are immediately replaced by births, and

(ii) There are no other entries or exits.

However in reality it is impossible to have these conditions. But, the reason for assuming the above two is that the analysis will be easier by keeping the virtual human population in mind. Such models can be applied to industrial items, where death of a person is

equivalent to the failure of an item or part and birth of a person is equivalent to replacement. Thus, organizations also face a fairly common situation. The following Mortality Theorem will make the conceptions clear.

Mortality Theorem: A large population is subject to a given mortality law for a very long period of time. All deaths are immediately replaced by births and there are no other entries or exits. Then the age distribution ultimately becomes stable and that the number of deaths per unit time becomes constant (which is equal to the size of the total population divided by the mean age at death).

Proof: For convenience, let each death occurs just before some time t=w, where 'w' is an integer and no member of the population can survive upto and beyond w+1 time units,

i.e. life span of any member lies between t=0 and t=w.

Let f(t) = number if births at time 't',

   p(x) = probability of member will die (fail) just before age x+1, i.e. at age 'x'.

Now f(t-x) = the number of births at time (t-x). The age of such newly born members who remain alive at time 't' will obviously be 'x'. This can be understood from the following Fig. 1.1.
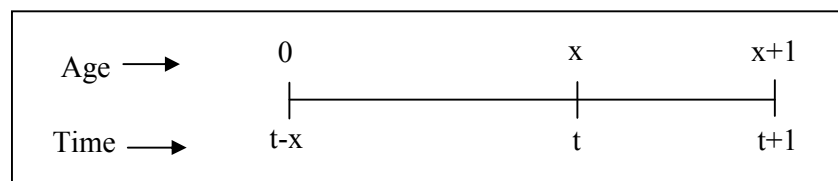

Fig. 1.1: Relation between age and time period

Hence, the expected number of deaths of such alive members at time 't' is p(x)f(t-x).

Therefore, the total number of deaths at time 't' will be

$$= \sum_{x=0}^{w} f(t-x)p(x), \text{ t=w, w+1, w+2, ...}$$

Also, total number of births at time (t+1) = f(t+1).

Since all deaths at time 't' are replaced immediately by births at time (t+1), therefore

$$f(t+1) = \sum_{x=0}^{w} f(t-x)p(x), \qquad \text{t=w,w+1,...} \qquad \text{---(1.17)}$$

The difference equation (1.17) in 't', may be solved by substituting f(t) $= A\alpha^t$ where A is some constant and $|\alpha| < 1$

Then difference equation (1.17) becomes

$$A\alpha^{t+1} = A\sum_{x=0}^{w} \alpha^{t-x} p(x)$$

On dividing by $A\alpha^{t-w}$, we get $\alpha^{w+1} = \sum_{x=0}^{w} \alpha^{w\text{-}x} p(x)$ or $\alpha^{w+1} - \sum_{x=0}^{w} \alpha^{w\text{-}x} p(x) = 0$

$$\text{Or } \alpha^{w+1} - \left[\alpha^{w} p(0) + \alpha^{w-1} p(1) + \alpha^{w-2} p(2) + \cdots + p(w)\right] = 0 \qquad \text{--- (1.18)}$$

Since the sum of all probabilities is unity, so

$$\sum_{x=0}^{w} p(x) = 1 \text{ or } 1 - \sum_{x=0}^{w} p(x) = 0 \text{ or}$$

$$1\text{-}[p(0)+p(1)+p(2)+...+p(w)]=0 \qquad \text{---(1.19)}$$

Now comparing equations (1.18) and (1.19), it is found that one solution of (1.18) is $\alpha_0 = 1$. But the polynomial equation (1.18) must have (w+1) total number of roots. Let the remaining roots be denoted

by $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_w$, consequently, the solution of difference equation (1.17) will be of the form:

$$f(t) = A_0 + A_1\alpha_1^t + A_2\alpha_2^t + \ldots + A_w\alpha_w^t \qquad \text{---(1.20)}$$

Where $A_0, A_1, \ldots, A_w$ are constants whose value can be determined from the age distribution at some given point in time. Further, it can be observed that the absolute value of all the remaining roots is less than unity i.e.

$|\alpha_i| < 1$ for i= 1,2,3,...,w.

Hence, $\alpha_1^t, \alpha_2^t, \ldots, \alpha_w^t$ tends to zero as t→∞. Consequently, equation (1.20) becomes f(t)= A_0, which shows that the number of deaths per unit time (as well as the number of births) is constant and equal to A_0.

To show that the age distribution ultimately becomes stable:

Let P(x) =the probability of members remain alive longer than 'x' time units.

Then,   P(x)= 1- P(survivor will die before attaining the age x)

$$= 1\text{-}[p(0)+p(1)+\ldots+p(x\text{-}1)] \text{ and } P(0)=1 \qquad \text{---(1.21)}$$

Since the number of births and deaths have become constant, each equal to A_0, the expected number of survivors of age 'x' is also stable at A_0 P(x).

As the deaths are replaced immediately (i.e. the number of births are always equal to the number of deaths), the size N of total population remains constant, i.e.

$$N = A_0 \sum_{x=0}^{w} P(x) \quad \text{or} \qquad \text{---(1.22)}$$

$$A_0 = \frac{N}{\sum_{x=0}^{w} P(x)} \qquad \text{---(1.23)}$$

Now the number of survivors aged 0,1,2,3,…… can be calculated from the equation (1.22) as $A_0, A_0 P(1), A_0 P(2)$, and so on

Finally, if the denominator in (1.23) i.e. $\sum_{x=0}^{w} P(x)$, is equivalent to mean age at death, then the age distribution will ultimately become stable.

To Prove this, $\sum_{x=0}^{w} P(x) = \sum_{x=0}^{w} P(x) \Delta(x)$ [$\because \Delta(x)=(x+1)-x$ =1 by finite differences]

$$= \left[P(x).x \right]_0^{w+1} - \sum_{x=0}^{w} (x+1) \Delta p(x)$$

$$= [P(w+1).(w+1) -0] - \sum_{x=0}^{w} (x+1) \Delta p(x) \qquad \text{---(1.24)}$$

But, P(w+1) = 1-p(0) –p(1) –p(2)-…-p(w) [from equ.(1.21)]

$\qquad$ = 0   [by virtue of eqn (1.19)]

And $\quad \Delta P(x) = P(X+1) - P(x)$

$\qquad$ = [1-p(0) –p(1)- p(2)-…- p(x)]- [1-p(0) –p(1)- p(2)-…- p(x-1)]

$\qquad$ = -p(x)

Therefore, substituting the simplified values of P(w+1) and ΔP(x) in equation (1.24) to obtain.

$$\sum_{x=0}^{w} P(x) \ = 0+ \sum_{x=0}^{w}(x+1)\,\text{p(x)} = \sum_{y=1}^{w+1}(y)p(y-1) \qquad \text{[setting x+1 = y]}$$

$$= \sum_{y=1}^{w+1} \text{y x prob.[that age at death is y]}$$

= mean (expected) age at death.

The theorem is thus proved.

1.10.2 Case 2: Group Replacement Of Items That Fail Completely

There are certain items viz. Light bulbs that either work or fail completely. In some cases a system made up of a big number of similar low cost items that are increasingly prone to failure with age. While replacing such failed items, always a set-up cost will be there for replacement. The said set-up cost is independent of the number of items to be replaced and hence it may be advantageous to replace entire group of items at fixed intervals. Such a policy is referred as group replacement policy and found attractive when the value of any individual item is so less and the cost of keeping records for age of individual items is not justifiable.

Group Replacement Policy:

Group replacement policy is defined in the following theorem and later it is explained by numerical example.

<u>Theorem</u>:

(a) One should group replace at the end of tth Period if the cost of individual replacements for the tth Period is greater than the average cost per period through the end of tth period.

(b) One should not group replace at the end of tth Period if the cost of individual replacement at the end of tth period is less than the average cost per period through the end of tth period.

<u>Proof</u>:

It is proposed to,

(i) replace all items in group simultaneously at fixed interval 't', whether they have failed or not, and

(ii) continue replacing failed items immediately as and when they fail.

Let N_t = number of units failing during time 't'

$\quad$ N=Total number of units in the system

$\quad$ C(t)=Cost of group replacement after time period 't'

$\quad$ C_1 = Cost of replacing an Individual item

$\quad$ C_2 = Cost of replacing an item in group

Then C(t) = $C_1[N_1 + N_2 + ... + N_{t-1}] + C_2 N$

Therefore, average cost per unit period will be

$$F(t) = \frac{C(t)}{t} = \frac{C_1[N_1 + N_2 + ... + N_{t-1}] + C_2 N}{t} \qquad\qquad \text{---(1.25)}$$

Now in order to determine the replacement age 't', the average cost per unit period [C(t)/t = F(t), say] should be minimum.

The condition for minimum of F(t) is

$$\Delta F(t-1) < 0 < \Delta F(t) \qquad\qquad\qquad \text{---(1.26)}$$

Now, $F(t) = F(t+1) - F(t) = \dfrac{C(t+1)}{t+1} - \dfrac{C(t)}{t} = \dfrac{C(t) + C_1 N_t}{t+1} - \dfrac{C(t)}{t}$

$$= \dfrac{tC_1 N_t - C(t)}{t(t+1)} = \dfrac{C_1 N_t - C(t)/t}{(t+1)} \qquad \text{---(1.27)}$$

which must be greater than zero for minimum F(t), that is

$$C_1 N_t > C(t)/t \qquad \text{---(1.28)}$$

Similarly, from ΔF(t-1) <0, $\qquad C_1 N_{t-1} < C(t)/t \qquad$ ---(1.29)

Thus from equations (1.28) and (1.29), the group replacement policy is completely established.

ILLUSTRATIVE EXAMPLE:

for a certain type of light bulbs(1000 Nos.), following mortality rates have been observed:

Week	: 1	2	3	4	5
Percent failing by the end of week	: 10	25	50	80	100

Each bulb costs Rs.10 to replace an individual bulb on failure. If all bulbs were replaced at the same time in group it would cost Rs. 4 per bulb. It is under proposal to replace all bulbs at fixed intervals of time, whether or not the bulbs have burnt out. And also it is to continue replacing immediately burnt out bulbs. Determine the time interval at which all the bulbs should be replaced?

SOLUTION:

Let p_i = the probability that a new light bulb fails during the ith week of its life.

Thus p_1 = the probability of failure in 1st week = 10/100 = 0.10

p_2 = the probability of failure in 2nd week = (25-10)/100 =0.15

p_3 = the probability of failure in 3rd Week = (50-25)/100 = 0.25

p_4 = the probability of failure in 4th week = (80-50)/100 = 0.3

p_5 = the probability of failure in 5th week = (100-80)/100 = 0.2

Since the sum of all the above probabilities is unity, the further probabilities p_6, p_7, p_8 and so on, will be zero. Thus, all light bulbs are sure to burnout by the 5th week.

Furthermore, it is assumed that bulbs that fail during a week are replaced just before the end of that week.

Let N_i = the number of replacements made at the end of the ith week. And let all 1000 bulbs are new initially. Thus,

$N_0 = N_0$ =1000

$N_1 = N_0 p_1 = 1000 \times 0.10$ =100

$N_2 = N_0 p_2 + N_1 p_1 = 1000 \times 0.15 + 100 \times 0.10$ =160

$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1 = 1000 \times 0.25 + 100 \times 0.15 + 160 + 0.10$ =281

$N_4 = N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1$ =377

$N_5 = N_0 p_5 + N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 p_1$ =350

$N_6 = 0 + N_1 p_5 + N_2 p_4 + N_3 p_3 + N_4 p_2 + N_5 p_1$ =230

$N_7 = 0 + 0 + N_2 p_5 + N_3 p_4 + N_4 p_3 + N_5 p_2 + N_6 p_1$ =286

It has been found that expected number of bulbs failing in each week increases until 4th week and then decreases until 6th week and again starts increasing. Thus, the number of failures or replacements will continue to oscillate till the system settles down to a steady state. In

steady state the proportion of bulbs burnt out in each week is reciprocal of their average life.

Individual replacement:

The mean age of bulbs $= 1 \times p_1 + 2 \times p_2 + 3 \times p_3 + 4 \times p_4 + 5 \times p_5$

= 1 x 0.1 + 2 x 0.15 +3 x 0.25 +5 x 0.30 +5 x 0.20 = 3.35 Weeks,

The number of failures in each week in steady state =1000/3.35 = 299

And the cost of replacing bulbs individually on failure

=10x299   (at the rate of Rs. 10 per bulb)

= Rs. 2990

Group replacement:

The replacement of all 1000 bulbs at the same time in bulk costs Rs. 4 per bulb and replacement of an individual bulb on failure costs Rs. 10. Costs of replacement of all bulbs simultaneously are calculated in the Table 1.2.

End of week	Cost of individual replacement	Total cost of group replacement (Rs.)	Average cost per week (Rs.)
1	100 x 10=1000	1000 x 4 + 100 x 10=5000	5000.00
2	160 x 10=1600	5000 + 160 x 10=5000	3300.00
3	281 x 10=2810	6600 + 281 x 10=9410	3136.67
4	377 x 10=3770	9410 + 377 x 10=13180	3295.00

Table 1.2: Cost of replacement of bulbs for Bulbs example

The cost of individual replacement in the 4th week is greater than the average cost for 3 weeks.

Therefore the optimal replacement decision is to replace all the bulbs at the end of every 3 weeks. Otherwise the average cost from 4th week onwards will start increasing.

1.11 EQUIPMENT RENEWAL

The word renewal means either to rope in new equipment in place of old equipment or repair the old equipment so that the probability density function of its future lifetime will be equal to that of new equipment. The probability that a renewal takes place during the small time interval (t, t+δt) is called the renewal rate at time 't'. Here time 't' is measured from the time when the first machine was started.

The renewal rate of equipment is asymptotically reciprocal of the mean life of the equipment i.e. $h(t) = \dfrac{1}{\lambda}$ = reciprocal of mean life

Equipment renewal comes under major preventive maintenance activity which may include replacement of few parts/subsystems or conditioning the equipment. With respect to this detailed mathematical models are not discussed here as the main focus area is on blocks and block replacements.

Summary: The first chapter discusses the concepts and importance of maintenance in production environment. The two types of maintenance –Breakdown and Planned- are explained. The objective of maintenance work should be to strike a balance between the availability and the overall running costs. Also, the possibilities viz. reengineering the equipment, replacement of the equipment etc. to ensure the equipment delivers its normal performance, are discussed.

Also this chapter discusses two categories of replacement techniques for determining the best replacement strategies for the items that deteriorate with time and those do not deteriorate but fail suddenly. These models are discussed with respect to the parameters like maintenance cost, time and value of money.