Time dispersion parameters – It quantifies the channel parameters.

1. Mean excess delay
2. RMS delay spread
3. Max excess delay

1) Mean excess delay ($\bar{\tau}$) – It is the first moment of power delay profile.

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

a_k → path gain for power delay

2) RMS delay spread (σ_τ) – It is the square root of the second central moment.

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

Each multipath component travel different path length so time of arrival for each path is different

A single pulse will be spreaded in time when it reaches at receiver. This effect which spread the signal is called delay spread.

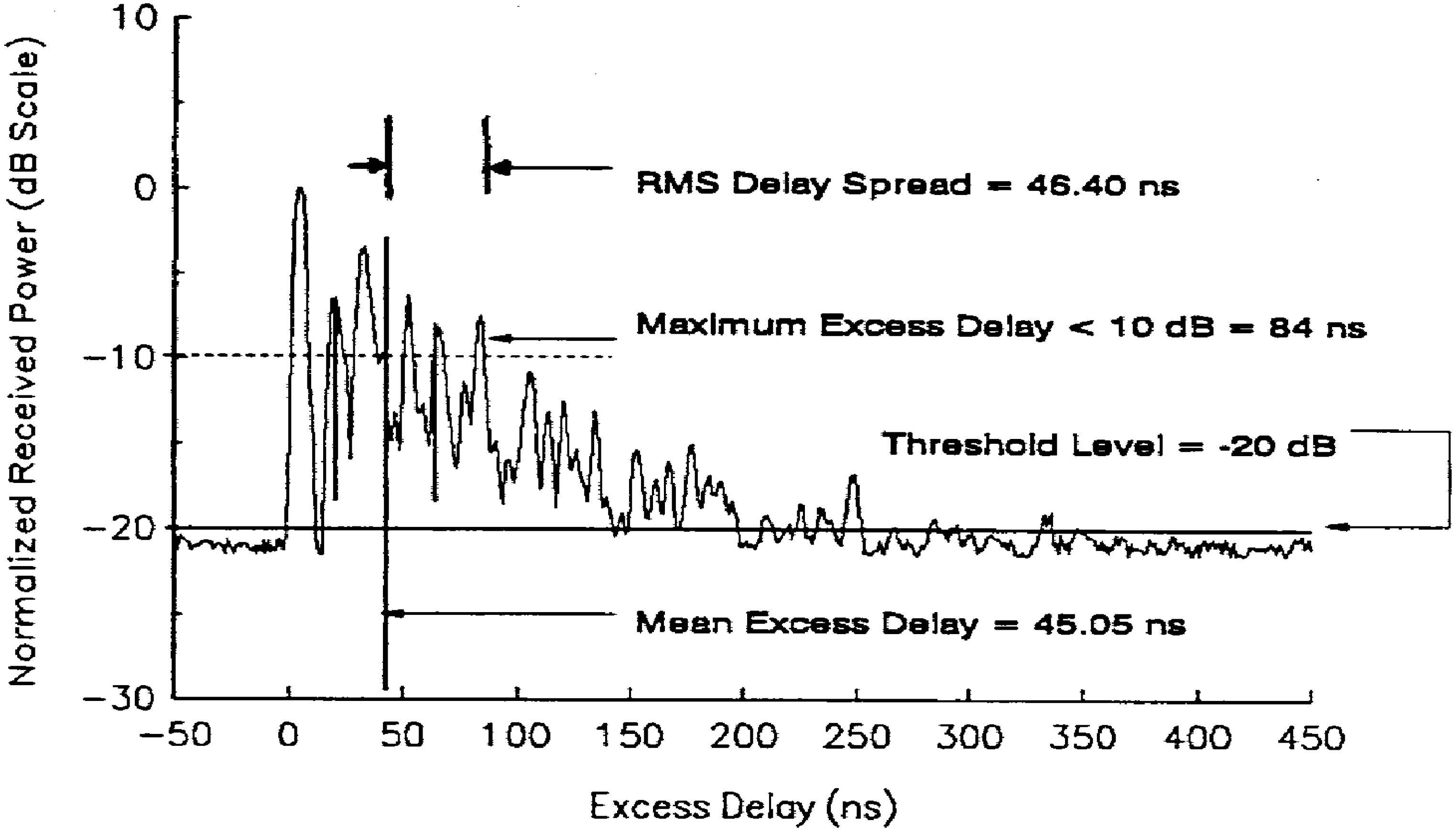
→ It characterizes time dispersiveness of channel.

3) Max excess delay (X dB)

Time delay during which multipath energy falls to X dB below maximum.

$$\text{M.E.D. (X dB)} = \tau_x - \tau_o$$

τ_o – Time at which first signal arrives at receiver



Ex.

$$\bar{\tau} = \frac{\sum P(\tau_k)\tau_k}{\sum P(\tau_k)} = \frac{1 \times 0 + 0.6 \times 2 + 4 \times 0.3}{1 + 0.6 + 0.3}$$

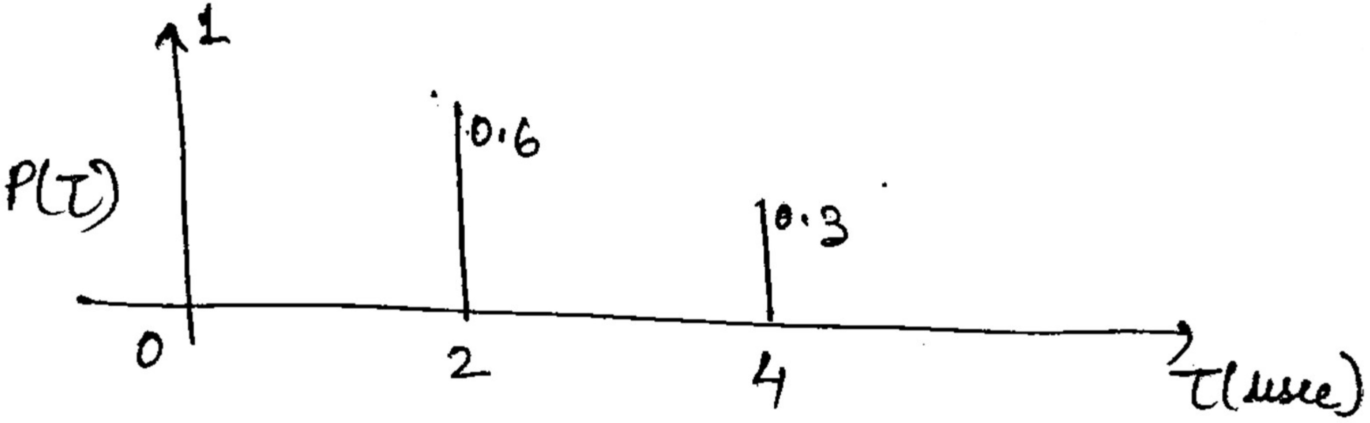
$$= 1.26\, \mu sec$$

$$\overline{\tau^2} = \frac{\sum P(\tau_k){\tau_k}^2}{\sum P(\tau_k)} = \frac{1 \times 0^2 + 0.6 \times 2^2 + 4^2 \times 0.3}{1 + 0.6 + 0.3}$$

$$= 3.79\, \mu sec^2$$

$$\sigma_\tau(rms\ delay\ spread) = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$= \sqrt{3.79 - (1.26)^2} = 1.48\, \mu sec$$



If BPSK modulation is used, what is maximum bit rate possible without use of any channel equalizer.

For no ISI

$$\frac{\sigma_\tau}{T_s} \leq 0.1$$

$$T_s \geq \frac{\sigma_\tau}{0.1} \geq 14.8\,\underline{\mu sec}$$

$$R_s = \frac{1}{T_s} = \frac{1}{14.8} = 67.56\,Kbps$$

Coherence bandwidth –

A statistical measure of the range of frequencies over which the channel can be considered flat

i.e.,

- Channel which passes all spectral components with equal gain and linear phase.
- It represents correlation between two fading signal envelopes at f_1 and f_2.
- It is a function of RMS delay spread.

Two frequencies that are larger than coherence B.W. faded independently.

It is useful in diversity technique

Multiple copies of same message are sent using different frequencies, these frequencies are separated by more than coherence bandwidth.

If we define coherence B.W. (B_c) as range of frequencies over which correlation is above 90% (0.9).

$$B_c = \frac{1}{50\,\sigma_\tau}$$

for more than 50% correlation

$$B_c = \frac{1}{5\,\sigma_\tau}$$

Ex. For a certain multipath channel let

σ_τ = 1.37 μsec

for 50% correlation

$$B_c = \frac{1}{5\ \sigma_\tau} = \frac{1}{5 \times 1.37} = 146\ KHz$$

Since, B_c > 30 KHz AMPS (Analog Mobile Phone System) will work without equalizer.

But it is not enough for GSM because 200 KHz is requirement for channel.

Coherence Time and Doppler Spread –

Doppler Spread = B_D = max doppler shift

$$f_{D_{max}} = \frac{v}{\lambda} = \cos\theta \Big|_{max}$$

If bandwidth of baseband signal is much greater than B_D then effect of Doppler spread is negligible at receiver (This is a slow fading)

Coherence time is statistical measure of time for which impulse response is time invariant.

If Symbol period > coherence time

Then channel will change during transmission of signal, hence there will be distortion at receiver.

$$T_c \approx \frac{1}{f_m}$$

$$T_c = \sqrt{\frac{9}{16\,\pi\, f_m^2}} = \frac{0.423}{f_m}$$

fm → max frequency shift due to doppler spread

Two signals arrived at receiver with a time spread grater than T_c are affected differently by channel.

Large coherence time $\Longrightarrow$ channel changes slowly

Type of Small-Scale Fading (Based on multipath time delay spread):

1. Flat Fading
2. Frequency Selective Fading

Flat Fading,

- $B_S << B_C \Longrightarrow \sigma_\tau << T_S$
- Rayleigh, Ricean distribution.
- Spectral characteristic of f_x signal preserved.

Frequency Selective Fading,

- $B_S > B_C \Longrightarrow \sigma_\tau >> T_S$
- ISI occurs.
- Spectral characteristics of f_x signal not preserved.

Type of Small-Scale Fading (Based on doppler spread B_D):

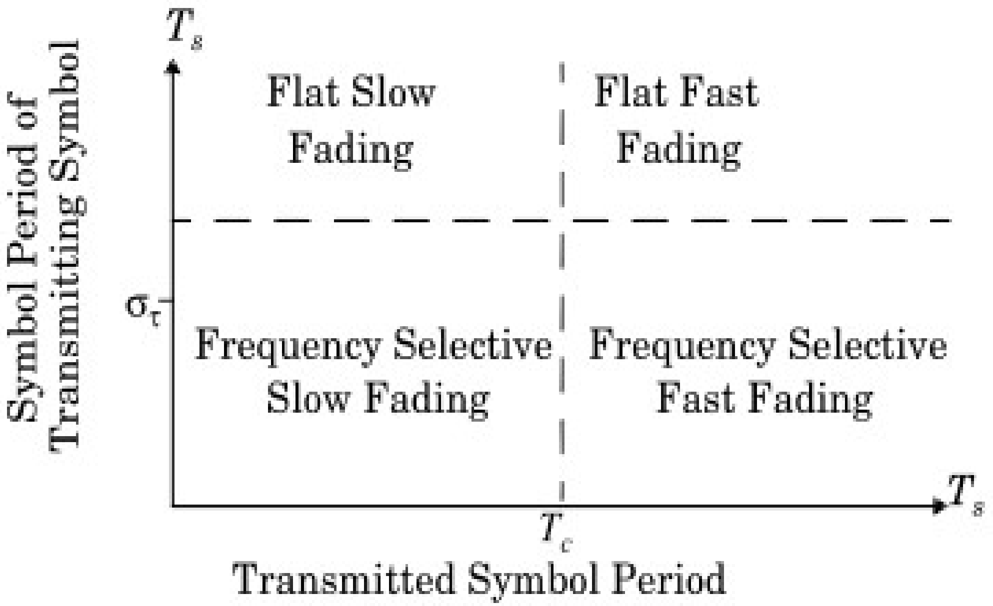
1. Fast fading
2. Slow fading

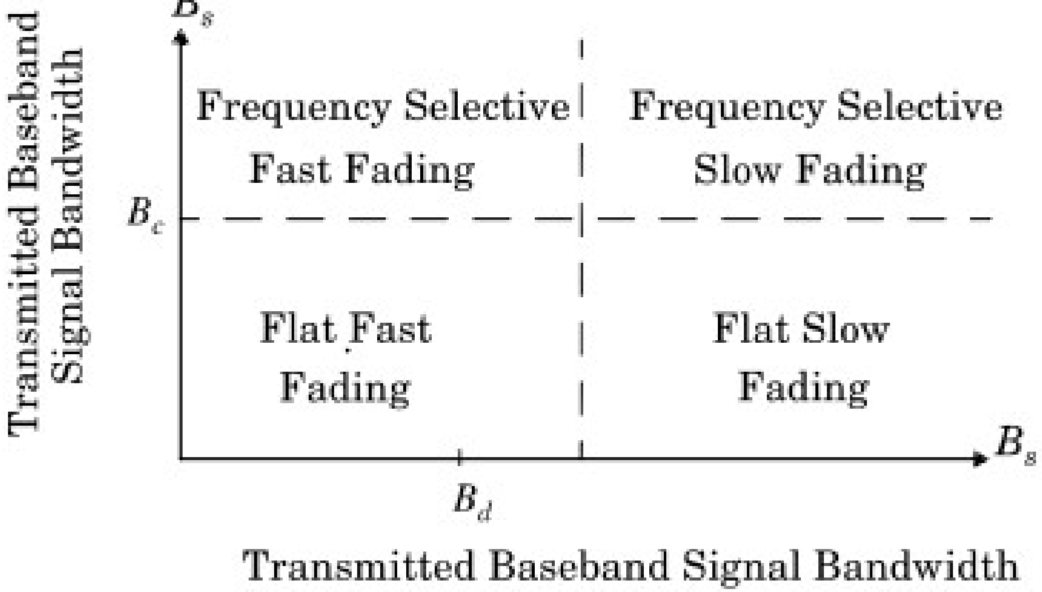
Fast fading,

- High doppler spread.
- $T_C < T_S$
- Channel varies faster than baseband signal variation.

Slow fading,

- Low doppler spread.
- $T_C > T_S$
- Channel variations are smaller than baseband signal variation.


Symbol Period of Transmitting Symbol
T_s
Flat Slow Fading
Flat Fast Fading
σ_τ
Frequency Selective Slow Fading
Frequency Selective Fast Fading
T_s
T_c
Transmitted Symbol Period


Transmitted Baseband Signal Bandwidth
B_s
Frequency Selective Fast Fading
Frequency Selective Slow Fading
B_c
Flat Fast Fading
Flat Slow Fading
B_s
B_d
Transmitted Baseband Signal Bandwidth