FLOW OVER NOTCHES AND WEIRS

9.1. DEFINITIONS

Notch. A **notch** *may be defined as an opening provided in the side of a tank or vessel such that the liquid surface in the tank is below the top edge of the opening*. A notch may be regarded as an *orifice* with the water surface *below its upper edge*. It is generally made of *metallic plate*. *It is used for measuring the rate of flow of a liquid through a small channel or a tank.*

Weir. A **weir** *may be defined as any regular obstruction in an open stream over which the flow takes place*. It is made of *masonry or concrete.* The conditions of flow in the case of a weir are practically the same as those of a rectangular notch. That is why, a notch is sometimes called as a weir and vice versa.

Weirs may be used for *measuring the rate of flow of water in rivers or streams.*

— *Nappe or vein*. The sheet of water flowing through a notch or over a weir is known as the *nappe* or *vein*.

— *Sill or crest*. The top of the weir over which the water flows is known as the *sill* or *crest.*

- ❖ The main difference between a notch and a weir is that the *notch is of small size*, but the weir is of a *bigger one*. moreover a notch is usually made in a *plate*, whereas a weir is usually made of *masonry or concrete*.

9.2. TYPES/CLASSIFICATION OF NOTCHES AND WEIRS

9·2·1 Types of Notches

There are several types of notches, depending upon their shapes. However, the following are important from subject point of view:

1. Rectangular notch,
2. Triangular notch,
3. Trapezoidal notch, and
4. Stepped notch.

9·2·2 Types of Weirs

There are several types of weirs depending upon their shapes, nature of discharge, width of crest or nature of crest. However, the following are important from subject point of view:

1. *According to shape:*

(*i*) Rectangular weir, and

(*ii*) Cippoletti weir.

2. *According to nature of discharge:*

(*i*) Ordinary weir, and

(*ii*) Submerged or drowned weir.

3. *According to the width of crest:*

(*i*) Narrow-crested weir, and

(*ii*) Broad-crested weir.

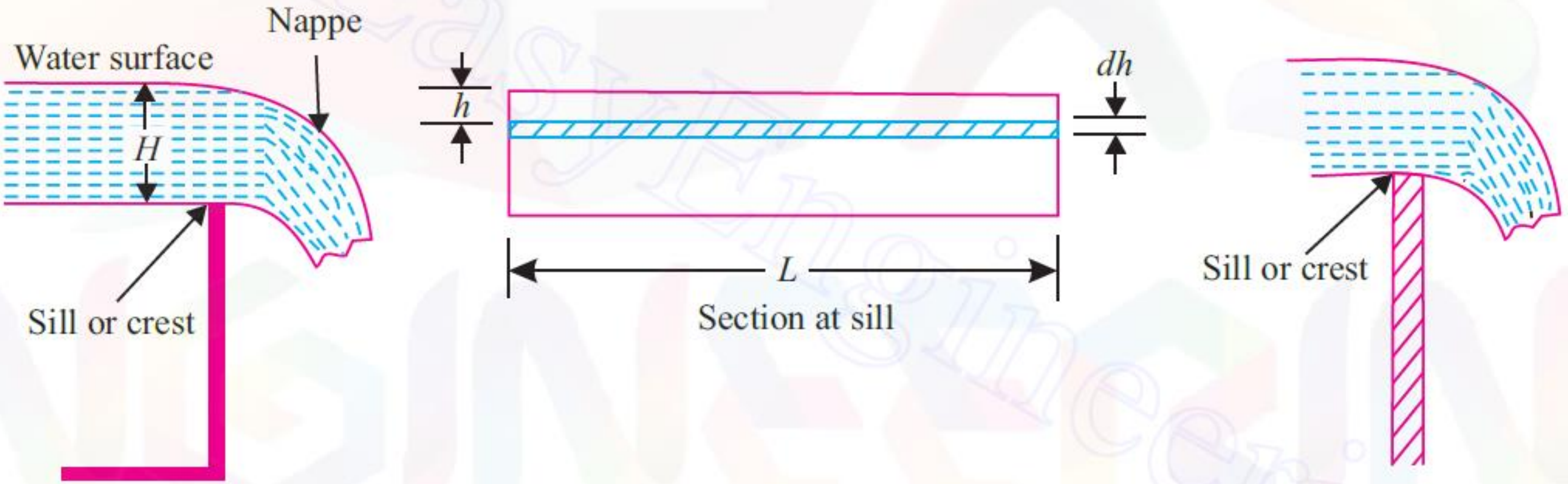
4. *According to the nature of crest:*

(*i*) Sharp-crested weir, and

(*ii*) Ogee weir.

9.3. DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 9·1.



(a) Rectangular notch

(b) Rectangular weir

Let, H = Height of water above sill of the notch,

L = Length of notch or weir, and

C_d = Co-efficient of discharge.

Let us consider a horizontal strip of water of thickness dh at a depth h from the water level as shown in Fig. 9·1.

$$\text{Area of strip} = L \times dh$$

Theoretical velocity of water flowing through strip

$$= \sqrt{2gh}$$

The discharge through the strip,

$$dQ = C_d \times \text{area of strip} \times \text{theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad \text{...}(i)$$

The total discharge, over the whole notch, may be found out by integrating the above equation within the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H (h)^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= 2/3 \times C_d \times L \times \sqrt{2g}\, (H)^{3/2}$$

i.e.

$$Q = \frac{2}{3} C_d \,.\, L \sqrt{2g}\, (H)^{3/2} \qquad \ldots(9\cdot1)$$

9.4. DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

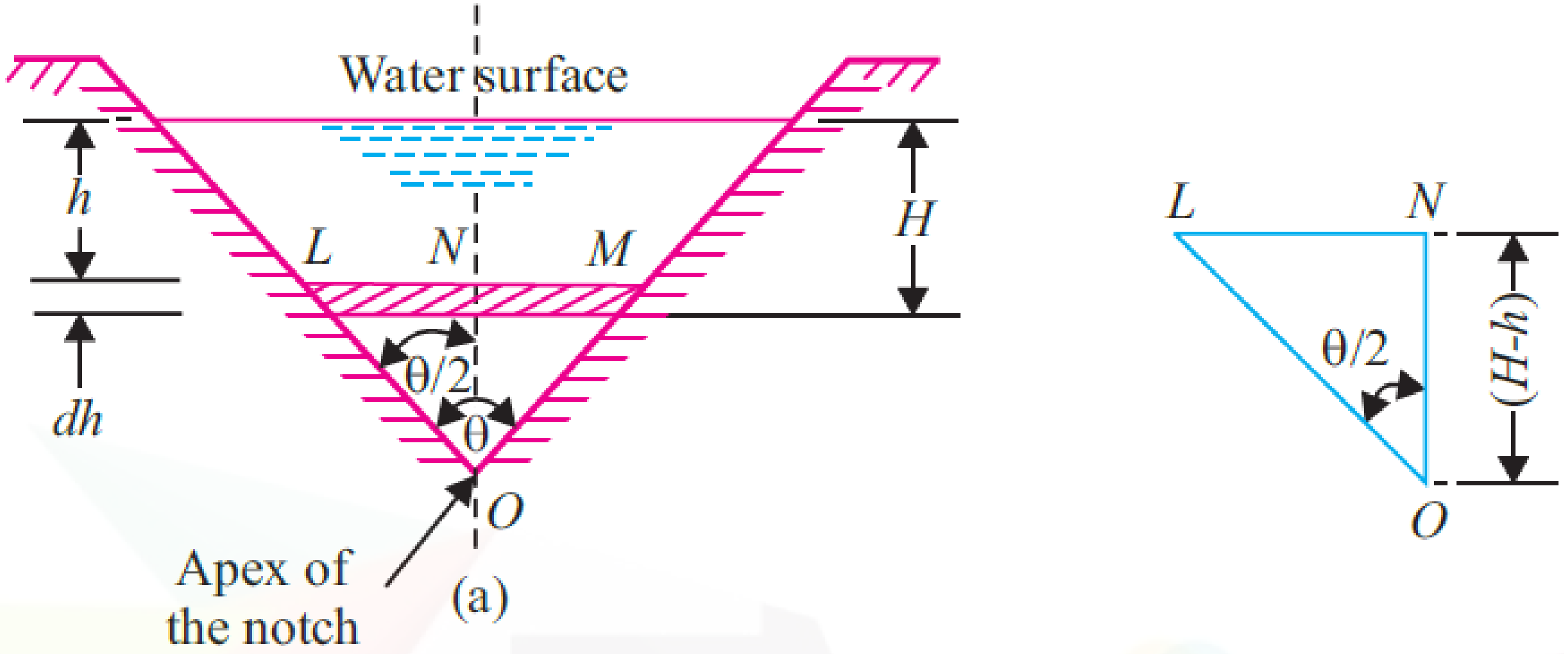
Refer to Fig. 9·2. *A* triangular notch is also called a *V*-notch.

Let, H = Head of water above the apex of the notch,

θ = Angle of the notch, and

C_d = Co-efficient of discharge.

Consider a horizontal strip of water of thickness dh, and at a depth h from the water surface as shown in Fig. 9·2.



From Fig. 9·2 (*b*), we have:

$$\tan\frac{\theta}{2} = \frac{LN}{ON} = \frac{LN}{H-h}$$

$$\therefore \quad LN = (H-h)\tan\frac{\theta}{2}$$

$$\text{Width of strip} = LM = 2LN = 2\,(H-h)\,\tan\frac{\theta}{2}$$

$$\therefore \quad \text{Area of the strip} = 2\,(H-h)\,\tan\frac{\theta}{2}\times dh$$

We know that theoretical velocity of water through the strip

$$= \sqrt{2gh}$$

∴ Discharge through the strip,

$$dQ = C_d \times \text{area of strip} \times \text{theoretical velocity}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2}\times dh \times \sqrt{2gh}$$

The total discharge, over the whole notch, may be found out by integrating the above equation, within the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \times 2(H - h) \tan \frac{\theta}{2} \times \sqrt{2gh} \cdot dh$$

$$= 2\, C_d \sqrt{2g} \tan \frac{\theta}{2} \int_0^H (H - h) \sqrt{h} \cdot dh$$

$$= 2\, C_d \sqrt{2g} \tan \frac{\theta}{2} \int_0^H \left[Hh^{1/2} - h^{3/2} \right] dh$$

$$= 2\, C_d \sqrt{2g} \tan \frac{\theta}{2} \left[\frac{H \cdot h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$= 2C_d \sqrt{2g} \tan\frac{\theta}{2}\left[\frac{2}{3} H \cdot H^{3/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2\, C_d \sqrt{2g} \tan\frac{\theta}{2}\left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2\, C_d \sqrt{2g} \tan\frac{\theta}{2}\left[\frac{4}{15} H^{5/2}\right]$$

$$= \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2} \qquad \text{...(9·2)}$$

For a *right angled V-notch*, if $C_d = 0{\cdot}6$,

$$\left(\theta = 90^\circ, \therefore \tan\frac{\theta}{2} = 1\right)$$

Then, $$Q = \frac{8}{15} \times 0\cdot6 \sqrt{2 \times 9\cdot81} \times 1 \times H^{5/2}$$

$$= 1{\cdot}417\, H^{5/2} \qquad \text{...(9·3)}$$

Advantages of a triangular notch over a rectangular notch:

A triangular notch claims the following *advantages* over a rectangular notch:

1. For a right angled *V*-notch or weir the expression for the computation of discharge is *very simple*.
2. For low discharges a triangular notch gives *more accurate results* than a rectangular notch.
3. In a given triangular notch, only one reading *i.e.*, head (*H*) is required to be taken for the measurement of discharge.
4. Ventilation of a triangular notch is not necessary.
5. The same triangular notch can measure a wide range of flows accurately.

9.5. DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

Fig. 9·3 shows a trapezoidal notch or weir which is a combination of a rectangular and a triangular notch or weir. As such the discharge over such a notch or weir will be the *sum of the discharges over the rectangular and triangular notches or weirs.*

Let, H = Height of water over the notch,

L = Length of the rectangular portion (or crest) of the notch.,

C_{d1} = Co-efficient of discharge for the rectangular portion, and

C_{d2} = Co-efficient of discharge for the triangular portion.

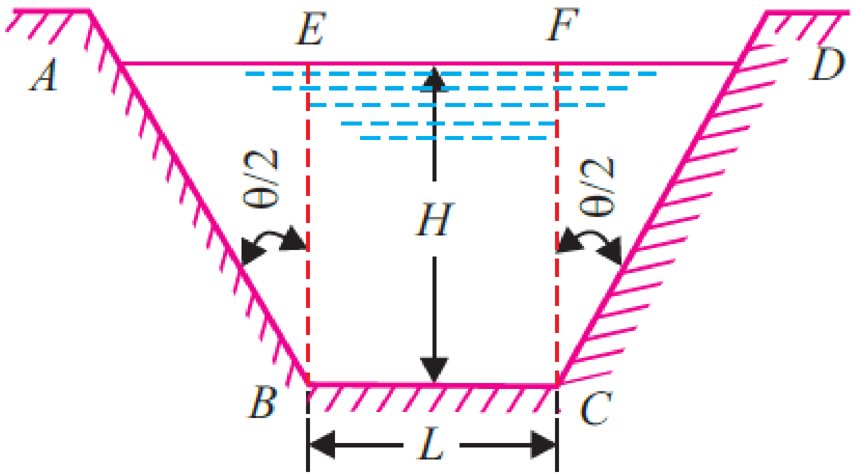


Fig. 9.3 The trapezoidal notch.

The discharge through the rectangular portion *BCFE* is given by (Eqn. 9·1),

$$Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g}\; H^{3/2}$$

The discharge through two triangular notches *ABE* and *FCD* is equal to the discharge through a single triangular notch of angle θ and is given by [Eqn. 9.2],

$$Q_2 = \frac{8}{15} C_d \sqrt{2g}\; \tan\frac{\theta}{2} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir *ABCDA*,

$$Q = Q_1 + Q_2$$

$$= \frac{2}{3} C_{d1} L \sqrt{2g}\; H^{3/2} + \frac{8}{15} C_{d2} \sqrt{2g}\; \tan\frac{\theta}{2} \times H^{5/2} \qquad \text{...(9·4)}$$

9.6. DISCHARGE OVER A STEPPED NOTCH

A **stepped notch** is *a combination of rectangular notches* as shown in Fig. 9·5. The discharge through a stepped notch is equal to the sum of the discharges through the different rectangular notches.

Consider a stepped notch as shown in Fig. 9·5.

Let, H_1 = Height of water above sill of notch 1,

L_1 = Length of notch 1,

H_2, L_2 = Corresponding values for notch 2,

H_3, L_3 = Corresponding values for notch 3, and

C_d = Co-efficient of discharge for all notches.

The discharge over the notch 1,

$$Q_1 = \frac{2}{3} C_d \, . \, L_1 \sqrt{2g} \; H_1^{3/2}$$

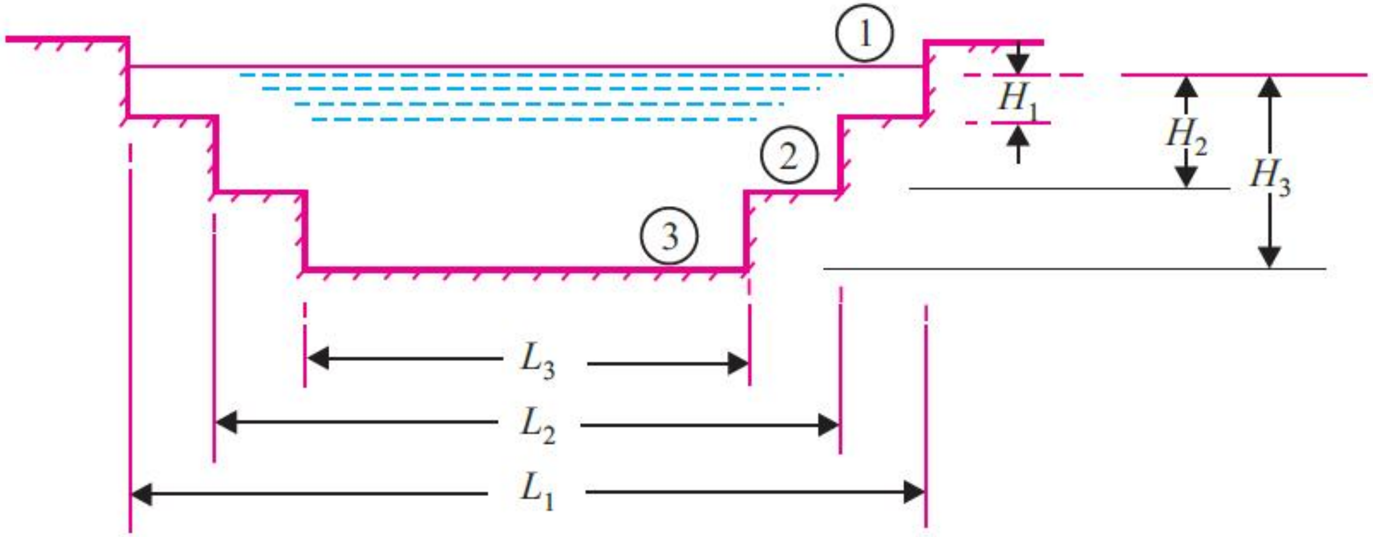


Fig. 9.5. The stepped notch.

Similarly, discharge over the notch 2,

$$Q_2 = \frac{2}{3} C_d \, . \, L_2 \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right]$$

and, discharge over the notch 3,

$$Q_3 = \frac{2}{3} C_d \cdot L_3 \sqrt{2g} \left[H_3^{3/2} - H_2^{3/2} \right]$$

$\therefore$ Total discharge, $Q = Q_1 + Q_2 + Q_3$

9.7. EFFECT ON DISCHARGE OVER A NOTCH OR WEIR DUE TO ERROR IN THE MEASUREMENT OF HEAD

The discharge over a rectangular notch or weir is proportional to $H^{3/2}$ and over a triangular notch or weir is proportional to $H^{5/2}$, where H is the height of liquid surface above the sill of the notch or weir. As such the accurate measurement of head H is quite essential in order to obtain an accurate value of the discharge over the notch or weir. However, if an error is introduced in the measurement of the head it will affect the computed discharge. The following cases of error in measurement of head will be considered:

(*i*) For rectangular notch or weir.

(*ii*) For triangular notch or weir.

(*i*) Rectangular Notch or Weir :

The discharge for a rectangular notch or weir is given by (Eqn. 9·1),

$$Q = \frac{2}{3} C_d . L \sqrt{2g}\ H^{3/2}$$

$$= KH^{3/2} \qquad ...(i)$$

$$\left(\text{where } K = \frac{2}{3} C_d . L \sqrt{2g}\right)$$

Differentiating the above equation, we get:

$$dQ = K \times 3/2 \times H^{1/2}\ dH \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get:

$$\frac{dQ}{Q} = \frac{K \times 3/2 \times H^{1/2}\ dH}{KH^{3/2}} = \frac{3}{2}\frac{dH}{H} \qquad ...(9·5)$$

Eqn. (9·5) shows that an *error of 1% in measuring H will produce* 1·5 % *error in discharge over a rectangular notch or weir.*

(*ii*) Triangular Notch or Weir :

The discharge over a triangular notch or weir is given by (Eqn. 9·2),

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2} \quad \text{...}(iii)$$

$$= K \times H^{5/2}$$

$$\left(\text{where, } K = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\right)$$

Differentiating Eqn. (*iii*), we get:

$$dQ = K \times 5/2 \times H^{3/2}\, dH \quad \text{...}(iv)$$

Dividing (*iv*) by (*iii*), we get:

$$\frac{dQ}{Q} = \frac{K \times 5/2 \times H^{3/2}\, dH}{K \times H^{5/2}} = \frac{5}{2}\frac{dH}{H} \quad \text{...}(9\cdot6)$$

Eqn. (9·6) shows that an error or *1% in measuring H will produce 2·5% error in discharge over a triangular notch or weir.*

9.8. VELOCITY OF APPROACH

The velocity with which the water approaches or reaches the weir or notch before it flows over it is known as '**velocity of approach**'. Thus if V_a is the velocity of approach, then an additional head $H_a\left(=\frac{V_a^2}{2g}\right)$ due to the velocity of approach, is acting on water flowing over the notch or weir. Then initial and final height of water over the notch or weir will be $(H + H_a)$ and H_a respectively.

The velocity of approach (V_a) is determined by finding the discharge over the weir or notch neglecting velocity of approach,

Let, Q = Discharge over weir or notch, and

A = Cross-sectional area of channel on the upstream side of the weir or notch.

Then the velocity of approach,

$$V_a = \frac{Q}{A}$$

This velocity of approach is used to find an additional head $\left(H_a = \frac{V_a^2}{2g}\right)$. Again the discharge is calculated and above process is repeated for more accurate discharge.

Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} C_d \,.\, L \sqrt{2g}\left[(H_1 + H_a)^{3/2} - (H_a)^{3/2}\right] \quad \text{...(9·7)}$$

9.9. EMPIRICAL FORMULAE FOR DISCHARGE OVER RECTANGULAR WEIR

The discharge over a rectangular weir,

$$Q = \frac{2}{3} C_d . L \sqrt{2g}\ H^{3/2} \qquad \text{... \textit{without} velocity of approach ...(}i\text{)}$$

$$= \frac{2}{3} C_d . L \sqrt{2g} \left[(H + H_a)^{3/2} - H_a^{3/2} \right] \qquad \text{... \textit{with} velocity of approach ...(}ii\text{)}$$

The equations (*i*) and (*ii*) are applicable to the weir or notch for which the *crest/sill length is equal to the width of the channel*; this type of weir is called **Suppressed weir.**

When the weir is not suppressed, the effect of end contractions is considered.

1. Francis's Formula:

On the basis of experimental analysis Francis established that:

— The end contraction decreases the effective length of the crest of weir and hence decreases the discharge.

— Each end contraction reduces the crest length by 0·1 H, where H is the head over the weir.

For a rectangular weir there are *two end contractions only* and hence *effective length*

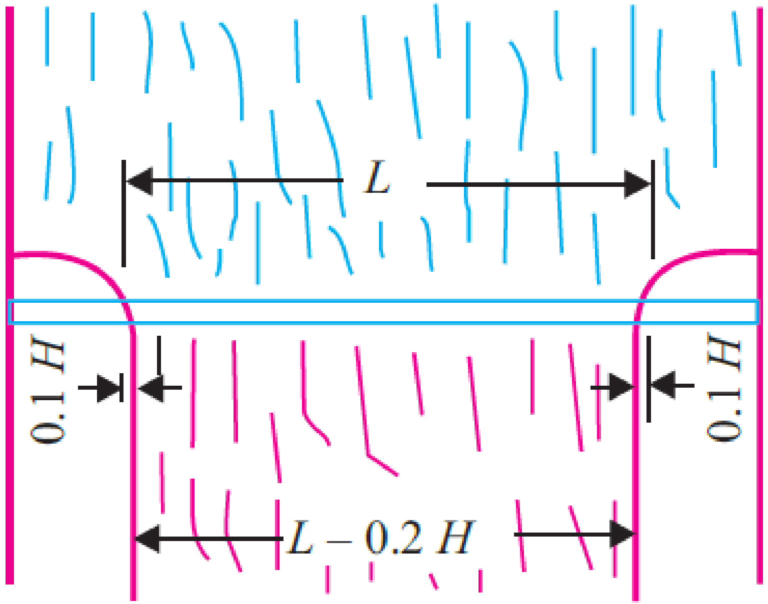


Fig. 9.7

$$= L - 0{\cdot}1 \times 2 \times H = L - 0{\cdot}2\,H$$

and discharge,

$$Q = \frac{2}{3} \times C_d \times (L - 0{\cdot}2\,H) \times \sqrt{2g}\,H^{3/2} \qquad \ldots(9{\cdot}8)$$

[If there are n end contractions, we may write the empirical formula proposed by Francis as:

$$Q = \frac{2}{3} \times C_d \times (L - 0{\cdot}1\, nH) \times \sqrt{2g}\; H^{3/2} \quad \text{... 9·8 (a)}$$
]

When $C_d = 0{\cdot}623$ and $g = 9{\cdot}81$ m/s^2, then:

$$Q = \frac{2}{3} \times 0.623 \times (L - 0.2H) \times \sqrt{2 \times 9.81}\; H^{3/2}$$

$$= 1{\cdot}84\,(L - 0{\cdot}2\,H)\,H^{3/2} \quad \text{...(9·9)}$$

— When end contractions are *suppressed*, we have:

$$Q = 1{\cdot}84\, LH^{3/2} \quad \text{...(9·10)}$$

(When end contractions are *suppressed*, the value of n is taken as *zero*.)

When velocity of approach is considered, we have:

$$Q = 1{\cdot}84\, L\left[(H + H_a)^{3/2} - H_a^{3/2}\right] \quad \text{...(9·11)}$$

2. Bazin's Formula:

Bazin's formula for the discharge (Q) over a rectangular weir is given as follows:

$$\left.\begin{aligned} Q &= m \times L \times \sqrt{2g} \times H^{3/2} \\ m &= \frac{2}{3} \times C_d = 0\cdot405 + \frac{0\cdot003}{H} \end{aligned}\right\} \quad ...(9\cdot12)$$

where, H = Height of water over the weir.

When velocity of approach is considered, we have:

where,

$$\left.\begin{aligned} Q &= m_1 \times L \times \sqrt{2g} \times (H + H_a)^{3/2} \\ m_1 &= 0\cdot405 + \frac{0\cdot003}{(H + H_a)} \end{aligned}\right\} \quad ...(9\cdot13)$$

9.10. CIPPOLETTI WEIR OR NOTCH

The Cippoletti weir is *trapezoidal weir*, having side slopes of 1 horizontal to 4 vertical as shown in Fig. 9·8. By providing slope on the sides, an increase in discharge through the triangular portions (*AED* and *FBC*) is obtained; without this slope the weir would be a rectangular one, and due to end contraction, the discharge would decrease. Thus the advantage of this weir is that the *factor of end contraction is not required* (while using Francis's formula).

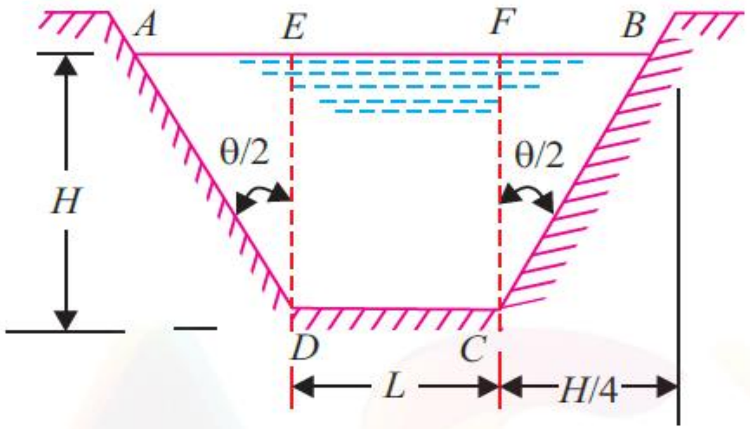


Fig. 9.8 Cippoletti weir.

Let us split the trapezoidal weir into the following:

(*i*) Rectangular weir, and

(*ii*) Triangular notch

The discharge over a rectangular weir (with two end contractions),

$$Q_1 = \frac{2}{3} \times C_d \times (L - 0 \cdot 2\ H) \sqrt{2g}\ H^{3/2} \quad \ldots(i)$$

and discharge over the triangular notch,

$$Q_2 = \frac{8}{15} C_d \sqrt{2g}\ \tan \frac{\theta}{2} \times (H)^{5/2} \quad \ldots(ii)$$

∴ Total discharge,

$$Q = Q_1 + Q_2$$

$$= \frac{2}{3} C_d (L - 0 \cdot 2\ H) \sqrt{2g}\ H^{3/2} + \frac{8}{15} C_d \sqrt{2g}\ \tan \frac{\theta}{2} \times H^{5/2} \quad \ldots(9 \cdot 14)$$

To avoid the factor of end contraction, Cippoletti gave the formula for discharge,

$$Q = \frac{2}{3} C_d L \sqrt{2g} \times H^{3/2} \quad \text{...(9·15)}$$

Equating the eqns. (9·14) and (9·15), we get:

$$\frac{2}{3} C_d L \sqrt{2g}\ H^{3/2} = \frac{2}{3} C_d (L - 0\cdot 2H) \sqrt{2g}\ H^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2}$$

Dividing both sides by $\frac{2}{3} C_d \sqrt{2g}\ H^{3/2}$, we have:

$$L = L - 0.2H + \frac{4}{5} \tan\frac{\theta}{2} \times H$$

or,
$$\frac{4}{5} \tan\frac{\theta}{2} \times H = 0.2H$$

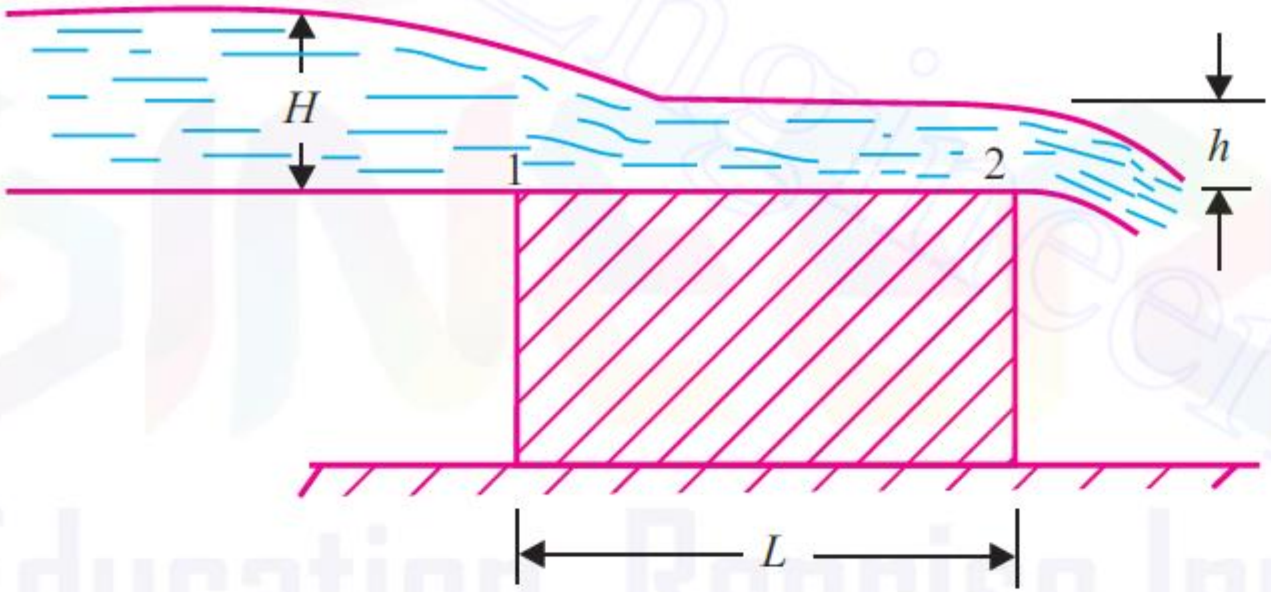
$\therefore$
$$\tan\frac{\theta}{2} = 0\cdot 2 \times \frac{5}{4} = \frac{1}{4}$$

The above relation indicates that in a trapezoidal weir having side slopes 1 horizontal to 4 vertical the factor of end contraction is *not* required for discharge, while using Francis's formula.

9.11. DISCHARGE OVER A BROAD CRESTED WEIR

Fig. 9·9 shows a broad-crested weir. Let 1 and 2 be the upstream and downstream ends of the weir respectively.

Let,
H = Head of water in the upstream side of the weir,
h = Head of water on the downstream side of the weir,
v = Velocity of the water on the downstream side of the weir,
L = Length of the weir, and
C_d = Co-efficient of discharge.



Applying Bernoulli's equation at 1 and 2, we get:

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$\therefore$
$$\frac{v^2}{2g} = H - h$$

or,
$$v = \sqrt{2g(H - h)}$$

$\therefore$ The discharge over weir,
$$\begin{aligned} Q &= C_d \times \text{area of flow} \times \text{velocity} \\ &= C_d \times L \times h \times v \\ &= C_d \times L \times h \times \sqrt{2g(H - h)} \\ &= C_d \times L \times \sqrt{2g}\,\sqrt{Hh^2 - h^3} \qquad \ldots(9{\cdot}16) \end{aligned}$$

The discharge will be *maximum*, if $(Hh^2 - h^3)$ is maximum.

$$\text{or,} \quad \frac{d}{dh}(Hh^2 - h^3) = 0$$

$$\text{or,} \quad 2hH - 3h^2 = 0$$

$$\text{or,} \quad 2H = 3h$$

$$\therefore \quad h = \frac{2}{3}H$$

Substituting the value of h in eqn. (9·16), we get:

$$Q_{\max} = C_d \times L \times \sqrt{2g}\sqrt{H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{9}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\,\sqrt{\frac{4}{27}\,H^3}$$

$$= C_d \times L \times \sqrt{2g} \times \frac{2}{3}\,H\,\sqrt{\frac{H}{3}}$$

$$= \frac{2}{3\sqrt{3}}\,C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= 0 \cdot 3849 \times C_d \times L \times \sqrt{2 \times 9 \cdot 81} \times H^{3/2}$$

$$= 1 \cdot 705 \times C_d \times L \times H^{3/2} \quad \text{...(9·17)}$$

9.12. DISCHARGE OVER A NARROW-CRESTED WEIR

In case of a narrow-crested weir, $2L < H$. This weir is similar to a rectangular weir or notch and hence, Q is given by:

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \quad \text{...(9·18)}$$

9.13. DISCHARGE OVER AN OGEE WEIR

In the Fig. 9·10 is shown an Ogee weir, in which the crest of the weir rises upto maximum height of 1·115 H and then falls as shown (where, H = height of water above inlet of the weir). The discharge over an Ogee weir is the same as that of a rectangular weir and is given by:

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad ...(9\cdot18)$$

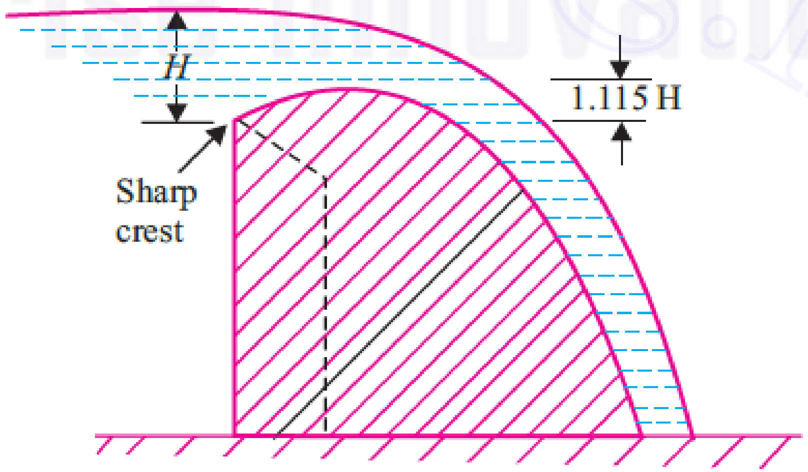


Fig. 9.10. An Ogee weir.

9.14. DISCHARGE OVER SUBMERGED OR DROWNED WEIR

A *weir is said to be* **submerged** *or* **drowned weir** *if the water level on its downstream side is above its crest*. Such a weir is shown in Fig. 9·11. The total discharge over the weir is obtained by dividing the weir into *two parts*. *The portion between upstream and downstream water surfaces may be treated as* **free weir** and portion between downstream water surface and crest as a **drowned weir.**

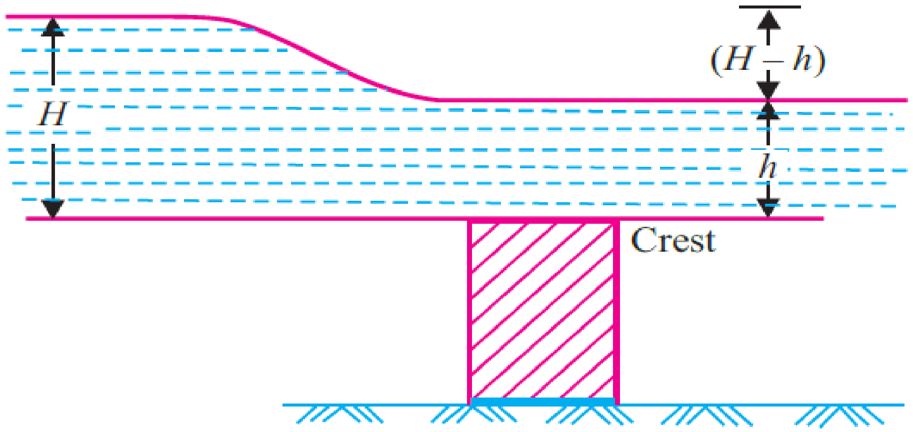


Fig. 9.11. Submerged weir.

Let, H = Height of water on the upstream side of the weir, and

h = Height of water on the downstream side of the weir.

Then, Q_1 = Discharge over upper portion

$$= \frac{2}{3}.C_{d1}.L.\sqrt{2g}\ (H-h)^{3/2}$$

and, Q_2 = Discharge through drowned portion

$= C_{d2} \times$ area of flow $\times$ velocity of flow

$$= C_{d2}.L.h.\sqrt{2g\ (H-h)}$$

where, C_{d1} and C_{d2} are the respective discharge co-efficients.

$\therefore$ Total discharge, $Q = Q_1 + Q_2$

$$= \frac{2}{3}.C_{d1}.L.\sqrt{2g}\ (H-h)^{3/2} + C_{d2}.L.h.\sqrt{2g\ (H-h)} \quad \ldots(9\cdot19)$$

9.15. TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH RECTANGULAR AND TRIANGULAR WEIRS OR NOTCHES

(a) Rectangular weir or notch :

Consider a reservior or a tank provided with a rectangular weir or notch in one of its sides.

Let, A = Uniform cross-sectional area of the tank,

L = Length of crest of the weir or notch,

H_1 = Initial height of liquid above the crest of notch,

H_2 = Final height of liquid above the crest of notch,

C_d = Co-efficient of discharge, and

T = Time required in seconds to lower the height of liquid from H_1 to H_2.

Further, let h = The height of liquid surface above the crest of weir or notch at any instant, and

dh = The fall of liquid surface in a small time dT.

Then,

$$-A.dh = Q \times dT$$

$$= \frac{2}{3} C_d . L . \sqrt{2g} . h^{3/2} \, dT$$

(Negative sign indicates that as T increases, h decreases.)

or,

$$dT = \frac{-A \, dh}{\frac{2}{3} C_d . L . \sqrt{2g} . h^{3/2}}$$

To obtain total time T, the above eqn. is integrated between the limits H_1 to H_2.

$$\therefore \qquad \int_0^T dT = \int_{H_1}^{H_2} \frac{-A dh}{\frac{2}{3} C_d . L . \sqrt{2g} \times h^{3/2}}$$

$$\text{or,} \qquad T = \frac{-A}{\frac{2}{3} C_d . L . \sqrt{2g}} \int_{H_1}^{H_2} h^{-3/2} \, dh$$

$$= \frac{-3A}{2 C_d . L . \sqrt{2g}} \left[\frac{h^{-3/2+1}}{(-3/2)+1} \right]_{H_1}^{H_2}$$

$$= \frac{-3A}{2 C_d . L . \sqrt{2g}} \left(-\frac{2}{1} \right) \left[\frac{1}{\sqrt{h}} \right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d . L . \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] \qquad \text{...(9·20)}$$

(*b*) Triangular weir or notch:

Consider a reservoir or a tank provided with a triangular weir or notch in one of its sides.

Let, A = Uniform cross-sectional area of the tank,

θ = Angle of the notch,

H_1 = Initial height of liquid above the apex of notch,

H_2 = Final height of liquid above the apex of notch, and

C_d = Co-efficient of discharge.

Further, let, h = The height of liquid surface above the crest of weir or notch at any instant; and,

dh = The fall of liquid surface in a small time dT.

Then,

$$-A.\,dh = Q \times dT$$

$$= \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2}\, dT$$

or,

$$dT = \frac{-A.dh}{\frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2}}$$

To obtain total time T, the above eqn. is integreated between the limits H_1 to H_2.

$$\therefore \qquad \int_0^T dT = \int_{H_1}^{H_2} \frac{-A.dh}{\frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2}}$$

or,

$$T = \frac{-A}{\frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-5/2}\, dh$$

$$= -\frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{h^{-3/2}}{-3/2}\right]_{H_1}^{H_2}$$

$$= \frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} (-2/3) \left[\frac{1}{h^{3/2}}\right]_{H_1}^{H_2}$$

$$= \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right] \qquad ...(9{\cdot}21)$$

THANKS