## DC-DC Converters (DC -Choppers)

A dc-to-dc converter, also known as dc chopper, is a static device which is used to obtain a variable dc voltage from a constant dc voltage source. Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc..... They are also used as dc voltage regulators.

Choppers are of two types: (1) Step-down choppers, and (2) Step-up choppers. In step-down choppers, the output voltage will be less than the input voltage, whereas in step-up choppers output voltage will be more than the input voltage.

## PRINCIPLE OF STEP-DOWN CHOPPER

Figure 1 shows a step-down chopper with resistive load. The thyristorin the circuit acts as a switch. When thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage waveform is as shown in Fig. 2.

Fig. 1 Chopper circuit.


Fig. 2 Chopper output voltage waveform, R - load.

## Methods of Control

The output dc voltage can be varied by the following methods.

* Pulse width modulation control or constant frequency operation.
* Variable frequency control.


## Pulse Width Modulation

- ton is varied keeping chopping frequency 'f' \& chopping period ' $T$ ' constant.
- Output voltage is varied by varying the ON time ton


## ANALYSIS OF A STEP-DOWN CHOPPER WITH R- <br> LOAD

Referring to Fig.2, the average output voltage $v_{o}$ can be found as
Let

$$
\begin{aligned}
\mathrm{T}=\text { control period } & =\mathrm{t}_{\mathrm{on}}+\mathrm{t}_{\text {off }} \\
v_{o} & =V_{a v}=\frac{1}{T} \int_{0}^{t_{o n}} V_{d} d t \\
V_{o} & =V_{d} \frac{\text { ton }}{T}=V_{d}(\gamma) \\
\text { where }, \gamma & =\frac{\text { ton }}{T}=\text { Duty cycle }
\end{aligned}
$$

- Maximum value of $y=1$ when $t_{\text {on }}=T \quad\left(t_{\text {off }}=0\right)$
- Minimum value of $\gamma=0$ when $\quad t_{\text {on }}=0 \quad\left(t_{\text {off }}=0\right)$

The output voltage is stepped down by the factor $\gamma$ $\left(0 \leq \mathrm{V}_{\mathrm{o}} \leq \mathrm{V}_{\mathrm{d}}\right)$. Therefore this form of chopper is a step down chopper.

The R.M.S. value of the output voltage $v_{o, r m s}=\sqrt{\frac{1}{T} \int_{0}^{\text {ton }} v_{0}{ }^{2} d t}=\sqrt{\gamma} v_{d}$

The Output power $=\frac{v_{o, r m s}{ }^{2}}{R}=\gamma \frac{v_{d}{ }^{2}}{R}$

Input current (Assume 100\% efficiency) $I_{a}=\frac{P}{V}=\frac{\gamma v_{d}{ }^{2} 1}{R \quad v_{d}}=\frac{\gamma v_{d}}{R}$
$\mathrm{f}=$ chopping frequency $=\left(\frac{1}{\operatorname{chopping} \operatorname{period}(T)}\right)=1 / \mathrm{T}$

The ripple factor, $R F$
It is a measure of the ripple content.

$$
R F=\sqrt{\left(\frac{V o_{\text {rms }}}{V o}\right)^{2}-1}=\sqrt{\frac{\gamma V_{d}^{2}}{\left.\frac{\gamma^{2} V_{d}^{2}}{2}\right)-1}}=\sqrt{\frac{1}{\gamma}-1}=\sqrt{\frac{1-\gamma}{\gamma}}
$$

Note1: In this type of chopper both the voltage and current are always positive, hence this chopper is called a single-quadrant Buck converter or class - A chopper.


Fig. 3 Single - quadrant operation
Note2: The chopper switch can also be implemented by using a power BJT, power MOSFET, GTO, and IGBT transistor. The practical devices have a finite voltage drop ranging from 0.5 V to 2 V , and for the sake of simplicity, the voltage drop of their power semiconductor devices are neglected.

Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V . The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms . Calculate, for resistive load of $10 \Omega$.
(a) The duty cycle $\gamma$.
(b) The average value of the output voltage $\mathrm{V}_{\mathrm{o}}$.
(c) The rms value of the output voltage $\mathrm{V}_{\text {orms }}$.
(d) The ripple factor RF.
(e) The output dc power.

Solution:
(a) $\mathrm{ton}=1 \mathrm{~ms}, \mathrm{~T}=2.5 \mathrm{~ms}$

$$
\gamma=\frac{\underline{t}_{\underline{o n}}}{T}=\frac{1 \mathrm{~ns}}{2.5 \mathrm{~ms}}=0.4
$$

(b) $\mathrm{V}_{\mathrm{av}}=\mathrm{V}_{\mathrm{o}}=\gamma \mathrm{V}_{\mathrm{d}}=0.4 \times 100=40 \mathrm{~V}$.
(c) $V_{\text {orms }}=\sqrt{\gamma} \bar{V}_{\mathrm{i}}=\mathrm{V} 0.4 \times 100=63.2 \mathrm{~V}$.
(d) $R F=\sqrt{\frac{1-\gamma}{\gamma}}=\sqrt{\frac{1-0.4}{0.4}}=1.225$
(e) $\quad I_{a}=\frac{V o}{R}=\frac{40}{10}=4 \mathrm{~A}$

$$
\mathrm{P}_{\mathrm{av}}=\mathrm{I}_{\mathrm{a}} \mathrm{~V}_{\mathrm{o}}=4 \times 40=160 \mathrm{~W}
$$

## STEP-DOWN CHOPPER WITH R-L LOAD

Consider a class-A chopper circuit with R-L load as shown in Fig.4.This is a step down chopper with one quadrant operation.

If we use the simplified linear analysis by considering that $T \ll \tau$, where ( $\mathrm{T}=\mathrm{t}_{\mathrm{on}}+\mathrm{t}_{\text {off }}$ ). In this case the current is continuous as shown in Fig.5.


Fig. 4


Fig. 5

## Referring to Fig.5:

- The current variation is almost linear and the current waveform becomes triangular.
- During the ON period , the equation govern the circuit is

$$
V_{d}=R \mathrm{i}+L \frac{d \mathrm{i}}{d t}
$$

Since $\frac{d \mathrm{i}}{d t}=$ constant, hence during ON period:

$$
\frac{d \mathrm{i}}{d t}=\frac{I_{2}-I_{1}}{t_{o n}}=\frac{\Delta I}{t_{o n}}
$$

Where $\Delta I$ is the peak - to -peak of the load current .Thus the equation of the current is given by:

$$
{\underset{1}{1}}_{\mathrm{i}}=I_{1}+\frac{\Delta \mathrm{I}}{\overline{\gamma T}} t \quad 0 \leq t \leq t_{\text {on }}
$$

Where

$$
\gamma=\frac{t o n}{T}
$$

During the off period:

$$
\frac{d \mathrm{i}}{d t}=\frac{I_{1}-I_{2}}{t_{o f f}}=-\frac{\Delta}{t_{o f f}}=-\frac{\Delta \mathrm{I}}{T-t_{n}}=-\frac{\Delta \mathrm{I}}{T-\gamma T}=-\frac{\Delta \mathrm{I}}{(1-\gamma) T}
$$

Hence, during the off the equation of the current is

$$
\mathrm{i}_{2}=I_{2}-\frac{\Delta \mathrm{I}}{(1-\overparen{V} T}\left(t-t_{n}\right) \quad t_{n} \leq t \leq T
$$

The average value of the output current is

$$
\begin{aligned}
& I_{a v}=\frac{1}{T}\left[\frac{1}{2} t_{o n}\left(I_{2}-I_{1}\right)+{\underset{2}{2}}_{t_{o f f}}\left(I_{2}-I_{1}\right)+I_{1} T\right] \\
& \quad I_{a v}=\frac{1}{2}\left(I_{2}+I_{1}\right)
\end{aligned}
$$

Example 2; An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with $5 \Omega$ resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.
(a) Find the average load voltage, current and the duty cycle of the chopper.
(b) What is the operating frequency $f$ ?
(c) Find the ripple current to maximum current ratio.

Solution:
(a) The average load voltage and current are:

$$
\begin{aligned}
\mathrm{V}_{\mathrm{av}} & =\mathrm{V}_{\mathrm{o}}=\gamma \mathrm{V}_{\mathrm{d}} \\
I_{a v} & =\frac{1}{2}\left(I_{2}+I_{1}\right)=\frac{9+10.2}{2}=9.6 A \\
I_{a v} & =\frac{V_{a v}}{R}=\frac{\gamma V_{d}}{R} \quad \text { or } \quad \gamma=\frac{\operatorname{Iav} R}{V_{\mathrm{i}}}=\frac{9.6 \times 5}{80}=0.6 \\
\mathrm{~V}_{\mathrm{av}} & =0.6 \times 80=48 \mathrm{~V} .
\end{aligned}
$$

(b) To find the operating (chopping) frequency:

During the ON period,

$$
\begin{equation*}
V_{d}=R \mathrm{i}+L \frac{d \mathrm{i}}{d t} \tag{1}
\end{equation*}
$$

Assuming $\quad \frac{d \mathrm{i}}{d t} \cong$ constant
$\frac{d \mathrm{i}}{d t} \cong \frac{\Delta}{t_{o n}}=\frac{10.2-9}{\gamma T}=\frac{1.2}{\gamma T}$
From eq.(1)

$$
L \frac{d \mathrm{i}}{d t} \cong V_{d}-I_{a v} R=80-5 x 9.6=32 V
$$

or $\quad \frac{d \mathrm{i}}{d t}=\frac{32}{L}=\frac{32}{0.4}=80 \mathrm{A.s}$
but $\frac{d \mathrm{i}}{d t}=\frac{1.2}{\gamma T}=80=\frac{1.2}{0.6 T}$
$\therefore \quad T=\frac{1.2}{0.6 \times 80}=25 \mathrm{~ms}$
Hence $\quad \mathbf{f}=\frac{1}{T}=\frac{1}{25 \times 10^{-3}}=40 \mathrm{~Hz}$
The maximum current $I_{m}$ occurs at $\gamma=1$,
$\therefore I_{m}=\frac{W d}{R}=\frac{1 \times 80}{5}=16 A$
Ripple current $\mathrm{I}_{\mathrm{r}}=\Delta \mathrm{I}=10.2-9=1.2 \mathrm{~A}$
$\therefore \frac{I_{\mathrm{r}}}{I_{m}}=\frac{1.2}{16}=0.075$ or $7.5 \%$.

## Input Current $I_{s}$

For the class-A chopper had shown in Fig.4, the On-state and OFF- state equivalent circuits are as depicted in Fig.6. When the thyristor is closed (during the ON period), the load current " i " rises from $I_{1}$ to $I_{2}$ and falls from $I_{2}$ to $I_{1}$ during the off period as shown in Fig.7(a). The input current $i_{s}$ flows during the ON period only as shown in Fig .7(b).


ON- State Equivalent CCT


OFF - State Equivalent CCT

Fig. 6


Fig. 7
The equation of the input current is

$$
\begin{array}{cl}
\mathrm{i}_{s}=\mathrm{i}_{1}=I+{ }_{1}+{\underset{\gamma}{\gamma T}}^{\Delta \mathrm{I}} t & 0 \leq t \leq t_{\text {on }} \\
\mathrm{i}_{s}=0 & t_{o n} \leq t \leq T
\end{array}
$$

The average value of the current drawn from the supply is simply found by,

$$
\begin{gathered}
I_{s(a v)}=\frac{1}{T}\left[\frac{1}{2} t_{o n}\left(I_{2}-I_{1}\right)\right]+\frac{t_{o n} I_{1}}{T} \\
\left.I_{s(a v)}=\frac{1}{T} \frac{1}{2} t_{o n}\left(I_{2}+I_{1}\right)\right]=\frac{t_{o n}}{2 T}\left(I_{2}+I_{1}\right)=\gamma I_{a v}
\end{gathered}
$$

## Minimum and Maximum Load Currents

The minimum current $\mathrm{I}_{1}$ and maximum current $\mathrm{I}_{2}$ can be found from the following two equations:

$$
\begin{aligned}
& I=I_{\min }=\frac{V_{o}}{R}-\frac{t_{\text {off }}}{2 L} V_{o} \\
& { }_{2}^{I}=I_{\max }=\frac{V_{o}}{R}+\frac{t_{\text {off }}}{2 L} V_{o}
\end{aligned}
$$

Where $V_{o}=V_{a v}$

## Note: The proof of these two equations is not needed

Example 3: A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a $10 \Omega$ resistive load. The inductive component of the load is 50 mH .For output average voltage of 50 V volts, find:
(a) The duty cycle
(b) ton
(c) The rms value of the output current
(d) The average value of the output current
(e) $I_{\text {max }}$ and $I_{\text {min }}$
(f) The input power
(g) The peak-to-peak ripple current.

## Solution:

(a)

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{av}}=\mathrm{V}_{\mathrm{o}}=\gamma \mathrm{V}_{\mathrm{d}} \\
& \quad \gamma=\frac{V_{a v}}{V_{d}}=\frac{50}{100}=0.5
\end{aligned}
$$

(b) $\quad T=1 / f=1 / 1000=1 \mathrm{~ms}$

$$
\gamma=\frac{t_{o n}}{T}
$$

$$
\mathrm{t}_{\mathrm{on}}=\mathrm{y} \mathrm{~T}=05 \times 1 \mathrm{~ms}=0.5 \mathrm{~ms} .
$$

(c) $\quad V_{\text {orms }}=\sqrt{\gamma} V_{\mathrm{i}}=\sqrt{0.5} \times 100=70.07 \mathrm{~V}$
(d) $\quad I_{a v}=\frac{V_{a v}}{R}=\frac{50}{10}=5 \mathrm{~A}$
(e) $\quad I_{\text {max }}=\frac{V_{a v}}{R}+\frac{t_{\text {off }}}{2 L} V_{a v}=\frac{50}{10}+\frac{(1-0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} \times 50$

$$
\begin{aligned}
& =5+0.25=5,25 \mathrm{~A} \\
I_{\text {min }}= & \frac{V_{\text {av }}}{R}-\frac{t_{\text {off }}}{2 L} V_{a v}=\frac{50}{10}-\frac{(1-0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} \times 50 \\
= & 5-0.25=4.75 \mathrm{~A}
\end{aligned}
$$

(f)

$$
\begin{gathered}
I_{s(a v)}=\frac{\gamma}{2}\left(I_{\min }+I_{\max }\right)=\gamma I_{a v}=0.5 \times 5=2.5 \mathrm{~A} \\
P_{\text {in }}=I_{s(a v)} V_{d}=2.5 \times 100=250 \mathrm{~W}
\end{gathered}
$$

(g)

$$
I_{-}=\Delta I=I_{\max }-I_{\min }=5.25-4.75=0.5 \mathrm{~A}
$$

Power Electronics

