## Kinematics and Mechanism (MEE-S203T)

Semester: 2021-22 (Odd Semester)

Time: 1.5 h
All questions are compulsory

## Section A

9 marks (9 questions of 1 mark each)

1. $A B C D$ is a four-link Mechanism. $A B=100 \mathrm{~mm}, B C=150$ $\mathrm{mm}, \mathrm{CD}=200 \mathrm{~mm}$ and $\mathrm{AD}=175 \mathrm{~mm}$. Can it act as a double-crank mechanism?
(a) No.
(b) Yes, if $A B$ is fixed
(c) Yes, if $B C$ is fixed
(d) Yes, if AD is fixed
2. The slider $A$, of an elliptical trammel shown in the figure moves towards $O$ with a velocity of $3 \mathrm{~m} / \mathrm{s}$ at the instant when $A B$ makes an angle of $40^{\circ}$ with the horizontal. Determine the velocity of the mid point of the link $A B$ at this instant

3. The number of instantaneous centres of rotation in a slider crank quick -return mechanism is $\qquad$ -.
4. A planer mechanism has 10 links and 12 rotary joints. Using Grubler's criterion, the number of degrees of freedom of the mechanism is $\qquad$ .
5. The magnitude of the velocity of any point on the kinematic link relative to the other point on the same kinematic link is the product of
(a) A square root of an angular velocity of the link and the distance between the two points under consideration
(b) An angular velocity of the link and the square of distance between the two points under consideration
(c) A square of an angular velocity of the link and the distance between the two points under consideration
(d) An angular velocity of the link and the distance between the two points under consideration
6. Match the degree of freedom of kinematic pair

| List-1 | List-II |
| :--- | :--- |
| 1. Spherical pair | A. 1 |
| 2. Cylindrical pair | B. 2 |
| 3. Screw pair | C. 3 |
| 4. Ball and socket joint |  |

7. For a crank and slotted lever quick return mechanism, $\alpha=$ $150^{\circ}$. Find the ratio of time of cutting stroke to time of return stroke.
8. Define: kinematic link, kinematic pair, kinematic chain.
9. Distinguish between mechanism and machine.

## Section B

9 marks (3 questions of 3 marks each)

1. For the configuration of a slider -crank mechanism, calculate the
(a) acceleration of the slider at B
(b) angular acceleration of the link AB OA rotates at $20 \mathrm{rad} / \mathrm{s}$ counter-clockwise

2. A mechanism in which $O A=Q C=100 \mathrm{~mm}, \mathrm{AB}=\mathrm{QB}=300$ mm and $C D=250 \mathrm{~mm}$. The crank OA rotates at 150 rpm in the clockwise direction. Determine the velocity of the slider at $D$.

## Section C

12 marks (2 questions of 6 marks each, Each question can have parts)

1. The crank OP of a crank- and slotted-lever mechanism rotates at 100 rpm in the counter-clockwise direction. Various lengths of the links are $\mathrm{OP}=90 \mathrm{~mm}, \mathrm{OA}=300$ $\mathrm{mm}, A R=480 \mathrm{~mm}$ and $R S=330 \mathrm{~mm}$. The slider moves along an axis perpendicular to $A O$ and is 120 mm from 0 .
(a) Determine velocity of the slider when the AOP is $135^{\circ}$ and quick return ratio.
(b) Determine angular velocity of link $A R, R S$.

2. PQRS is a four bar chain with link PS fixed. The lengths of the links are $P Q=62.5 \mathrm{~mm}$; $Q R=175 \mathrm{~mm}$; $\mathrm{RS}=112.5 \mathrm{~mm}$ and $\mathrm{PS}=200 \mathrm{~mm}$. The crank PQ rotates at $10 \mathrm{rad} / \mathrm{s}$ clockwise. Draw the velocity and acceleration diagram when angle QPS $=60^{\circ}$ and $Q$ and $R$ lie on the same side of PS.
(a) Find the angular velocity links $Q R$ and $R S$.

(a)
(c) State kennedy theorem. Locate all instantaneous centre of rotation for a four-bar mechanism.
(b) Find the angular acceleration of links QR and RS


Q2. $P Q R S$ is a four bar chain with link $P S$ fixed. The lengths of the links are $P Q=62.5 \mathrm{~mm} ; Q R=175 \mathrm{~mm} ; R S=112.5 \mathrm{~mm}$ and $P S=200 \mathrm{~mm}$. The crank PQ rotates at $10 \mathrm{rad} / \mathrm{s}$ clockwise. Draw the velocity and acceleration diagram when angle $\mathrm{QPS}=$ $60^{\circ}$ and $Q$ and $R$ lie on the same side of $P S$.
(a) Find the angular velocity links QR and RS.
(b) Find the angular acceleration of links QR and RS


200 mm

In $\triangle$ PQS,
$\cos 60^{0}=\frac{P Q^{2}+P S^{2}-Q S^{2}}{2 \times P Q \times P S}=\frac{62.5^{2}+200^{2}-Q S^{2}}{2 \times 62.5 \times 200}$
$Q S=177.2 \mathrm{~mm}$
$\cos \angle P Q S=\frac{P Q^{2}+Q S^{2}-P S^{2}}{2 \times P Q \times Q S}=\frac{62.5^{2}+177.2^{2}-200^{2}}{2 \times 62.5 \times 177.2}$

## $\angle P Q S=102.2^{0}$

$\frac{177.2}{\sin 60^{\circ}}=\frac{62.5}{\sin \angle P S Q}$
$\angle P S Q=17.79^{\circ}$
In $\triangle Q S R$
$\cos \angle R=\frac{Q R^{2}+R S^{2}-Q S^{2}}{2 \times Q R \times R S}=\frac{175^{2}+112.5^{2}-177.2^{2}}{2 \times 175 \times 112.5}$
$\angle R=72.44^{0}$
$\frac{Q S}{\sin \angle R}=\frac{112.5}{\sin \angle S Q R}=\frac{175}{\sin \angle Q S R}$
$\frac{177.2}{\sin 72.44^{0}}=\frac{112.5}{\sin \angle S Q R}$
$\angle S Q R=37.25^{\circ}$
$\angle Q S R+\angle S Q R+\angle R=180^{\circ}$

$$
\begin{aligned}
& \angle Q S R+37.25+72.44=180^{\circ} \\
& \angle Q S R=70.31^{\circ}
\end{aligned}
$$

$$
\angle Q=\angle P Q S+\angle S Q R
$$

$$
\angle Q=102.23^{0}+37.25^{0}=139.48^{0}
$$

$$
\angle S=\angle P S Q+\angle Q S R
$$

$$
\angle S=17.79+70.31=88.1
$$

$$
\angle P=60^{\circ}, \angle Q=139.48^{\circ}, \angle R=72.44^{\circ} \text { and } \angle S=88.1^{\circ}
$$


$V_{Q} \cos 49.48^{\circ}=V_{R} \cos 17.56^{\circ}$
$V_{R}=\frac{\omega_{P Q} \times P Q \times \cos 49.48^{0}}{\cos 17.56^{0}}=\frac{10 \times 0.0625 \times \cos 49.48^{0}}{\cos 17.56^{0}}=0.426 \mathrm{~m} / \mathrm{s}$
$V_{R}=0.426 \mathrm{~m} / \mathrm{s}$
$V_{R}=\omega_{R S} \times R S$
$\omega_{R S}=\frac{V_{R}}{R S}=\frac{0.426}{0.1125}=3.79 \mathrm{rad} / \mathrm{s}$
$\omega_{R S}=3.79 \mathrm{rad} / \mathrm{s}$
(1.5 marks)

Q1. For the configuration of a slider -crank mechanism, calculate the
(a) acceleration of the slider at B
(b) angular acceleration of the link AB

OA rotates at $20 \mathrm{rad} / \mathrm{s}$ counter-clockwise


## Acceleration of slider B:

$$
\begin{aligned}
& \vec{a}_{B}=\vec{a}_{A}+\vec{a}_{B / A} \\
& \left|\overrightarrow{\boldsymbol{a}}_{\boldsymbol{A}}\right|=\omega_{O A}^{2} \times O A \\
& =20^{2} \times 0.48 \\
& =192 \mathrm{~m} / \mathrm{s}^{2} \\
& \overrightarrow{\boldsymbol{a}}_{\boldsymbol{B} / A}=\bar{a}_{B / A}^{r}+\bar{a}_{B / A}^{t} \\
& \left|\bar{a}_{B / A}^{r}\right|=\omega_{A B}^{2} \times A B
\end{aligned}
$$

Find $\omega_{A B}$
Let $A B$ is link -3 , locate $I_{13}$.

$V_{A}=\omega_{A B} \times A P\left(\mathrm{P}\right.$ is point $\left.\mathrm{t}_{13}\right)$
$\omega_{A B}=\frac{V_{A}}{A P}=\frac{\omega_{O A} \times O A}{A P}$
Find AP,
In $\triangle$ OPB
$\cos 60^{\circ}=\frac{O B}{O P}$

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OP = 2x OB.
In \triangleOAB,
cos60}=\frac{O\mp@subsup{A}{}{2}+O\mp@subsup{B}{}{2}-A\mp@subsup{B}{}{2}}{2\timesOA\timesOB}=\frac{48\mp@subsup{0}{}{2}+O\mp@subsup{B}{}{2}-160\mp@subsup{0}{}{2}}{2\times480\timesOB
OB=1785 mm
(0.25 mm)
OP = 2 x 1785 mm
OA + AP = 3570 mm
480+AP = 3570 mm
AP = 3090 mm
\omega}\mp@subsup{\omega}{AB}{}=\frac{\mp@subsup{V}{A}{}}{AP}=\frac{\mp@subsup{\omega}{OA}{}\timesOA}{AP}=\frac{20\times480}{3090
\omega
(0.25 marks)
|\mp@subsup{\overline{a}}{B/A}{r}}|=\mp@subsup{\omega}{AB}{2}\timesAB=3.1\mp@subsup{1}{}{2}\times1.6=15.48\textrm{m}/\mp@subsup{\textrm{s}}{}{2}\quad(0.25 marks
|\mp@subsup{\overline{a}}{B/A}{t}|=\mp@subsup{\alpha}{AB}{}\timesAB=\mp@subsup{\alpha}{AB}{}\times1.6
\vec{\boldsymbol{a}}}\boldsymbol{B}=\mp@subsup{\vec{\boldsymbol{a}}}{A}{}+\mp@subsup{\vec{\boldsymbol{a}}}{B/A}{
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Acceleration of $B$ along horizontal direction. Therefore summation of these three vector along $y$-direction should be equal to zero.

$$
a_{B / A}^{r} \sin B+a_{B / A}^{t} \sin \left(90^{\circ}-B\right)=a_{A} \sin 60^{\circ}
$$

$15.48 \times \sin B+1.6 \times \alpha_{A B} \sin \left(90^{\circ}-B\right)=192 \sin 60^{\circ}$

Find angle B,

$$
\begin{aligned}
& \frac{1600}{\sin 60^{0}}=\frac{480}{\sin B} \\
& \sin B=0.26 \\
& \cos B=0.97 \\
& 15.48 \times 0.26+1.6 \times \alpha_{A B} \times 0.97=192 \sin 60^{0} \\
& \begin{array}{l}
\alpha_{A B}=104.6 \mathrm{rad} / \mathrm{s}^{2} \\
a_{B}=a_{B / A}^{r} \cos B-a_{B / A}^{t} \cos \left(90^{0}-B\right)+a_{A} \cos 60^{0} \\
a_{B}=15.48 \times 0.97-1.6 \times 104.6 \times 0.26+192 \times 0.5 \\
a_{B}=67.502 \mathrm{~m} / \mathrm{s}^{2}
\end{array} \quad(1 \text { marks }) \\
& (1 \text { marks })
\end{aligned}
$$

1. The crank OP of a crank- and slotted-lever mechanism rotates at 100 rpm in the counter-clockwise direction. Various lengths of the links are $O P=90 \mathrm{~mm}, \mathrm{OA}=300 \mathrm{~mm}, \mathrm{AR}=480 \mathrm{~mm}$ and $\mathrm{RS}=330 \mathrm{~mm}$. The slider moves along an axis perpendicular to AO and is 120 mm from O .
(a) Determine velocity of the slider when the AOP is $135^{\circ}$ and quick return ratio.
(b) Determine angular velocity of link AR, RS.


## Velocity approach method

## Step-1: Find the length AP

$\ln \triangle$ OPA,
$\cos 135^{0}=\frac{O P^{2}+O A^{2}-A P^{2}}{2 \times O P \times O A}$
$\cos 135^{0}=\frac{90^{2}+300^{2}-A P^{2}}{2 \times 90 \times 300}$
$A P^{2}=90^{2}+300^{2}-2 \times 90 \times 300 \cos 135^{\circ}$
$A P=369.16 \mathrm{~mm}$
(0.25 marks)
$\ln \triangle$ OPA,
Let $\angle \mathrm{OPA}=\ominus$
$\cos \theta=\frac{A P^{2}+O P^{2}-A O^{2}}{2 \times A P \times O P}$
$\cos \theta=\frac{369.16^{2}+90^{2}-300^{2}}{2 \times 369.16 \times 90}$
$\theta=35.07^{0}$
(0.25 marks)

Velocity of P perpendicular to $\mathrm{AP}=V_{P / 0} \cos \theta$
Velocity of $P$ perpendicular to $A P=$ Velocity of point $Q$ (coincident to point $P$, point $Q$ lie on link $A R$ )
Velocity of point $\mathrm{Q}=$ angular velocity of link $A R \times A Q$
$[A Q=A P]$
$V_{P / 0} \cos \theta=\omega_{A R} \times A P$
$\omega_{o P} \times O P \times \cos 35.07^{0}=\omega_{A R} \times A P$
$\omega_{A R}=\frac{\omega_{O P} \times O P \times \cos 35.07^{0}}{A P}$
$\omega_{A R}=\frac{\frac{2 \pi \times 100}{60} \times 90 \times \cos 35.07^{0}}{369.16}=2.08 \mathrm{rad} / \mathrm{s}$
(1 marks)

