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## Seperable polynomial:-

An irreducible polynomial $f(x)$ of degree $n$ over the field $F$ is said to be separable if it has all distinct roots

If roots are repeated then $f(x)$ is called inseparable polynomial.

## Minimal polynomial :-

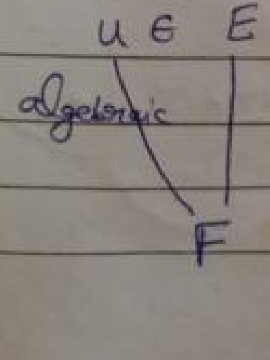
**Theorem:-** to find minimal polyn.

Let $E$ be an extension field of $F$. Let $u \in E$ be algebraic over $F$. Let $p(x) \in F[x]$ be a polynomial of the least degree such that $p(u) = 0$. then

(i) $p(x)$ is irreducible over $F$.

(ii) If $g(x) \in F[x]$ is such that ~~If $g(x) \in F[x]$~~ $g(u) = 0$, then $p(x)/g(x)$.

(iii) There is exactly one monic polynomial $p(x) \in F[x]$ of least degree such that $p(u) = 0$




$u \in E$ is algebraic over $F$ if $\exists$ a ~~...~~ polyn $p(x) \in F[x]$ s.t $p(u) = 0$

S K Jain

S.R. Nagpal

## Monic Polynomial :-

The monic irreducible polynomial in $F[x]$ of which $u$ is a root is called monic polyⁿ for $u$ over $F$.

**Q** Determine the minimal polyⁿ over $Q$ of the following elements

(a) $\sqrt{2}+5$

(b) $3\sqrt{2}+5$

(c) $\sqrt{-1+\sqrt{2}}$

(d) $\sqrt{2}-3\sqrt{3}$

(a) $u=\sqrt{2}+5$ $\qquad Q(\sqrt{2})$

$u-5=\sqrt{2}$ $\qquad Q$

$(u-5)^2-2=0 \Rightarrow u^2+25-10u-2=0$

$\Rightarrow u^2-10u+23=0$

$\therefore$ $P(x) = x^2-10x+23$ ✓ is minimal polyⁿ

(c) $u=\sqrt{-1+\sqrt{2}}$

$u^2 = -1+\sqrt{2}$

$u^2+1 = \sqrt{2}$

$(u^2+1)^2 = 2$

$u^4+1+2u^2 = 2$

$\therefore$ $p(x) = x^4+2x^2-1$

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Seperable elements:-

$$\begin{array}{c} E \\ | \\ F \end{array}$$

Let E be an extension of a field F. An element $\alpha \in E$ that is algebraic over F is called seperable over F if its minimal polyn over F is seperable.

Seperable Extension:-

An algebraic extension E of a field F is called seperable extension ~~of~~ if each element of E is seperable over F.

Remarks:- Any polynomial over a field of characteristic zero is seperable.

1. Thus if F is a field of characteristic 0 then any algebraic extension of F is seperable.

2. If F is finite field then each algebraic extension is seperable extension.

## Perfect field :-

A field F is called perfect if each of its algebraic extensions is seperable.

Examples:- ① Fields of characteristic 0
② finite fields

R, Q → perfect field.

Results: ① Every extension of Q is seperable.

② finite extension of a finite field is separable.

③ Infinite fields of characteristic $p>0$ have inseparable field extensions.
such fields are not in general perfect.