

Similar Matrices

Suppose A and B are matrices of order n over the same field F , then

B is said to be similar to A over field F if there exist an $n \times n$ invertible matrix P over F s.t. $B = P^{-1}AP$

OR

$$B = P^{-1}AP \Leftrightarrow PBP^{-1} = A$$

and we write $A \sim B$ ($\sim \rightarrow$ similar)

RESULTS- Relation of similarity is an equivalence relation on the set of all $n \times n$ square matrices over a field F

Proof:

[1] Reflexivity:- Let $A = [a_{ij}]_{n \times n}$; $a_{ij} \in F$
we can write as

$$A = I^{-1}AI \text{ always.}$$

So $A \sim A \Rightarrow \sim$ is reflexive.

[2] Symmetric

Let $A \sim B$, \therefore so there exist invertible matrix P s.t. $B = P^{-1}AP$

$$\Rightarrow PBP^{-1} = A$$

$$\therefore A = PBP^{-1}$$

$$A = (P^{-1})^{-1}BP^{-1}$$

$$\boxed{A = R^{-1}BR}$$

$$\text{Let } P^{-1} = R$$

$$\therefore B \sim A \text{ - (2)}$$

\therefore If $A \sim B$ then $B \sim A$, \therefore symmetric.

③ Transitivity - Let $A \sim B$ and $B \sim C$
T.P.T $A \sim C$.

$$A \sim B \Rightarrow \exists \text{ invertible } P \in F \text{ s.t. } B = P^{-1}AP$$

$$B \sim C \Rightarrow \exists \text{ invertible } Q \in F \text{ s.t. } C = Q^{-1}BQ$$

$$\begin{aligned} \text{now } C &= Q^{-1}BQ = Q^{-1}(P^{-1}AP)Q \\ &= (Q^{-1}P^{-1})A(PQ) \\ &= (PQ)^{-1}A(PQ) \\ C &= R^{-1}AR \end{aligned}$$

$\Rightarrow A \sim C$ proved, $R = PQ$
 $\iff P, Q$ are invertible $\iff R$ invertible

Questions- If A and B are two similar matrices then their determinant will be same.

SOME STANDARD RESULTS ON SIMILAR MATRICES ($P \Rightarrow Q$)

Contrapositive सत्य कर लो

RESULT ①
Statement:

If $A \sim B$ then "their determinant will be same" OR

$$\text{if } A \sim B \Rightarrow \det(A) = \det(B)$$

Converse need not be true.

Contrapositive is true.

Converse of Contrapositive is need not be true

Counter:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \det A = \det B \text{ but } A \not\sim B$$

RV of $A = 0, 0$, EV of $B = 0, 1$
Eigen values are not same so not diagonalizer

Counter के लिए I diagonal को zero इससे triangular को some Counter