Lecture:- 13th

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⊙ Method of solving ordinary differential equation of first order :-

$$\phi\left(x, y, \frac{dy}{dx}\right) = 0$$

There are four methods of solving ODE of first order-

1. Variable separable
2. Homogeneous ODE
3. Linear differential equation.
4. Exact ODE.

★ Solution of ODE:-

• **Definition:-** A relation b/w y & x that satisfy given ODE is known as solution of ODE.

⊙ **Complete solution:-** If no. of arbitrary constants in the solution of ODE is equals to the order of ODE, then that solution is known as complete solution.

Method-①:- Variable separable method.

1-Step:- $f_1(y)\cdot dy = f_2(x)dx$.

2-step:- $\int f_1(y)\,dy = \int f_2(x)\,dx + C$

3-step:- $y = f(x) + C$ {general equation}

Question:- Solve -

$$\frac{1}{x}\cdot dy - \frac{1}{y}\cdot dx = 0$$ {divide dx}

Sol:-

$$\Rightarrow \frac{1}{x}\cdot\frac{dy}{dx} - \frac{1}{y}\cdot = 0$$

$$\Rightarrow \frac{1}{x}\cdot\frac{dy}{dx} = \frac{1}{y}.$$

$$\boxed{y\cdot dy = x\cdot dx}$$ variable separated.

$$\int y\cdot dy = \int x\cdot dx + C$$

$$\boxed{\frac{y^2}{2} = \frac{x^2}{2} + C}$$ Ans.

Extra- {$2C = A$}

$$y^2 = x^2 + 2C$$

$$\boxed{y^2 = x^2 + A}$$ Ans

Question:- $x\cdot dy = y\cdot dx = ?$

Sol:- $\Rightarrow \frac{1}{y}\cdot dy = \frac{1}{x}\cdot dx \Rightarrow \int\frac{1}{y}\cdot dy = \int\frac{1}{x}\cdot dx + C$

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$\Rightarrow \log y = \log x + C \quad \{C = \log A\}$

$\log y = \log x + \log A$

$\log y = \log Ax.$

$\boxed{y = Ax}$ Ans

Question:- $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Sol:- $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\Rightarrow \int \frac{1}{1+y^2} dy = \int \frac{1}{1+x^2} . dx + C$

$\boxed{\tan^{-1} y = \tan^{-1} x + C}$ Ans

Question:- $e^{2x-3y} . dx + e^{2y-3x} . dy = 0$

Sol:- $e^{2x} . e^{-3y} . dx + e^{2y} . e^{-3x} dy = 0$ $\{$ divide $e^{-3y} . e^{-3x}\}$

$\Rightarrow \frac{e^{2x} . e^{-3y}}{e^{-3y} . e^{-3x}} . dx + \frac{e^{2y} . e^{-3x}}{e^{-3y} e^{-3x}} . dy = 0$

$\Rightarrow e^{5x} . dx + e^{5y} dy = 0$

$\Rightarrow \int e^{5y} . dy = \int -e^{5x} . dx$

$\Rightarrow \boxed{\frac{e^{5y}}{5} = -\frac{e^{5x}}{5} + C}$ A

or $\boxed{e^{5y} + e^{5x} = A}$ A Ans

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Question:- $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

Sol:- $\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 e^{-y}$

{divide e^{-y} both sides}

$\Rightarrow \frac{1}{e^{-y}} \cdot \frac{dy}{dx} = \frac{e^x \cdot e^{-y}}{e^{-y}} + \frac{x^2 e^{-y}}{e^{-y}}$

$\Rightarrow e^y \cdot \frac{dy}{dx} = (e^x + x^2)$

$\Rightarrow \int e^y \cdot dy = \int (e^x + x^2) dx$

$$\boxed{e^y = e^x + \frac{x^3}{3} + c}$$ Ans