Lecture:- 14th

Date:- 19/04/2022.

Homogeneous ordinary differential ~~funct~~ Equation:-

Homogeneous functions:- The function given by $z=f(x,y)$ is said to be homogeneous function of degree n if —

$$\left[f(tx,ty) = t^n f(x,y)\right]. \qquad \left[\begin{matrix} x \to tx \\ y \to ty \end{matrix}\right]$$

Example: (1) $f(x,y) = x^2+xy$.

$$f(tx,ty) = (tx)^2 + (tx)(ty)$$
$$= t^2x^2 + t^2xy$$
$$= t^2(x^2+xy)$$
$$= t^2 f(x,y)$$

So, this is homogeneous function of degree 2.

(2) Method-2nd:-

Homogeneous ODE:-

An ODE, $\boxed{\dfrac{dy}{dx} = \dfrac{f_1(x,y)}{f_2(x,y)}}$ is called homogeneous ODE of degree n, if each $f_1(x,y)$, $f_2(x,y)$ are homogeneous functions of same degree.

Procedure: How to solve ODE with HODE:-

If $\left[\dfrac{dy}{dx} = \dfrac{f_1(x,y)}{f_2(x,y)}\right]$ is H.O.D.E then, ★

2022/4/19 23:21

(i) ⇒ let $y = vx$
and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ } Put in (*) Equation.

(ii) ⇒ after the (i) step we get variable separable form then we have to solve it.

Example:- Solve $x^2y \cdot dx - (x^3+y^3)\,dy = 0$

Solution:- $x^2y\,dx = (x^3+y^3)dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2y}{x^3+y^3} = \frac{f_1}{f_2} \quad \text{—— (A) (Homogeneous Eq.)}$$

[∴ $y = vx$
$\frac{dy}{dx} = v + x\frac{dv}{dx}$]
Put in (A) Eq.

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2(vx)}{x^3+(vx)^3}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{v\cancel{x^3}}{\cancel{x^3}(1+v^3)}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{v}{(1+v^3)}$$

$$\Rightarrow x\cdot\frac{dv}{dx} = \frac{v}{(1+v^3)} - \frac{v}{1} \Rightarrow \frac{\cancel{v}-\cancel{v}-v^4}{(1+v^3)}$$

$$\Rightarrow x\cdot\frac{dv}{dx} = \frac{-v^4}{(1+v^3)}.$$

$$\Rightarrow \frac{1+v^3}{v^4}dv = -\frac{1}{x}\cdot dx.$$

$$\Rightarrow \int\frac{1}{v^4}dv + \int\frac{1}{v}dv = \int -\frac{1}{x}\cdot dx.$$

$$\Rightarrow -\frac{1}{3v^3} + \log v = -\log x + C$$

$$\boxed{-\frac{1}{3}\left(\frac{x}{y}\right)^3 + \log\left(\frac{y}{x}\right) = -\log x + C}$$

Ans

2022/4/19 23:21

Example:- Solve -

$$x\,dy - y\cdot dx = \sqrt{x^2+y^2}\cdot dx.$$

Solution:-

$$x\cdot dy - (y + \sqrt{x^2+y^2})\,dx = 0$$

$$x\cdot dy = (y+\sqrt{x^2+y^2})\,dx.$$

$$\left\{\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}\right\} \text{———} (A)$$

$$\left\{ y = vx, \quad \frac{dy}{dx} = v + x\cdot\frac{dv}{dx} \text{ Put} \right\}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{vx + \sqrt{x^2+(vx)^2}}{x}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{vx + x\sqrt{(1+v^2)}}{x}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = \frac{\cancel{x}(v+\sqrt{(1+v^2)})}{\cancel{x}}$$

$$\Rightarrow v + x\cdot\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$\Rightarrow x\cdot\frac{dv}{dx} = \cancel{v} + (\sqrt{1+v^2}) - \cancel{v}$$

$$\Rightarrow x\cdot\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\Rightarrow \int\frac{1}{x}\,dx = \int\frac{1}{\sqrt{1+v^2}}\,dv.$$

$$\Rightarrow \log x = \log\left[v + (\sqrt{1+v^2})\right]$$

$$\Rightarrow \log\left[\frac{y}{x} + \sqrt{1+(y/x)^2}\right] = \log x + C$$

$$\Rightarrow \log\left[y/x + \sqrt{1+(y/x)^2}\right] = \log x + \log C$$

$$\Rightarrow \boxed{\left[y/x + \sqrt{1+(y/x)^2}\right] = Cx.}$$

Ans

2022/4/19 23:21