Two Criterion of classification.

There are however many situation in which the response variable of interest may be affected by more than one factor so when two independent factors might have an affect on the response variable of interest, it is possible to design the test so that an analysis of variance can be used to test for the effects of the two factors simultaneously. Such a test is called two criterion (factors) analysis of variance. With two factor ANOVA we can test two sets of hypothesis with the same data at the same time.

In a two way classification the data are classified according to two different criteria or factors and proceed as following

(I.) Suppose we classify our sample of N value of x according to some quality A into K classes and according to another quality B into n classes so that $N = nK$. Let the sample variate value in ith A class and jth B class be x_{ij}.

Since the variance of a set of values is independent of the origin i.e. a shift of the origin does not affect variance calculations Again since we are concerned only with the ratio of two variances, any change of scale will not affect the value of this ratio Therefore in case large (numerically) values of x_{ij} we can shift the origin through a suitable quantity and also may change the scale to make easy calculations.

We now construct the table

		Quality A									
		1	2	3	—	—		k	S_j	S_j^2	$\sum_j x_{ij}^2$
Quality B	1	x_{11}	x_{12}		j			x_{1k}			
	2	x_{21}	x_{22}					x_{2k}			
	⋮										
	i	-	—	-	x_{ij}	-		—			
	n							x_{nk}			
S_i									$S = \sum S_i = \sum S_j$	$\sum S_j^2 =$	$\sum\sum x_{ij}^2$
S_i^2									$\sum S_i^2 =$		
$\sum_i x_{ij}^2$									$\sum\sum x_{ij}^2$		

(II) Calculate the following sum of squares:

(i) Total sum of squares (SST)

$$= \sum_i \sum_j (x_{ij} - \bar{x})^2 = \sum\sum x_{ij}^2 - \frac{S^2}{N}$$

with dof $(nk-1)$

(ii) Sum of squares b/w A classes (SSC)

$$= \sum_{i=1}^{k} n(\bar{x}_i - \bar{x})^2 = \frac{\sum S_i^2}{n_i} - \frac{S^2}{N}$$

with dof $(k-1)$

(iii) Sum of squares b/w B classes (SSR)

$$= \sum_{j=1}^{n} k(\bar{x}_j - \bar{x})^2 = \frac{\sum S_j^2}{n_j} - \frac{S^2}{N}$$

(IV) Residual sum of squares (SSE)

$$= SST - (SSC + SSR)$$

$$= \sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \bar{x}_j - \bar{x}_i + \bar{x})^2$$

$$= \sum \sum x_{ij}^2 - \frac{\sum S_i^2}{n_i} - \frac{\sum S_j^2}{n_j} + \frac{S^2}{N}$$

with dof $(n-1)(k-1)$

After calculating these sum of squares we draw the the following ANOVA-table for two criterion of classification

Source of Variation	Sum of Squares	Degrees of freedom	Mean Squares	F ratios
Between A Classes	SSC	$k-1$	$MSC = \frac{SSC}{k-1}$	$F = \frac{MSC}{MSE}$
Between B Classes	SSR	$n-1$	$MSR = \frac{SSR}{n-1}$	$F = \frac{MSR}{MSE}$
Residual	SSE	$(k-1)(n-1)$	$MSE = \frac{SSE}{(k-1)(n-1)}$	
Total	SST	$nk-1$		

CRITICAL VALUES of F

First we see the value of F form table for $v_1 = (k-1)$ & $v_2 = (k-1)(n-1)$ and compare with calculated value of b/w A classes and then find the value of F form table for $v_1 = (n-1)$ and $v_2 = (k-1)(n-1)$.