Chi-Square Distribution

The square of the standard normal variate Z i.e.

$$Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2$$

is called a chi-square variate with 1 degree of freedom. Generalising, we have that if X_i, $i = 1, 2 - n$ are n independent variables $N(\mu_i, \sigma_i^2)$, then

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{x_i - \mu_i}{\sigma_i}\right]^2$$

is a chi-square variate with n degree of freedom. and the distribution of this variate is called a chi-square distribution and is denoted by $\chi^2(v)$. The pdf of $X \sim \chi^2(v)$ with v degree of freedom is given by

$$f(x) = \frac{1}{2^{v/2} \sqrt{v/2}} e^{-x/2} x^{(n/2 - 1)}, \quad 0 \leq x \leq \infty$$

The χ^2 distribution has only one parameter, v the number of degrees of freedom. For very small number of degrees of freedom, the chi-square distribution is skewed to right and as the number of degrees of freedom increases the curve rapidly becomes more symmetrically so that for large values of v the chi-square distribution is closely approximated by the normal curve.

The chi-square distribution is probability distribution and the total area under the curve in each chi-square distribution is 1.

Properties of χ^2 distribution

① The mean and variance of χ^2 distribution are v and $2v$ respectively.

② $\mu_1 = 0$ $\mu_2 = 2v$, $\mu_3 = 8v$, $\mu_4 = 48v + 12v^2$

③ $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{64v^2}{8v^3} = \frac{8}{v}$ & $\beta_2 = \frac{\mu_4}{\mu_2^2} = 3 + \frac{12}{v}$

④ As the number of degrees of freedom $n \to \infty$, the χ^2- distribution tends to the normal distribution

⑤ If $X_1, X_2 \ldots X_m$ are m independent χ^2- variates with degree of freedom $v_1, v_2 \ldots v_m$ respectively, then $\sum X_i$ is also a χ^2- variate with $v_1 + v_2 + \ldots + v_m$ degree of freedom. The converse is also true.

Application of χ^2- distribution

The χ^2 test is one of the most popular statistical inference procedures. It is applied to a very large number of problems in practice which can be summed up as the following:

① χ^2-test as a test of goodness of fit: For examples suppose that we have fitted a binomial or a Poisson distribution to a given data of a sample We use the χ^2-distribution to test whether this fitting of the binomial or Poisson distribution to data is acceptable

② χ^2-test as a test of independence: For examples, suppose that a population has two characteristics or attributes or traits. χ^2-distribution can be used to test whether the two attributes are dependent (related) or independent, based on a random sample drawn from a population.

③ χ^2-test as a test of homogeneity: the χ^2-test of homogeneity is an extension of the chi-square test for independence. Tests of homogeneity are designed to determine whether two or more independent random samples are drawn from the same population or from different populations. For example, we may be interested in finding out wether or not universities students of various labels i.e. UG, PG, PhD feel the same in regards to the amount of work required by their profess i.e too much work, right amount of work or little work. We shall take hypothesis that the three samples come from the same population i.e. the three classification are homogeneous in so far as the opinion of three different groups of the students about the amount the work required by their professor is concerned. This also means there exists no difference in opinion among the three classes of people on the issue