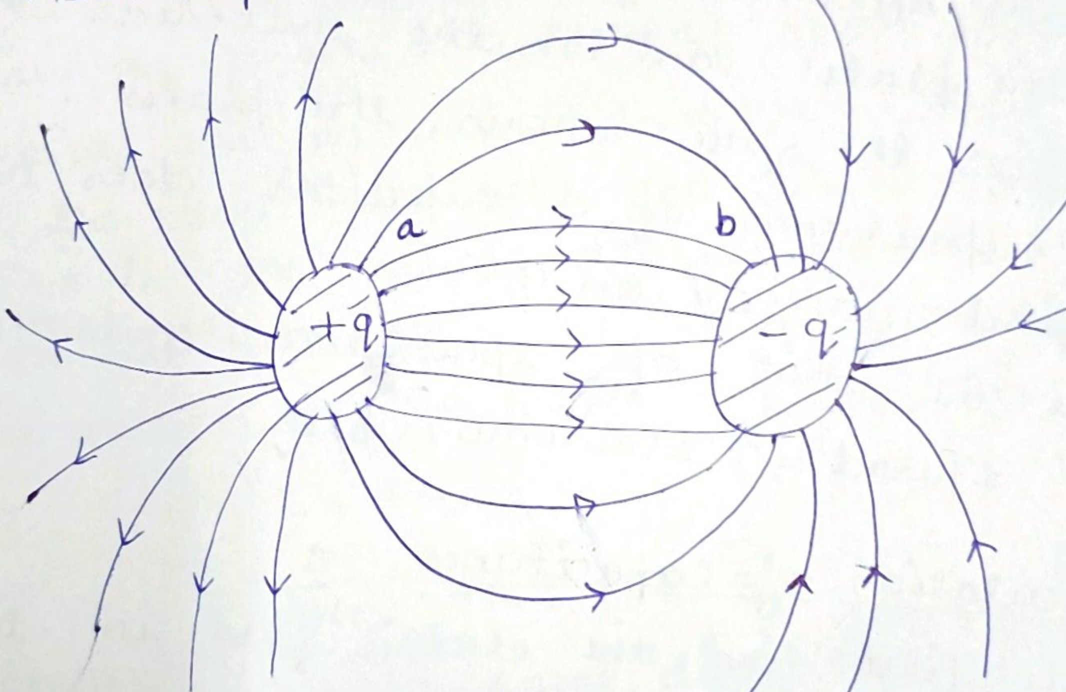
Capacitor → A capacitor is a device that stores energy in an electrostatic field. A capacitor can draw energy relatively slowly (over several seconds) from the battery, and it then can release the energy rapidly (within milliseconds) through the bulb.

Capacitors are also used to produce electric fields, such as the parallel-plate device that gives the very nearly uniform electric field that deflects beams of electrons in a TV or oscilloscope tube.



this is a generalized capacitor, consisting of two conductors a and b of arbitrary shape. No matter what their geometry, these conductors are called plates.

A capacitor is said to be charged if its plates carry equal and opposite charges +q and -q. q is not the net charge on the capacitor which is zero.

When we charge a capacitor, we find that the charge q that appears on the capacitor plates is always directly proportional to the potential difference ΔV b/w the plates $q \propto \Delta V$. The capacitance C is the constant of proportionality necessary to make this relationship into an equation.

$$q = C\Delta V \quad -①$$

the capacitance is a geometrical factor that depends on the size, shape, and separation of the plates and on the material that occupies the space between the plates.

the capacitance of a capacitor does not depend on ΔV or q.

the SI unit of capacitance is coulomb/volt

1 farad = 1 coulomb / volt

Calculation of capacitance →

① we first find the electric field in the region b/w the plates.

② then find out the potential difference b/w the plates by integrating the electric field.

③ then use equation

$$q = C\Delta V$$

$$C = \frac{q}{\Delta V}$$

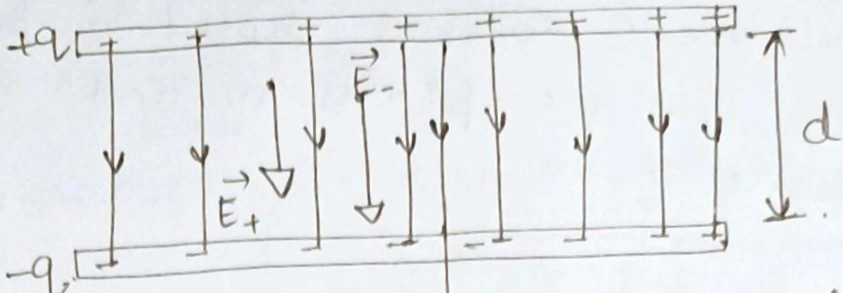
$$\Delta V = V_+ - V_-$$

$$= -\int_-^+ \vec{E} \cdot d\vec{s}$$

$$= \int_+^- \vec{E} \cdot d\vec{s} \quad -②$$

Note - capacitance C will always have positive value

A parallel-plate capacitor →



→ this shows the path of the integration.

Here we are considering a parallel plate capacitor, two flat plates are very large and very close together, the seperation d is very small than the length or width of the plates.

Here we are assuming that the electric field has the same magnitude and direction every where in the volume between the plates.

the net electric field is the sum of the fields due to the two plates- $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$E = E_+ + E_-$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Note-
E_+ and E_- have same directions.

$$\Delta V = \int_+^- E \, da = \int_+^- \frac{\sigma}{\epsilon_0} \, da$$

$$\Delta V = \int_+^- \frac{q}{A\epsilon_0} \, da \qquad \sigma = \frac{q}{A}$$

$\vec{E}$ and $d\vec{s}$ are parallel.

$$\boxed{\Delta V = \frac{q}{A\epsilon_0} d}$$

$$C = \frac{q}{\Delta V} = \frac{q}{qd/A\epsilon_0} = \frac{A\epsilon_0}{d}$$

By the formula $C = \frac{\epsilon_0 A}{d}$ we can see that the capacitance C does not depend on potential difference b/w the plates ΔV and charge on the plate q.

$$\boxed{\begin{array}{l} \epsilon_0 = 8.85 \times 10^{-12} \ F/m \\ \epsilon_0 = 8.85 \ pF/m \end{array}}$$

A Spherical capacitor →

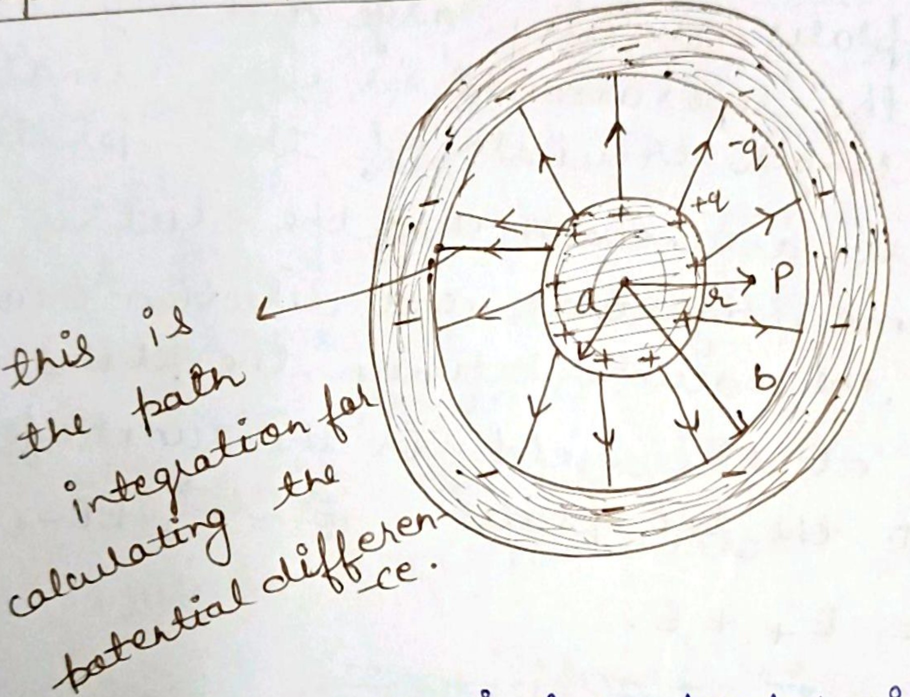


Figure (a)

Here we have a spherical conductor, in which the inner conductor is a solid sphere of radius a, and the outer conductor is a hollow spherical shell of inner radius b. We assume that the inner sphere carries a charge +q and that the outer sphere carries -q charge. Using Gauss's law analysis, the charge on the inner conductor resides on its surface and for the outer conductor the charge resides on its inner layer. So if we consider region >b then total E = 0 as total charge ⇒ q-q = 0

So the flux through the surface is zero.
In the region $a < r < b$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

the electric field depends only on the charge on the inner sphere.

$$\Delta V = \int_{+}^{-} E\,dS = \int_{+}^{-} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\,dr$$

$$\Delta V = \frac{1 \times q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Delta V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$\vec{E}\cdot d\vec{S} = E\,dS$ and $dS = dr$ (Here we are considering the path of integration is in the radial direction.)

$$C = q/\Delta V$$

$$\boxed{C = \frac{4\pi\epsilon_0 ab}{b-a}}$$

the Capacitance in the case of spherical capacitor, has the form of ϵ_0 times a quantity with the dimension of length.

A cylindrical capacitor – Figure (a) also represent the cross section of a cylindrical capacitor, in which the inner conductor is a solid rod of radius a carrying a charge +q uniformly distributed over its surface, and the outer conductor is a coaxial cylindrical shell of inner radius b carrying a charge of −q uniformly distributed over its inner surface. the capacitor has length L and we assume L >> b so that as was the case with the parallel plate capacitor.

Now, Applying Gauss's law-

$r < b$ no net charge so $E = 0$ every where.

Now $a < r < b$

$$E = \frac{1}{2\pi\epsilon_0} \frac{q}{Lr} \quad a < r < b$$

Here we have replaced the linear charge density λ with $\frac{q}{L}$ and the distance y with the radial coordinate r.

$$\Delta V = \int_{+}^{-} E\,ds = \int_a^b \frac{1}{2\pi\epsilon_0} \frac{q}{Lr}\,dr$$

$$\Delta V = \frac{1}{2\pi\epsilon_0} \frac{q}{L} \ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{\Delta V}{q} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$

this is the capacitance formula for the cylindrical geometry.

$$\boxed{C = \epsilon_0 \times \text{length (dimension)}}$$

<u>Energy storage in an electric field →</u>

work is done when two equal and opposite charges are separated. this energy is stored as electric potential energy in the system, and it can be recovered as kinetic energy if the charges are allowed to come together again. Similarly, a charged capacitor has stored in it an electrical potential energy U equal to the work W done by the external agent