Convection Heat Transfer

Introduction

- the controlling equation for convection is *Newton's Law of Cooling*

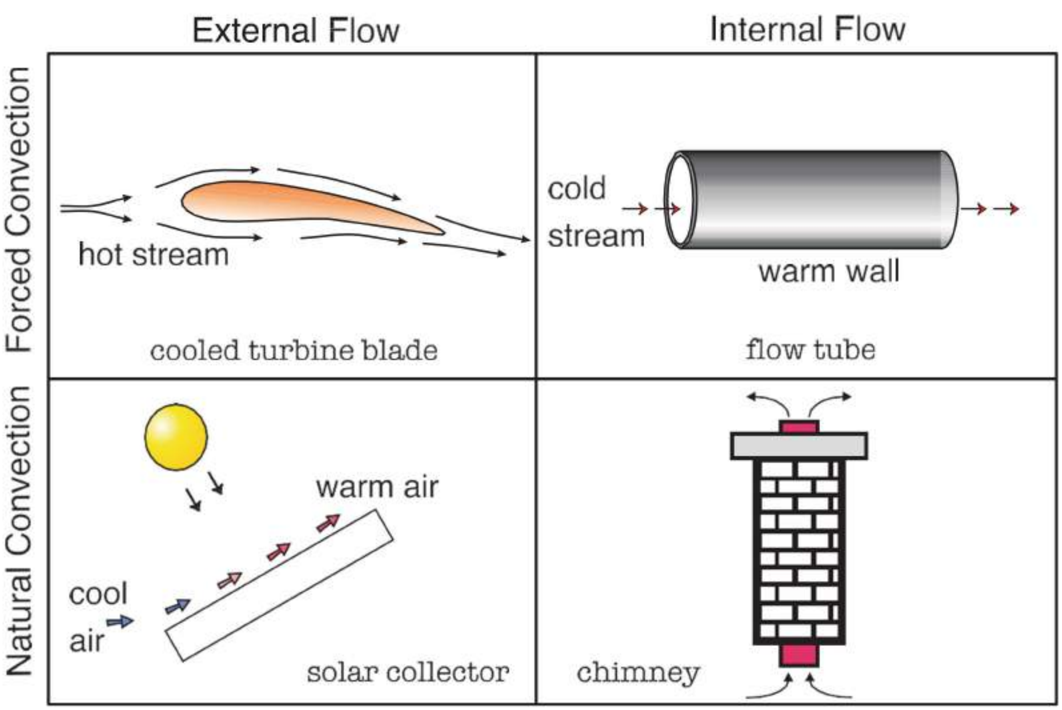
$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty) \qquad \Rightarrow \qquad R_{conv} = \frac{1}{hA}$$

A = total convective area, m^2

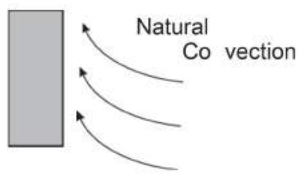
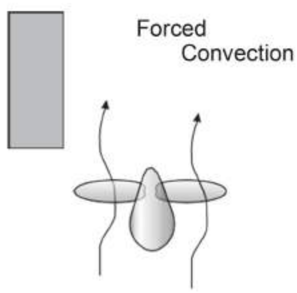
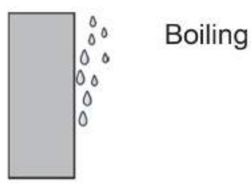
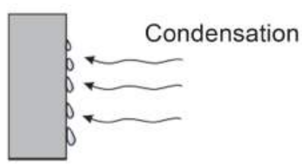
h = heat transfer coefficient, $W/(m^2 \cdot K)$

T_w = surface temperature, $^\circ C$

T_∞ = fluid temperature, $^\circ C$



The following table gives the range of heat transfer coefficient expected for different convection mechanisms and fluid types.

Process	$h\ [W/(m^2 \cdot K)]$
	• gases 3 - 20 • water 60 - 900
	• gases 30 - 300 • oils 60 - 1 800 • water 100 - 1 500
	• water 3 000 - 100 000
	• steam 3 000 - 100 000

Dimensionless Groups

It is common practice to reduce the total number of functional variables by forming dimensionless groups consisting of relevant thermophysical properties, geometry, boundary and flow conditions.

Prandtl number: $Pr = \nu/\alpha$ where $0 < Pr < \infty$ ($Pr \rightarrow 0$ for liquid metals and $Pr \rightarrow \infty$ for viscous oils). A measure of ratio between the diffusion of momentum to the diffusion of heat.

Reynolds number: $Re = \rho U L/\mu \equiv UL/\nu$ (forced convection). A measure of the balance between the inertial forces and the viscous forces.

Peclet number: $Pe = UL/\alpha \equiv RePr$

Grashof number: $Gr = g\beta(T_w - T_f)L^3/\nu^2$ (natural convection)

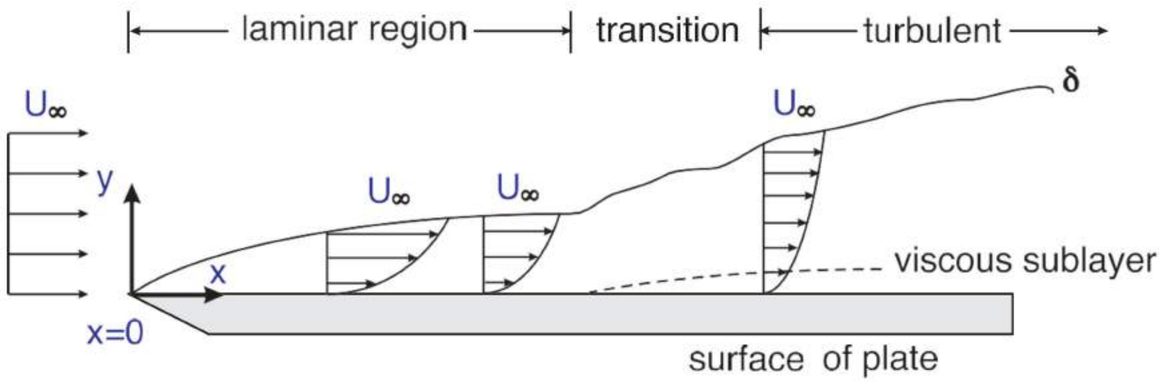
Rayleigh number: $Ra = g\beta(T_w - T_f)L^3/(\alpha \cdot \nu) \equiv GrPr$

Nusselt number: $Nu = hL/k_f$ This can be considered as the dimensionless heat transfer coefficient.

Stanton number: $St = h/(U\rho C_p) \equiv Nu/(RePr)$

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.



- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

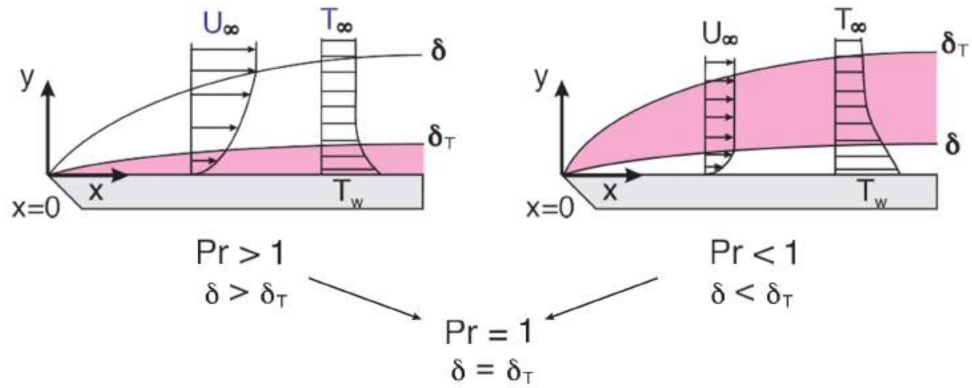
$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

  - for flow over a flat plate $Re_{cr} \approx 500,000$

Boundary Layers

Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer



- the velocity of the fluid progressively increases away from the wall until we reach approximately $\mathbf{0.99}\, U_\infty$ which is denoted as the $\boldsymbol{\delta}$, the *velocity boundary layer thickness*.

Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

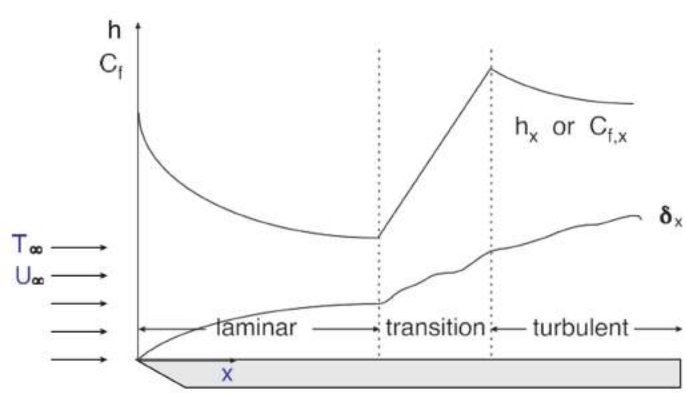
$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

Flow Over Plates

We will use *empirical correlations* based on experimental data where

$$C_{f,x} = C_1 \cdot Re^{-m}$$

$$Nu_x = C_2 \cdot Re^m \cdot Pr^n = \frac{h_x \cdot x}{k_f}$$



1. Laminar Boundary Layer Flow, Isothermal (UWT)

The local values of the skin friction and the Nusselt number are given as

$$C_{f,x} = \frac{0.664}{Re_x^{1/2}}$$

$$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$ ⇒ local, laminar, UWT, $Pr \geq 0.6$

An average value of the skin friction coefficient and the heat transfer coefficient for the full extent of the plate can be obtained by using the mean value theorem.

$$C_f = \frac{1.328}{Re_L^{1/2}}$$

$$Nu_L = \frac{h_L L}{k_f} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$ ⇒ average, laminar, UWT, $Pr \geq 0.6$

For low Prandtl numbers, i.e. liquid metals

$$Nu_x = 0.565\, Re_x^{1/2}\, Pr^{1/2}$$ ⇒ local, laminar, UWT, $Pr \leq 0.6$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

The local skin friction is given as

$$C_{f,x} = \frac{0.0592}{Re_x^{0.2}}$$ ⇒ local, turbulent, UWT, $Pr \geq 0.6$

$$Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}$$ ⇒ local, turbulent, UWT, $0.6 < Pr < 100, \quad 500,000 \leq Re_x \leq 10^7$

$$C_{f,x} = \frac{0.074}{Re_x^{0.2}}$$ ⇒ average, turbulent, UWT, $Pr \geq 0.6$

$$Nu_L = 0.037\, Re_L^{0.8}\, Pr^{1/3}$$ ⇒ average, turbulent, UWT, $0.6 < Pr < 100, \quad 500,000 \leq Re_x \leq 10^7$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

$$Nu_L = \frac{h_L L}{k} = (0.037\, Re_L^{0.8} - 871)\, Pr^{1/3}$$

⇒ average, combined, UWT, $0.6 < Pr < 60$, $500,000 \leq Re_L \leq 10^7$

4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$Nu_x = 0.453\, Re_x^{1/2}\, Pr^{1/3}$$

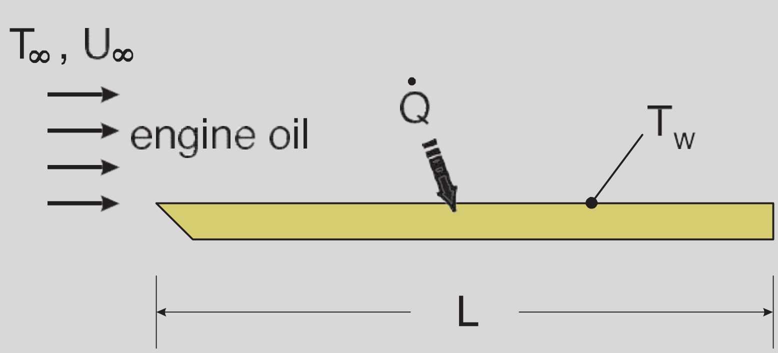
⇒ local, laminar, UWF, $Pr \geq 0.6$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$Nu_x = 0.0308\, Re_x^{4/5}\, Pr^{1/3}$$

⇒ local, turbulent, UWF, $Pr \geq 0.6$

Example 1: Hot engine oil with a bulk temperature of $60\ ^\circ C$ flows over a horizontal, flat plate $5\ m$ long with a wall temperature of $20\ ^\circ C$. If the fluid has a free stream velocity of $2\ m/s$, determine the heat transfer rate from the oil to the plate if the plate is assumed to be of unit width.



Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Bernstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$Nu_D = 0.3 + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

⇒ average, UWT, $Re_D < 10^7$, $0 \le Pr \le \infty$, $Re_D \cdot Pr > 0.2$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 12-3 are commonly used, where

$$Nu_D \approx \frac{\overline{h} D}{k} = C\, Re_D^m Pr^{1/3}$$

⇒ see Table 12-3 for conditions

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

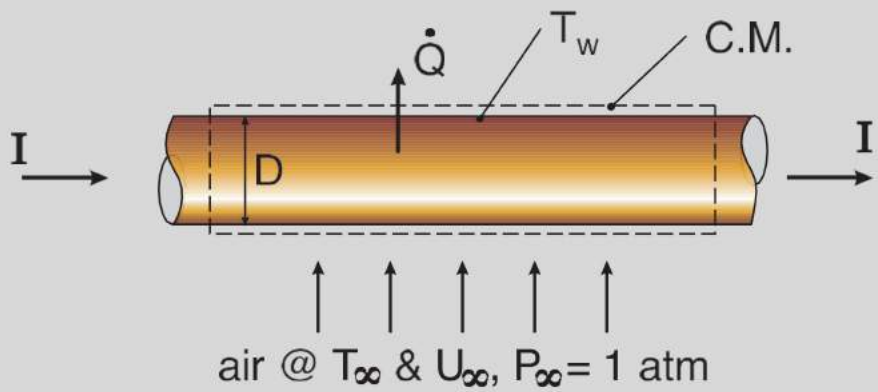
For flow over an isothermal sphere of diameter D

$$Nu_D = 2 + \left[0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

⇒ average, UWT, $0.7 \le Pr \le 380$, $3.5 < Re_D < 80,000$

where the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on the free stream temperature, T_∞ and the dynamic viscosity of the fluid at the surface, μ_s, is based on the surface temperature, T_s. All other properties are based on T_∞.

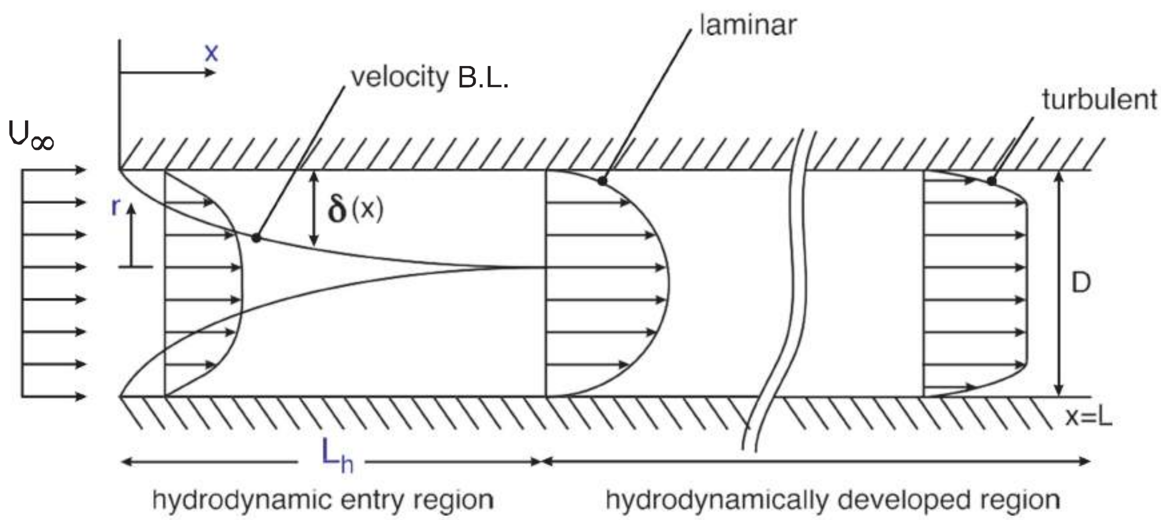
Example 2: An electric wire with a $1\ mm$ diameter and a wall temperature of $325\ K$ is cooled by air in cross flow with a free stream temperature of $275\ K$. Determine the air velocity required to maintain a steady heat loss per unit length of $70\ W/m$.



Internal Flow

The *mean velocity* and Reynolds number are given as

$$U_m = \frac{1}{A_c}\int_{A_c} u\, dA = \frac{\dot{m}}{\rho_m A_c} \qquad Re_D = \frac{U_m D}{\nu}$$



Hydrodynamic (Velocity) Boundary Layer

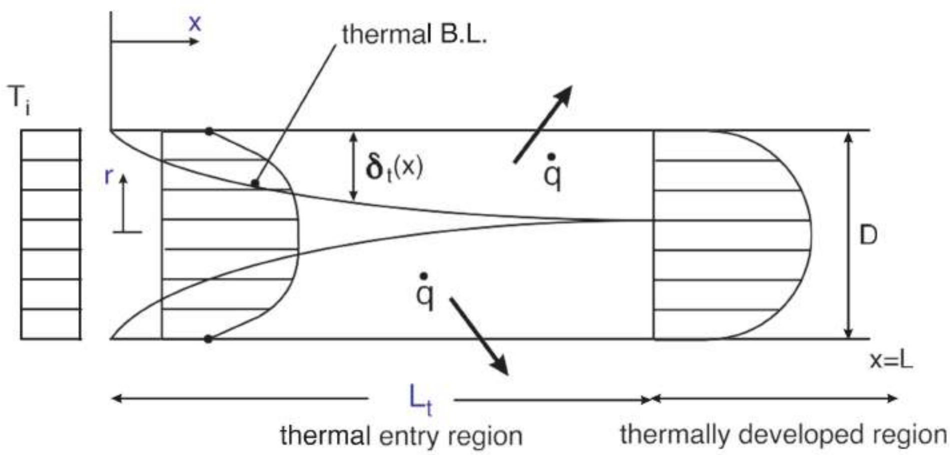
- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x\left(\frac{U_m x}{\nu}\right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

- the hydrodynamic entry length can be approximated as

$$L_h \approx 0.05 Re_D D \qquad \text{(laminar flow)}$$

Thermal Boundary Layer



- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D = Pr L_h \qquad \text{(laminar flow)}$$

- for turbulent flow $L_h \approx L_t \approx 10D$

Wall Boundary Conditions

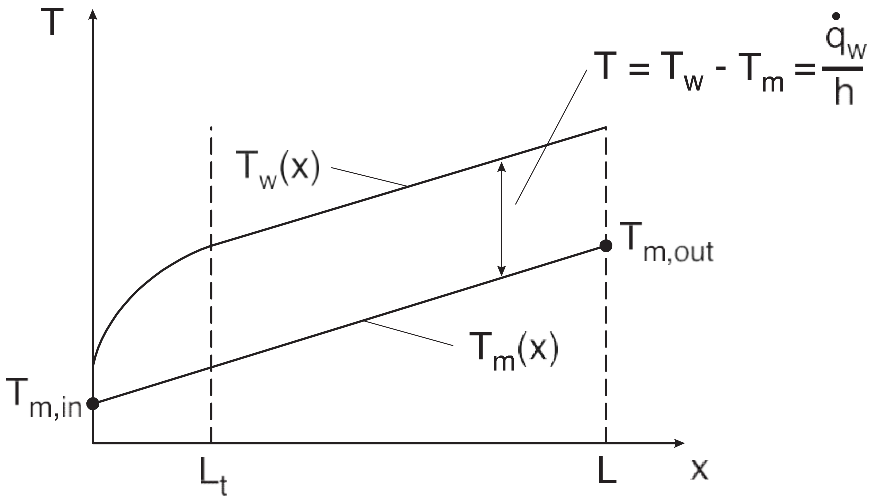
1. Uniform Wall Heat Flux: Since the wall flux $q^{\cdot}_w$ is uniform, the local mean temperature denoted as

$$T_{m,x} = T_{m,i} + \frac{q^{\cdot}_w A}{\dot{m} C_p}$$

will increase in a linear manner with respect to x.

The surface temperature can be determined from

$$T_w = T_m + \frac{q^{\cdot}_w}{h}$$



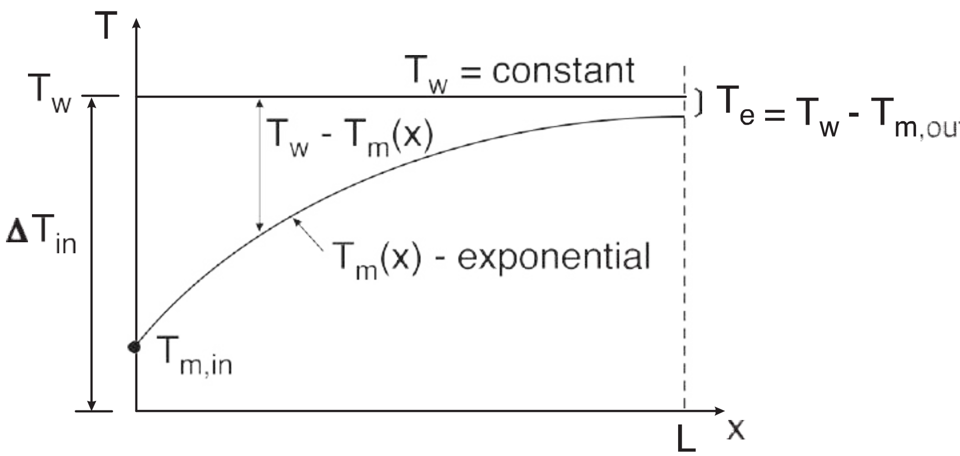
2. Isothermal Wall: The outlet temperature of the tube is

$$T_{out} = T_w - (T_w - T_{in})\exp[-hA/(\dot{m}C_p)]$$

Because of the exponential temperature decay within the tube, it is common to present the mean temperature from inlet to outlet as a log mean temperature difference where

$$\dot{Q} = hA\Delta T_{ln}$$

$$\Delta T_{ln} = \frac{T_{out} - T_{in}}{\ln \dfrac{T_w - T_{out}}{T_w - T_{in}}} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})}$$



1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$$\boxed{Nu_D = 3.66} \Rightarrow \text{fully developed, laminar, UWT, } L > L_t \;\&\; L_h$$

$$\boxed{Nu_D = 4.36} \Rightarrow \text{fully developed, laminar, UWF, } L > L_t \;\&\; L_h$$

$$\boxed{Nu_D = 1.86\left(\frac{Re_D Pr D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}} \Rightarrow \begin{array}{l}\text{developing laminar flow, UWT,}\\ Pr > 0.5\\ L < L_h \text{ or } L < L_t\end{array}$$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction with Table 10-4 to determine the Reynolds number and in turn the Nusselt number.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}(T_{m,in} + T_{m,out})$$

except for μ_s which is evaluated at the wall temperature, T_s.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Boelter equation (Eq. 13-68) can be used

$$\boxed{Nu_D = 0.023\, Re_D^{0.8}\, Pr^n} \Rightarrow \begin{array}{l}\text{turbulent flow, UWT or UWF,}\\ 0.7 \leq Pr \leq 160\\ Re_D > 2,300\\ n = 0.4 \text{ heating}\\ n = 0.3 \text{ cooling}\end{array}$$

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

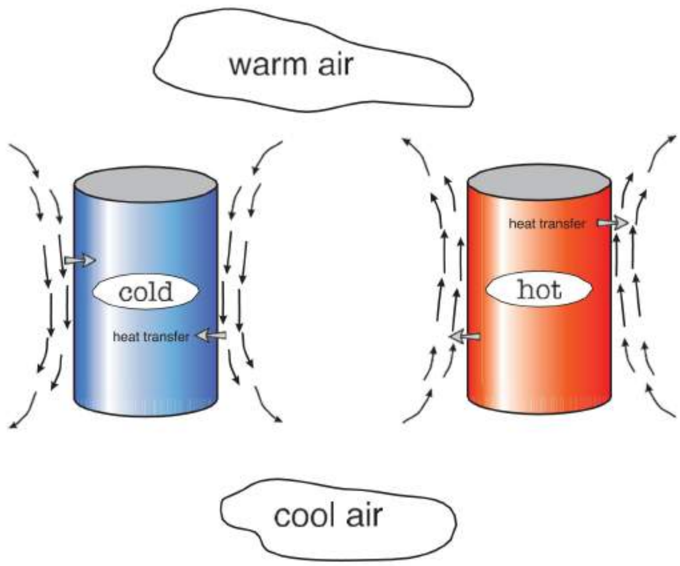
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}(T_{m,in} + T_{m,out})$$

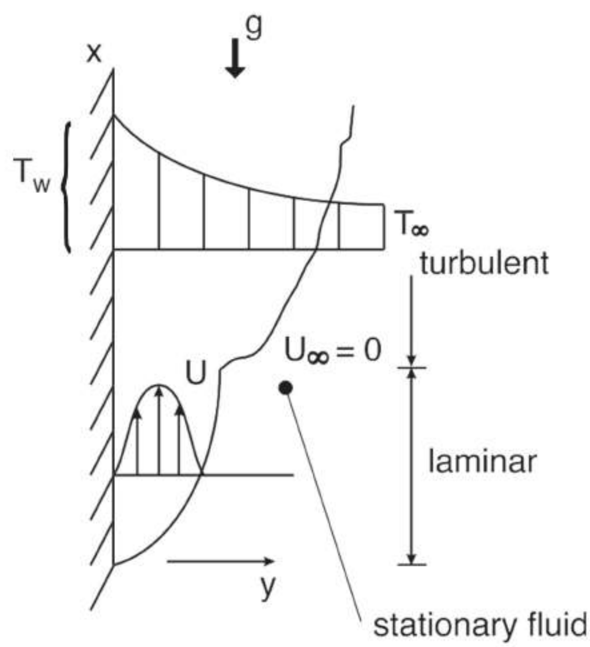
Natural Convection

What Drives Natural Convection?

- fluid flow is driven by the effects of buoyancy



Natural Convection Over Surfaces



- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero

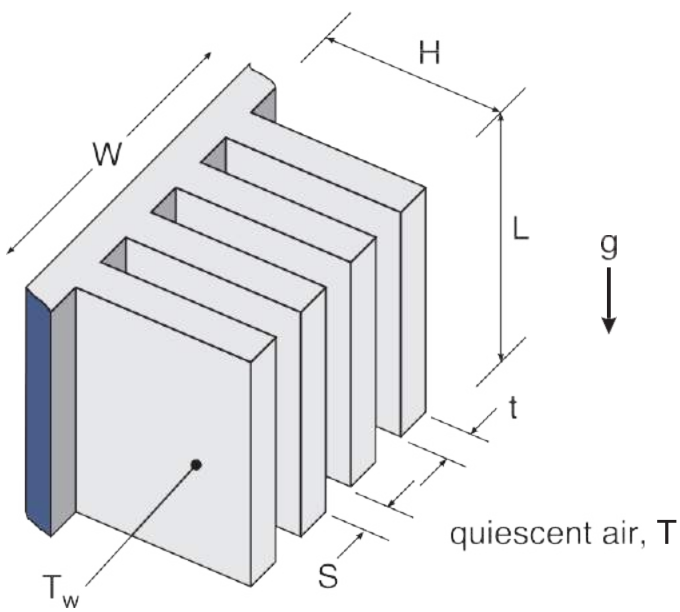
Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$\boldsymbol{Nu = f(Gr, Pr) \equiv C Gr^m Pr^n} \qquad \text{where} \quad \boldsymbol{Ra = Gr \cdot Pr}$$

- $\boldsymbol{C}$ depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- $\boldsymbol{m}$ depends on type of flow (laminar or turbulent)
- $\boldsymbol{n}$ depends on the type of fluid and type of flow
- Table 14-1 should be used to find Nusselt number for various combinations of geometry and boundary conditions
  - for ideal gases $\boldsymbol{\beta = 1/T_\infty, \ (1/K)}$
  - all fluid properties are evaluated at the film temperature, $\boldsymbol{T_f = (T_w + T_\infty)/2}$.

Natural Convection From Plate Fin Heat Sinks



The average Nusselt number for an isothermal plate fin heat sink with natural convection can be determined using

$$\boldsymbol{Nu_S = \frac{hS}{k_f} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}}\right]^{0.5}}$$

A basic optimization of the fin spacing can be obtained as follows: For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{S^3 L}{Ra_S} \right)^{1/4} = 2.714 \left(\frac{L}{Ra_L^{1/4}} \right)$$

with

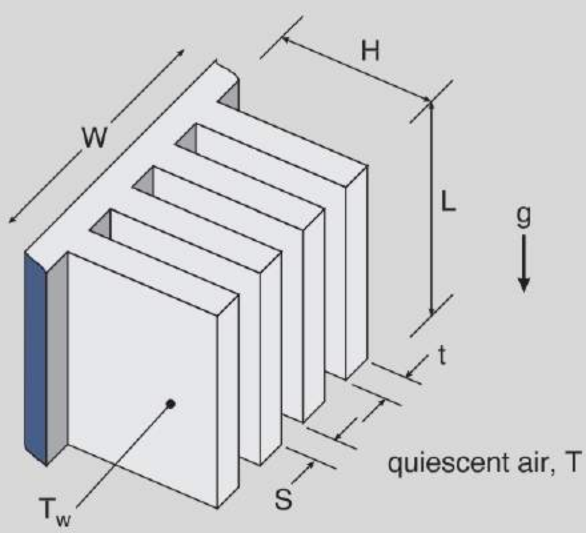
$$Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} Pr$$

The corresponding value of the heat transfer coefficient is

$$h = 1.307 k_f / S_{opt}$$

All fluid properties are evaluated at the film temperature.

Example 3: Find the optimum fin spacing, S_{opt} and the rate of heat transfer, $\dot{Q}$ for the following plate fin heat sink cooled by natural convection.



Given:

$$W = 120\ mm \qquad H = 24\ mm$$
$$L = 18\ mm \qquad t = 1\ mm$$
$$T_w = 80\ ^\circ C \qquad T_\infty = 25\ ^\circ C$$
$$P_\infty = 1\ atm \qquad \text{fluid} = \text{air}$$

Find: S_{opt} and the corresponding heat transfer, $\dot{Q}$