Gauss - Jordan Method -

This is another popular method used for solving linear equations.
This method uses the process of elimination of variables, but there is a major difference b/w them. In Gauss Jordan method, a variable is eliminated from all other rows than the below the pivot equation.
This process eliminates all the off-diagonal terms producing a diagonal matrix rather than a triangular matrix.
Consequently, we obtain the values of unknowns directly from the b vector, without employing back - substitution.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Result of Gauss elimination

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a'_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

Result of Gauss-Jordan Elimination

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b'_1 \\ b''_2 \\ b'''_3 \end{bmatrix}$$

Ex-① Solve
$$2x_1 + 4x_2 - 6x_3 = -8$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12$$

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Using By Gauss- Jordan

Sol. Step-1 Normalise first eq. by dividing it by 2 (pivot element) the result is.

$$x_1 + 2x_2 - 3x_3 = -4$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12$$

Step2 eliminate x_1 from second equation. (subtracting 1 time the first equation from it)

Similarly, eliminate x_1 from the third equation by subtracting 2 times the first eq. from it.

the result is.

$$x_1 + 2x_2 - 3x_3 = -4$$
$$0 + x_2 + 4x_3 = 14$$
$$0 - 8x_2 + 4x_3 = -4$$

Step-3- Normalise the second equation.

Step-4- following similar approach, eliminate x_2 from first and third equations, this gives.

$$x_1 + 0 - 11x_3 = -32$$
$$0 + x_2 + 4x_3 = 14$$
$$0 + 0 + 36x_3 = 108$$

Step-5- Normalise the third equation.

$$x_1 + 0 - 11x_3 = -32$$
$$0 + x_2 + 4x_3 = 14$$
$$0 + 0 + x_3 = 3$$

Step-6- Eliminate x_3 from the first and second eqns. we get-

$$x_1 + 0 + 0 = 1$$
$$0 + x_2 + 0 = 2$$
$$0 + 0 + x_3 = 3$$

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Iterative Methods-

(1). Jacobi iteration method

(2) Gauss-Seidal iteration method

(3) Successive over relaxation method.

Jacobi Iteration Method-

Let us consider a system of n equations in n unknowns.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \quad -(1)$$
$$\vdots \qquad\qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$$x_1 = \frac{b_1 - (a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n)}{a_{11}}$$

$$x_2 = \frac{b_2 - (a_{21}x_1 + a_{23}x_3 + \cdots + a_{2n}x_n)}{a_{22}} \quad -(2)$$

$$\vdots \qquad\qquad \vdots$$

$$x_n = \frac{b_n - (a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn-1}x_n)}{a_{nn}}$$

these new values are again used to compute the next set of x values.

In general, an iteration for x_i can be obtained from the ith eq. as follows.

$$x_i^{(k+1)} = \frac{b_i - (a_{i1}x_1^{(k)} + a_{ii-1}x_{i-1}^{(k)} + a_{ii+1}x_{i+1}^{(k)} + \cdots + a_{in}x_n^{(k)})}{a_{ii}} \quad -(3)$$

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Ex- Obtain the solution of the following system using the Jacobi iteration method-

$2x_1 + x_2 + x_3 = 5$

$3x_1 + 5x_2 + 2x_3 = 15$

$2x_1 + x_2 + 4x_3 = 8$

first we solve for unknowns on the diagonal

$$x_1 = \frac{5 - x_2 - x_3}{2}$$

$$x_2 = \frac{15 - 3x_1 - 2x_3}{5}$$

$$x_3 = \frac{8 - 2x_1 - x_2}{4}$$

we assume the initial values of x_1, x_2, and x_3 to be zero, we get,

$$x_1^{(1)} = \frac{5}{2} = 2.5, \quad x_2^{(1)} = \frac{15}{5} = 3, \quad x_3^{(1)} = \frac{8}{4} = 2$$

For the second iteration, we have-

$$x_1^{(2)} = \frac{5 - 3 - 2}{2} = 0$$

$$x_2^{(2)} = \frac{15 - 3 \times 2.5 - 2 \times 2}{5} = \frac{3.5}{5} = 0.7$$

$$x_3^{(2)} = \frac{8 - 2 \times 2.5 - 3}{4} = 0$$

after third iteration.

$$x_1^{(3)} = \frac{5 - 0.7}{2} = 2.15$$

$$x_2^{(3)} = \frac{15 - 3 \times 0 - 2 \times 0}{5} = 3$$

$$x_3^{(3)} = \frac{8 - 2 \times 0 - 0.7}{4} = 1.825$$

after fourth iteration-

$x_1^{(4)} = 0.0875$

$x_2^{(4)} = 1.225$

$x_3^{(4)} = 0.175$

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