Interference of Light

Wave theory of light

The wave theory of light was first proposed by Dutch physicist Christiaan Huygens in 1678. During that period of time everyone beleived in Newton's corpuscular theory of light which satisfactorily explained the phenomena of reflection, refraction and rectilinear propagation of light. But Newton's theory of light fails to explain the phenomena like interference, diffraction and polarization of light.
With the wave theory of light Huygens was able to explain satisfactorily the phenomena of reflection, refraction and total internal reflection. Moreover Huygens theory predicted that the velocity of light in a medium will be less than the velocity of light in free space which is just converse of the prediction made from Newton's corpuscular theory of light.

Huygens principle of wave propagation

Huygens theory is essentially based on geometrical construction which allows us to determine the shape of a wavefront at any time if the shape of the wavefront at an earlier time is known.

According to Huygens principle each point source is a centre of disturbance from which light spreads out in all directions setting the other particles in vibration

Wave front

A wavefront is the locus of all points which are in same phase and have same amplitude.

For example if we drop a small stone in a pool of water, circular ripples spread out from the pt. of impact, each point on the circumference of the circle oscillates with same amplitude and same phase and thus we have a circular wave front.

Generally wave fronts are of three types

1. Spherical wave front : The spreading of light waves from a pt. source in a uniform isotropic medium is shown in the figure. In isotropic medium the light waves travel in all directions and hence the locus of all points which have the same amplitude and are in same phase is a sphere. Thus each spherical surface concentric with the source is a wave front. At large distances from the source, a small portion of the spherical wave front can be considered as plane wave front.

2. Cylindrical wave front

If we have a light source in the form of a narrow slit, then the wave front will be cylindrical with slit as axis. As shown in the figure the wave front of a narrow slit OO' is of cylindrical shape and all points on the cylindrical surface will be in shape phase and also having same amplitude.

At large distances from the source the radius of cylindrical wave front is quite large then any portion of the cylindrical wave front can be treated as plane wave front.

Interference of Light

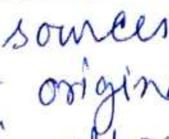
The phenomenon of interference occurs when light waves superimpose with each other. When two light waves of same frequency and having a constant phase difference [suppose $I_1 = I_{01} \sin(kx - \omega t)$ and $I_2 = I_{02} \sin(kx - \omega t + \delta)$ where $k = \frac{2\pi}{\lambda}$ (wave no.) and $\delta = k\Delta x$ = const. phase difference] traverse simultaneously in the same region of a medium and superimpose with each other, then there is a modification in the intensity of light in the region of superposition which is different from the sum of the intensities due to individual waves at that point. This modification in the intensity of light resulting from the superposition of two or more waves of light is called interference of light. When the resultant amplitude is the sum of the amplitudes due to waves, the interference is known as constructive interference where as at certain points the resultant amplitude is equal to the difference of two amplitudes, the interference is known as destructive interference.

Coherent Sources

Two sources of light are said to be coherent if they emit light waves which have always a constant phase difference between them. It means that two sources must emit radiation of the same wave length.

Types of interference

① **Division of wave front** : In this category coherent sources are obtained by dividing the wavefront originating from a common source. Here a beam is allowed to fall on two closely spaced holes and the two beams emanating from the holes interfere. This method is known as division of wave front. The instrument used to obtain interference by division of wavefront are Fresnel's biprism, Fresnel's mirror, Lloyd's mirror etc.

Fig. 

② **Division of amplitude** : In this method the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus two coherent beams are produced by division of amplitude. These beams travel different paths are finally brought together to produce interference. The interference in thin films, Newton's rings are obtained by division of amplitude.

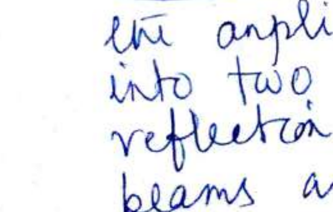
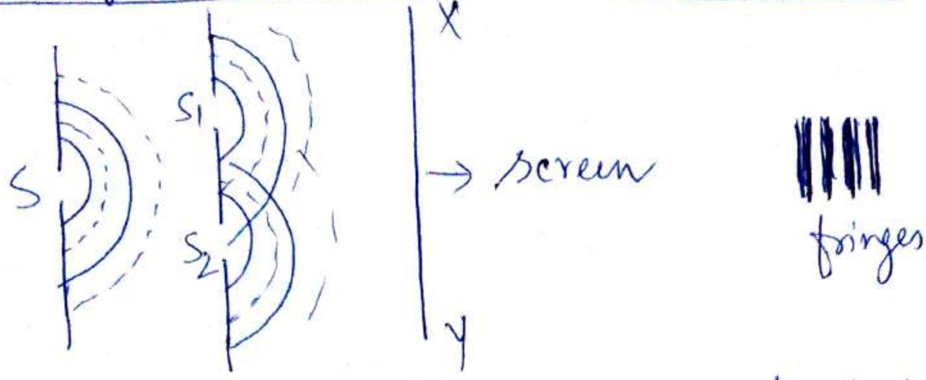


Fig.

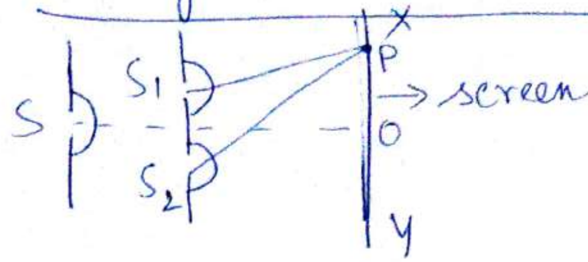
Interference by division of wavefront
Young's double slit experiment



In 1801 Thomas Young demonstrated experimentally the phenomenon of interference of light. He allowed the sunlight to pass through a pinhole S and then at some distances through two very closedly spaced pin holes S_1 and S_2 separated by an opaque space. The interference pattern was observed on a screen XY. He observed few colored bright and dark bands on the screen. These are known as fringes. Now a days, to increase the brightness, the pin holes S_1 and S_2 are replaced by slits and to increase the no. of fringes sunlight is replaced by monochromatic light. The interference pattern consists of equally spaced bright and dark fringes.

The formation of interference fringes on the screen can be explained on the basis of wave theory of light.

Analytical treatment of interference



Let S be the source of monochromatic light of wavelength λ. S_1 and S_2 are two pin holes or slits which are situated at equidistant from the sources. The waves arriving at S_1 and S_2 from S will be in phase at all time and having same frequency. The secondary waves coming out from S_1 and S_2 superimpose with each other and interference pattern is formed on the screen. We have to investigate the intensity at P on the screen XY.

Let a_1 and a_2 be the amplitudes of two waves coming from S_1 and S_2 respectively.

The displacements y_1 and y_2 due to the waves from S_1 and S_2 are given by

$$y_1 = a_1 \sin \omega t \quad \text{—} (1)$$

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{—} (2)$$

where δ is the phase difference between the two waves reaching at pt. P at an instant t.

According to the superposition the two waves the resultant displacement is

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$\therefore y = a_1 \sin\omega t + a_2 (\sin\omega t \cos\delta + \cos\omega t \sin\delta)$

$y = (a_1 + a_2\cos\delta)\sin\omega t + (a_2 \sin\delta)\cos\omega t$ — (3)

Suppose $a_1 + a_2\cos\delta = A\cos\phi$ — (4)

$a_2 \sin\delta = A\sin\phi$ — (5)

where A and ϕ are unknown constants.

Then from eqns. (4), (5) and (3) we get

$y = A\cos\phi \sin\omega t + A\sin\phi\cos\omega t$

$\therefore \boxed{y = A\sin(\omega t + \phi)}$ — (6)

Hence the resultant vibration at P is simple harmonic vibration of amplitude A and phase ϕ.

Squaring and adding eqns. (4) and (5) we get

$(a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2 = A^2(\cos^2\phi + \sin^2\phi)$

or, $A^2 = a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta$

or $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

The resultant intensity at P is proportional to the square of the amplitude ie $I = A^2$

$\therefore$ Thus $\boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$ — (7)

$= a_1^2 + a_2^2 + 2a_1a_2\cos 2\frac{\delta}{2}$

$= a_1^2 + a_2^2 + 2a_1a_2\left(2\cos^2\frac{\delta}{2} - 1\right)$

$\therefore \boxed{I = (a_1 - a_2)^2 + 4a_1a_2\cos^2\frac{\delta}{2}}$ — (8)

The intensity will vary according to the variation of $\cos^2\frac{\delta}{2}$

$\therefore$ The phase difference δ is given by

$$\delta = \frac{2\pi}{\lambda} \times (\text{path diff} = \Delta)$$

$$\boxed{\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)}$$

Condition for maximum intensity

From eq^n. (7) $I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$ → it is clear that I will be maximum when $\cos\delta = 1$

$\therefore$ phase diff $\boxed{\delta = 2n\pi}$, $n = 0, 1, 2, \ldots$

and the path diff $\Delta = \frac{\lambda}{2\pi}\delta = \frac{\lambda}{2\pi} 2n\pi = n\lambda$

i.e. the maximum intensity at pt. P can be obtained if

$$\boxed{\begin{aligned}\text{phase diff} &= \delta = 2n\pi \\ \text{path diff} &= \Delta = n\lambda\end{aligned}}$$ when $n = 0, 1, 2, 3 \ldots$

The value of maximum intensity is

$$I_{max} = A^2 = a_1^2 + a_2^2 + 2a_1a_2 \qquad \text{since } \cos\delta = +1$$

$$\therefore \boxed{I_{max} = (a_1 + a_2)^2} \quad \text{—(9)}$$

Obviously $I_{max} > a_1^2 + a_2^2$.

Thus the resultant intensity is greater than the sum of intensities due to individual waves.

Condition for minimum intensity

The resultant intensity I is minimum at P when $\cos\delta$ is minimum ie $\cos\delta = -1$ that is

$$\boxed{\begin{array}{l}\text{when phase diff. } \delta = (2n+1)\pi \\ \text{and path diff } \Delta = \left(n+\frac{1}{2}\right)\lambda\end{array}}, \quad n = 0, 1, 2, \ldots$$

since $\Delta = \frac{\delta x}{2\pi}$

The value of minimum intensity is

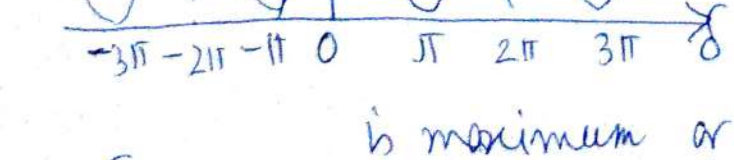
$$I_{min} = A^2 = a_1^2 + a_2^2 - 2a_1a_2$$

$$\boxed{I_{min} = (a_1 - a_2)^2}$$

Special case when $a_1 = a_2$ (same amplitudes)

$$\boxed{\begin{array}{l} I_{max} = a^2 + a^2 + 2a.a = 4a^2 \\ I_{min} = a^2 + a^2 - 2a.a = 0 \end{array}}$$

Intensity distribution curve

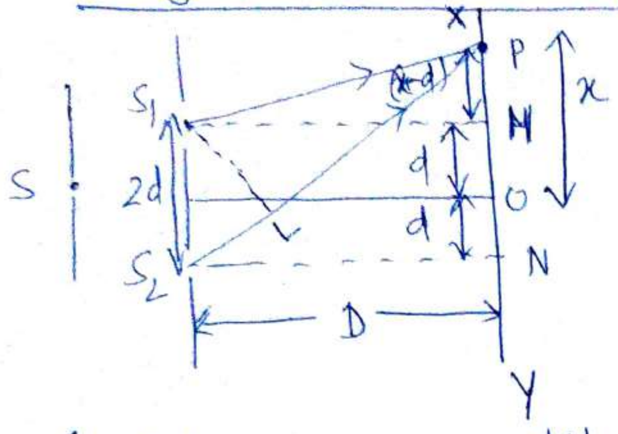


A curve showing the variation of intensity with phase is called intensity distribution curve.

From the above eq^ns. it is clear that intensity is maximum or bright at pts. when $\delta = 2n\pi$, $n = 0, 1, 2 \ldots$ and is $4a^2$. Intensity is minimum or dark at pts. when $\delta = (2n+1)\pi$, $n = 0, 1, 2 \ldots$

(OP10)

Fringe Width in Young's Double ~~St~~ Slit Expt.



Let S be a narrow slit which is illuminated by the monochromatic light of wavelength λ. The two slits S_1 and S_2 are ~~separed~~ separated by a distance 2d. Let a screen XY be placed at a distance D from the coherent sources S_1 and S_2. Waves coming out from S_1 and S_2 are intercepted on the screen XY. Now we will find out the positions of maxima (bright fringe) and minima (dark fringe) on the screen.

Let us consider a pt. P which is at a distance x from the pt. O as shown in the figure. Draw S_1M and S_2N perpendiculars from S_1 and S_2 on the screen. Join S_1P and S_2P.

The path difference between the two waves reaching at pt. P from S_1 and S_2 slits is given by

$$\Delta = S_2P - S_1P$$

In right angled triangle S_2PN

$$(S_2P)^2 = (S_2N)^2 + (NP)^2 = \left[D^2 + (x+d)^2\right]$$

$$\therefore \; S_2P = \left[D^2 + (x+d)^2\right]^{1/2} = D\left[1 + \frac{(x+d)^2}{D^2}\right]^{1/2}$$

If $D >> x$ and d, we get

$$S_2P = D\left[1 + \frac{(x+d)^2}{2D^2}\right] = D + \frac{(x+d)^2}{2D}$$

Similarly in S_1MP right angled triangle

$$S_1P = \left[(S_1M)^2 + (MP)^2\right]^{1/2} = \left[D^2 + (x-d)^2\right]^{1/2}$$

or, $$S_1P = D + \frac{(x-d)^2}{2D}$$

$\therefore$ The path difference

$$\Delta = S_2P - S_1P = D + \frac{(x+d)^2}{2D} - D - \frac{(x-d)^2}{2D}$$

$$= \frac{1}{2D}\left[(x+d)^2 - (x-d)^2\right]$$

$$\therefore \boxed{\Delta = \frac{4xd}{2D} = \frac{2d}{D}x}$$

$\therefore$ Phase difference between the two waves reaching at P is given by

$$\text{phase diff} = \frac{2\pi}{\lambda} \times \text{path diff.}$$

$$\boxed{\therefore \delta = \frac{2\pi}{\lambda} \cdot \frac{2d}{D}x = \frac{4\pi d}{\lambda D}x}$$

Position of bright fringes

To get bright fringes, the path diff.

$$\Delta = n\lambda$$

$$\therefore \frac{2d}{D}x = n\lambda$$

$$\therefore \boxed{x = \frac{n\lambda D}{2d}}, \quad n = 0, 1, 2 \ldots$$

So the n^{th} bright fringe is obtained at a distance

$$x_n = \frac{n\lambda D}{2d} \quad \text{from the pt. O.}$$

At $n=0$, we get the central bright fringe

$$x_1 = \frac{\lambda D}{2d} \quad \text{for } n=1$$

$$x_2 = \frac{2\lambda D}{2d} \quad \text{for } n=2$$

$$x_3 = \frac{3\lambda D}{2d} \quad \text{for } n=3$$

$\therefore$ The fringe width of the dark fringe

$$\omega = x_2 - x_1 = x_3 - x_2 = \cdots = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$

$$\therefore \boxed{\omega = \frac{\lambda D}{2d}} \Rightarrow \text{width of dark fringe}$$

Position of dark fringe

To get a dark fringe at P the path difference must satisfy the following relation

$$\Delta = \left(n+\tfrac{1}{2}\right)\lambda$$

$$\therefore \frac{2dx}{D} = \left(n+\tfrac{1}{2}\right)\lambda$$

$$\therefore \boxed{x = \frac{(2n+1)\lambda D}{4d}}$$

The above eqn gives the position of the n^{th} dark fringe from pt. O.

$$\therefore \boxed{x_n = \frac{(2n+1)\lambda D}{4d}}, \quad n = 0, 1, 2 \ldots$$

The position of the 1st, 2nd, 3rd dark fringes are obtained at

$$x_1 = \frac{3\lambda D}{4d}, \quad n=1$$

$$x_2 = \frac{5\lambda D}{4d}, \quad n=2$$

$$x_3 = \frac{7\lambda D}{4d}, \quad n=3$$

$$- \quad - \quad - \qquad -$$

$$- \quad - \quad - \qquad -$$

∴ The width of the bright fringe is

$$\boxed{\omega = x_2 - x_1 = \frac{\lambda D}{2d}}$$

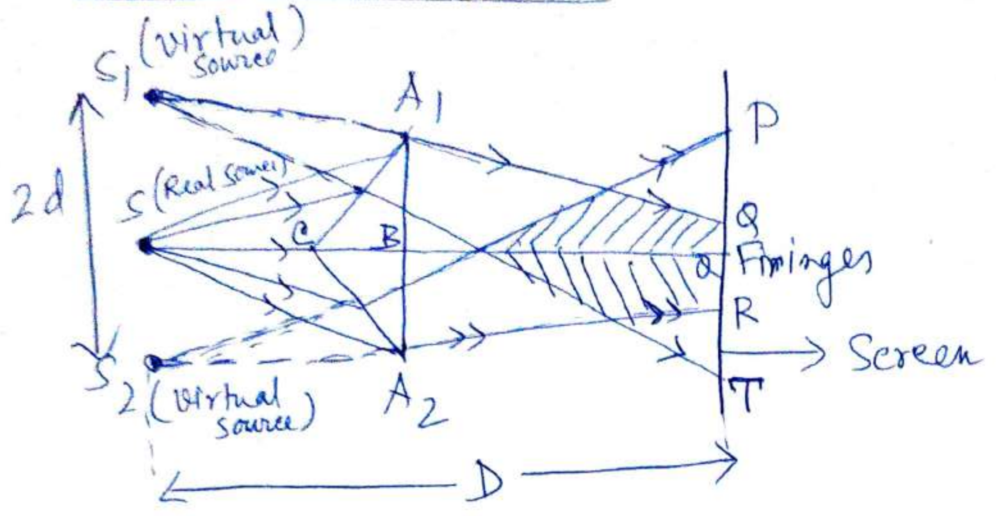
Thus we see that the bright and dark fringes are of equal width which is given by

$$\boxed{\omega = \frac{\lambda D}{2d}}$$

∴ The wavelength of the incident wave is given by

$$\boxed{\lambda = \frac{\omega 2d}{D}}$$

Fresnel's Biprism



Fresnel devised another simple arrangement for the production of interference pattern. He used a biprism. The biprism consists of two acute angled prisms with their bases in contact. It is actually simple prism of obtuse angle 179° and whose acute angles are very small ($\alpha = 0.5°$) of each. C is the refracting edge of the prism which is placed parallel to the plane of the slit S. S is illuminated with monochromatic light of wavelength λ. It is allowed to fall symmetrically on the biprism. The light going through the prism A_1BC appears in a cone S_1QT and the light going through A_2BC appears in a cone S_2PR. So S_1 and S_2 are virtual images of S. These two virtual images S_1 and S_2 act as the coherent sources. In the common shaded region the coherent beam overlap and interference pattern is obtained on the screen placed at distance D from the slit. In the region QR the interference fringes have equal width.

This arrange is equivalent to Young's double slit expt. with S_1 and S_2 acting as two slits. Suppose the separation between the two virtual sources (ie two slits S_1 and S_2) is 2d and the distance between slit and the screen is D. The position of the nth bright and nth dark fringes from the centre pt. O is given by the relations

$$x_n = \frac{n\lambda D}{2d} \longrightarrow \text{for bright fringe}$$

and $$x_n = \frac{(2n+1)\lambda D}{4d} \longrightarrow \text{for dark fringe}$$

∴ The fringe width is obtained by

$$\boxed{\omega = \frac{\lambda D}{2d}}$$

Determination of wavelength of light

The biprism arrangement can be used to determine the wavelength of an almost monochromatic light like ~~se~~ sodium light. Light from sodium lamp illuminates the slit S and interference fringes can be easily view through the eyepiece. The fringe width ω can be determined by means of a micrometer attached to the eyepiece. Once ω is ~~kno~~ known, λ can be determined from the formula

$$\lambda = \frac{\omega\, 2d}{D}$$

The distance 2d between the two virtual sources can be obtained by placing a convex lens between the biprism and the eyepiece. For a fixed position of the eyepiece there will be two positions of the lens where where the real images of the two virtual sources S_1 and S_2 can be seen in the eyepiece.

Let d_1 be the separation between the real images of S_1 and S_2 for the first position of the lens and d_2 be the separation between the real images of S_1 and S_2 for the 2nd position of the lens.

Then it can be easily shown that

$$\text{Magnification} = \frac{v}{u} = \frac{d_1}{2d} \quad \text{—(I)} \quad \left(\begin{array}{l} d_1 = \text{image length} \\ 2d = \text{object length} \end{array}\right)$$

As the two positions of the lens are conjugate ~~to each~~ so for the 2nd position of the lens ($u \Rightarrow v$ and $v \Rightarrow u$)

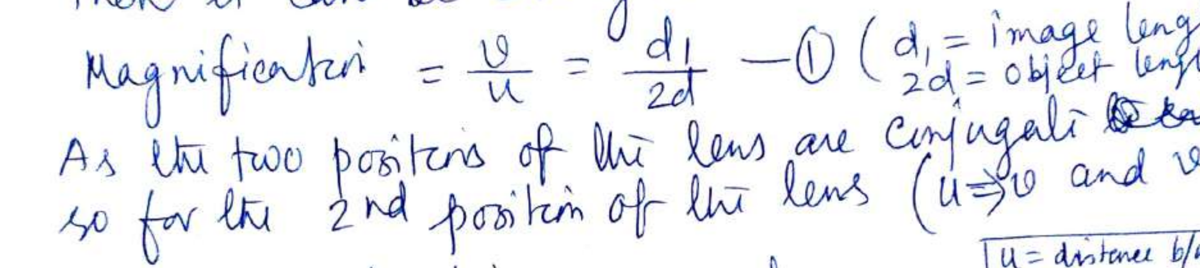
$$\text{Again Magnification } \frac{u}{v} = \frac{d_2}{2d}$$

u = distance b/w slit and lens
v = distance b/w eyepiece and lens.

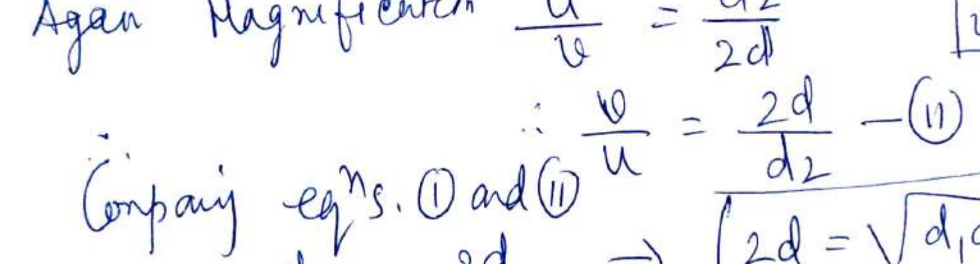
$$\therefore \frac{v}{u} = \frac{2d}{d_2} \quad \text{—(II)}$$

Comparing eq^{n}s. (I) and (II)

$$\frac{d_1}{2d} = \frac{2d}{d_2} \Rightarrow \boxed{2d = \sqrt{d_1 d_2}} \Rightarrow \text{Distance between two virtual source}$$



First position of the lens when it is near the biprism

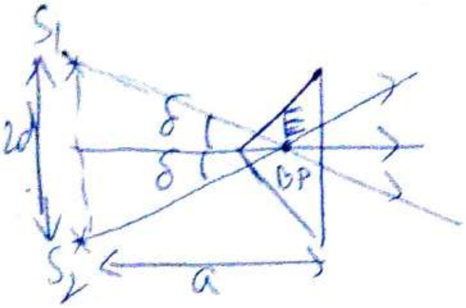


2nd position of lens

Determination of 2d (distance b/w two virtual sources by deviation method

In this method, 2d (distance b/w two virtual sources S_1, S_2) can be determined using the fact that for a prism of very small refracting angle, the deviation δ produced is given by

$$\delta = (\mu - 1)\alpha$$, α = refracting angle
μ = refractive index of the material of prism



The total angle $\angle S_1 E S_2 = 2\delta$

$$\therefore 2\delta = 2(\mu - 1)\alpha$$

From the figure

$$2d = 2\delta . a$$

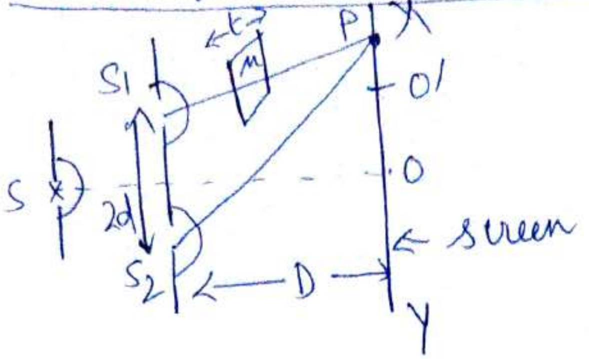
where a is the distance between slit and the biprism

$$\therefore 2d = 2(\mu - 1)\alpha a$$

$$\therefore \boxed{2d = 2a(\mu - 1)\alpha}$$

$\tan\delta = \frac{d}{a}$
if δ is small
$\tan\delta \approx \delta$
$\therefore \delta = \frac{d}{a}$
$\therefore d = \delta a$
$\therefore 2d = 2\delta a$

Determination of the thickness of a thin sheet of transparent material (Displacement of fringes)



Let us now discuss the change in the interference pattern produced by introducing a thin transparent plate in the path of one of the two interfering beams as shown in the figure.

Let a thin transparent sheet (like mica) of thickness t and refractive index μ be introduced in the path of the wave from the slit S_1. It is obvious that the light reaching at the pt. P from S_1 has to traverse a distance 't' in the plate and a distance $(S_1P - t)$ in the air. Thus the time required for the light to reach from S_1 to the pt. P is given by

$$\text{Time}(T) = \frac{S_1P - t}{c} + \frac{t}{v}$$

where v is the of light in mica sheet and 't' is its thickness

since $\mu = \frac{c}{v}$

$$= \frac{S_1P - t}{c} + \frac{\mu t}{c}$$

$$= \frac{1}{c}\left[(S_1P - t) + \mu t\right]$$

$$= \frac{1}{c}\left[S_1P + (\mu - 1)t\right] \quad \text{--- (1)}$$

Thus by introducing a thin plate the effective optical path is increased by an amount $(\mu - 1)t$.

In the absence of the sheet, let O be the position of the central bright fringe, which corresponds to equal optical path from S_1 and S_2. Now in presence of the mica plate, the optical paths (S_1O, S_2O) become unequal so the two waves coming from S_1 and S_2 do not arrive at O simultaneously, therefore the central fringe is shifted to a pt. O' so that at O' two optical paths become equal. Such a shift results for all fringes.

Now the effective path difference at any pt. P on the screen is

$$\Delta' = S_2P - [S_1P + (\mu-1)t]$$
$$= S_2P - S_1P - (\mu-1)t \quad \text{--- (2)}$$

If 2d is the separation between the two slits, D is the distance of the screen from the slits and x_n is the position of the n^{th} bright fringe at P, then the path difference in the absence of the mica plate is

$$\Delta = S_2P - S_1P = \frac{2d}{D} x_n^{ab} \quad \text{--- (3)}$$

Thus the effective path difference is

$$\Delta' = S_2P - S_1P - (\mu-1)t$$
$$\boxed{\therefore \Delta' = \frac{2d}{D} x_n^{ab} - (\mu-1)t}$$

If the pt. P is now the centre of the n^{th} bright fringe then the cond^n of bright fringe

$$\Delta' = n\lambda$$
$$\therefore \frac{2d}{D} x_n^{ab} - (\mu-1)t = n\lambda$$
$$\therefore x_n^{ab} = \frac{D}{2d}[n\lambda + (\mu-1)t] \quad \text{--- (4)}$$

In the absence of the mica plate i.e. when $t=0$, the position of the n^{th} bright fringe from O is

$$x_n^{ab} = \frac{D}{2d} n\lambda$$

[$\Delta = n\lambda$ cond^n for bright fr.]

$\therefore$ The displacement of the nth bright fringe

$$x_0 = x_n - x_n^{ab}$$

$$= \frac{D}{2d}\left[n\lambda + (\mu-1)t\right] - \frac{D}{2d}n\lambda$$

$$\boxed{\therefore\ x_0 = \frac{D}{2d}(\mu-1)t} \quad (5)$$

It is clear from eqn (5) that x_0 is independent of order n, that is the displacement is same for all bright fringes (ie for $n = 0, 1, 2 \ldots$). Similar expression can also be obtained for dark fringes. Thus the introduction of the transparent sheet in the path of one of the wave simply shift the entire fringe system through a distance $\frac{D}{2d}(\mu-1)t$ towards the side on which the sheet has been introduced.

Thickness of Mica Sheet

If by the introduction of a mica sheet, the central fringe moves through a distance formerly occupied by nth bright fringe ie $x_n^{ab} = \frac{D}{2d}n\lambda$

$$\therefore \frac{D}{2d}n\lambda = \frac{D}{2d}(\mu-1)t$$

$$\therefore n\lambda = (\mu-1)t$$

$$\therefore \boxed{t = \frac{n\lambda}{\mu-1}}$$

thickness