# Angle Modulation

- We can encode information as either
  - Time varying phase
  - Time varying frequency
- Both result in angle modulation
- Both are very closely related
- Easy to do with an sdr, just another complex envelope!

# Instantaneous Frequency

- In general, the frequency of a signal at an instant in time depends on the entire signal.
- For generalized sinusoids, we can use a simpler approach. Suppose

  $$\varphi(t) = A\cos\theta(t)\,.$$

  Then $\theta(t)$ is the *generalized angle*. For a true sinusoid,

  $$\theta(t) = \omega_c t + \theta_0\,,$$

  linear with slope $\omega_c$ and offset $\theta_0$
- The generalized angle is *not* limited to $[0, 2\pi]$. Wrapping introduces discontinuities.
- Phase unwrapping is easy at an IF, where the phase changes are small from on sample to the next.

# Instantaneous Frequency (cont.)

- Instananeous frequency is derivative of generalized angle:

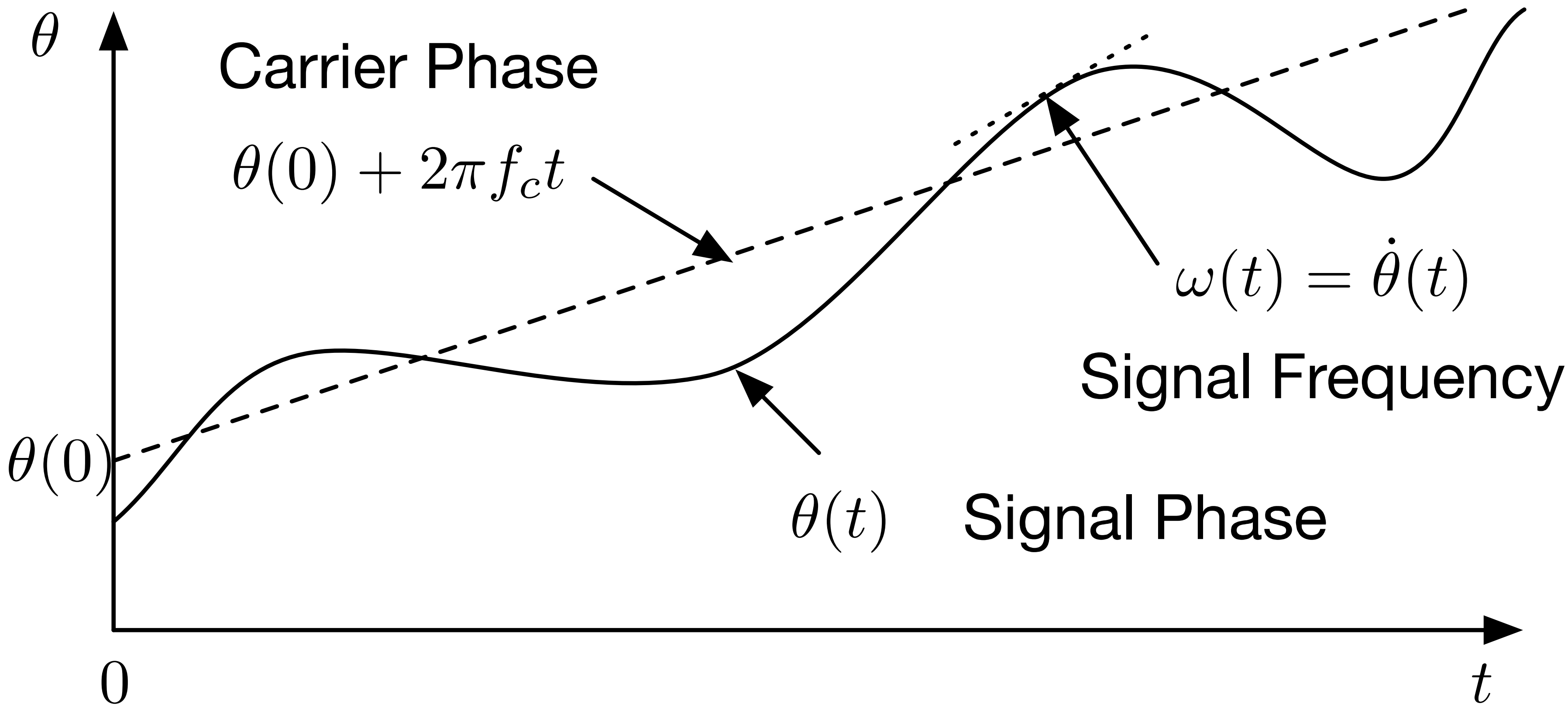
$$\omega_i(t) = \frac{d\theta}{dt} = \theta'(t)$$

- The phase is just the integral of the frequency

$$\theta(t) = \int_{-\infty}^{t} \omega_i(u)\, du = \theta(0) + \int_0^t \omega_i(u)\, du$$

- We can modulate a generalized sinusoid by using a signal $m(t)$ to vary either $\theta(t)$ or $\omega_i(t)$.
- In either case, the frequency of the modulated signal changes as a function of $m(t)$.

# Instantaneous Frequency (cont.)




- Dashed line is the carrier phase
- Solid line is the phase of the transmitted signal
- Slope of the solid line is the instantaneous frequency of the transmitted signal.

# Phase Modulation (PM)

- In PM, *phase* is varied *linearly* with $m(t)$:

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

  which produces a transmitted signal

$$\varphi_{\mathrm{PM}}(t) = \cos\left(2\pi f_c t + k_p m(t)\right)$$

- The instantaneous frequency is

$$\omega_i(t) = \frac{d\theta}{dt} = 2\pi f_c + k_p \dot{m}(t)$$

- If $m(t)$ varies rapidly, then the frequency deviations are larger.
- The bandwidth of the signal is determined by $\dot{m}(t)$, similar to AM.

# Frequency Modulation (FM)

- In FM, *frequency* is is varied *linear* in $m(t)$:

$$\omega_i(t) = 2\pi f_c + k_f m(t)$$

  which produces a signal

$$\varphi_{\mathrm{FM}}(t) = \cos\big((2\pi f_c + k_f m(t))t\big)$$

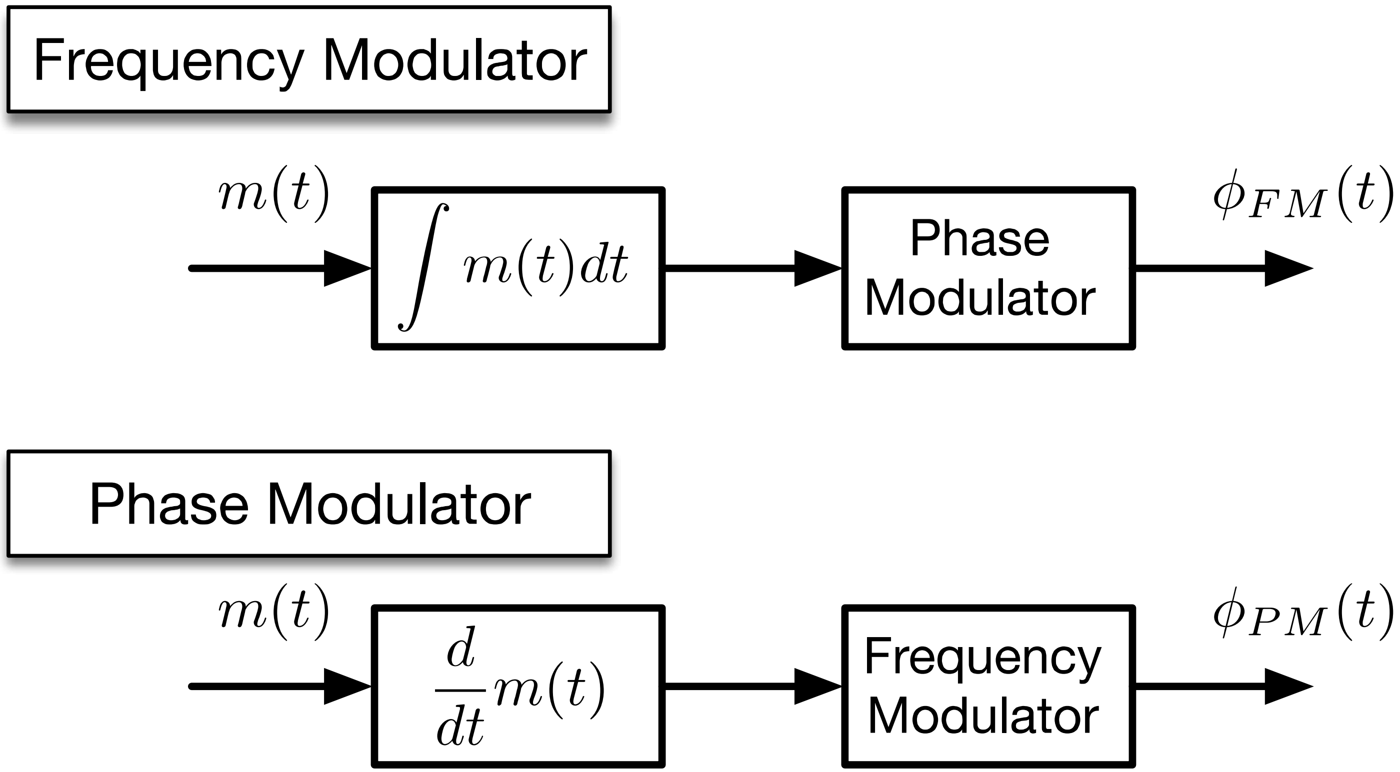
- The bandwidth of the signal is determined by the *amplitude* of $k_f m(t)$ (obvious), and also the bandwidth of $m(t)$ (less obvious).
- The angle is

$$\theta(t) = \int_{-\infty}^{t} \big(2\pi f_c + k_f m(t)\big)\, du = 2\pi f_c t + k_f \int_{-\infty}^{t} m(u)\, du$$

- We could apply this $\theta(t)$ to a phase modulator, and get exactly the same effect as applying $\omega_i(t)$ to a frequency modulator.

# Relationship Between FM and PM

- Phase modulation of $m(t)$ = frequency modulation of $\dot{m}(t)$.
- Frequency modulation of $m(t)$ = phase modulation of $\int m(u)\,du$.




- We can produce both types of modulation with either modulator.
- Direct digital synthesis (DDS) chips will do this for you.

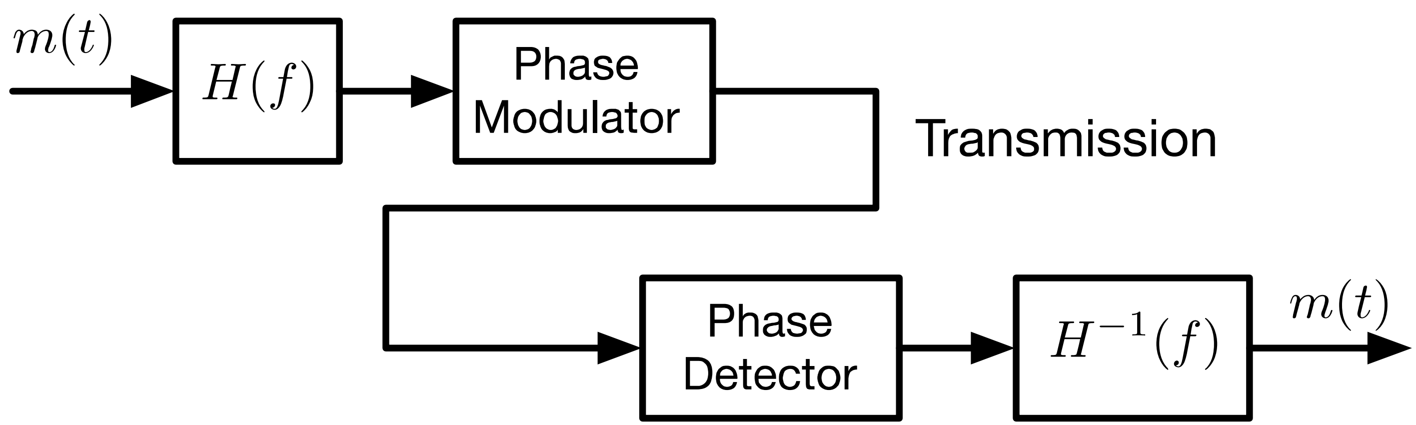
# Generalized Angle Modulation

- We can generalize modulation by convolving the message signal with an impulse response $h(t)$

$$\varphi_{\mathrm{EM}}(t) = A\cos\left(2\pi f_c t + h(t) * m(t)\right)$$

  This is a filter with a transfer function $H(f)$.

- We recover $m(t)$ from phase by using inverse filter $H^{-1}(f)$. E.g., for FM the inverse of integration is differention.




- PM ($h(t) = k_p \delta(t)$) and FM ($h(t) = k_f u(t)$) are special cases.

- Also used for pre-emphasis to improve noise characteristics, and pulse shaping to reduce signal bandwidth.