
SCALES

Dimensions of large objects must be reduced to accommodate on standard size drawing sheet. This reduction creates a scale of that reduction ratio, which is generally a fraction & such a scale is called Reducing Scale and the ratio is called Representative Factor.

Representative Fraction:

The ratio of the dimension of the object shown on the drawing to its actual size is called the Representative Fraction (RF).

$$RF = \frac{\text{Dimension of Drawing}}{\text{Dimension of object (actual dimensions)}} = \frac{\text{Length of the object in the drawing}}{\text{actual length}}$$

Ex . When a 1 cm long line in a drawing represents 1 meter length of the object

$$R.F = \frac{1\text{cm}}{1\text{m}} = \frac{1\text{cm}}{1 \times 100\text{cm}} = \frac{1}{100}$$

**For computing R.F, the numerator and denominator should be in same units

Metric Measurements:

10 millimetres (mm)	= 1 centimetre(cm)
10 centimetres (cm)	= 1 decimetre(dm)
10 decimetre (dm)	= 1 metre(m)
10 metres (m)	= 1 decametre (dam)
10 decametre (dam)	= 1 hectometre (hm)
10 hectometres (bm)	= 1 kilometre (km)
1 hectare	= 10,000 m ²

Necessity-

It is not convenient, always, to draw drawings of the object to its actual size. e.g. Buildings. Hence scales are used to prepare drawing at

- Full size
- Reduced size
- Enlarged size

Types of Scales:

• *Engineers Scale* :

The relation between the dimension on the drawing and the actual dimension of the object is mentioned numerically (like 10 mm = 15 m).

• *Graphical Scale*:

Scale is drawn on the drawing itself. This takes care of the shrinkage of the engineer's scale when the drawing becomes old.

Types of Graphical Scale:

- Plain Scale
- Diagonal Scale
- Vernier Scale
- Comparative scale

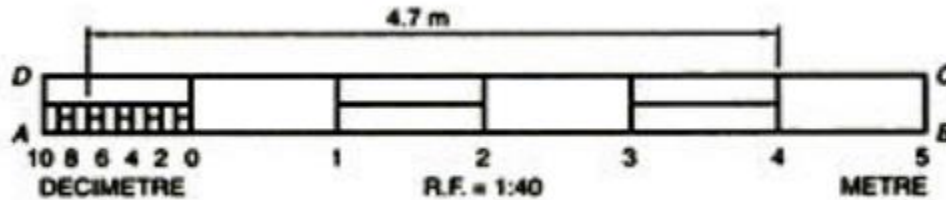
A) Plain Scales:

A plain scale is simply a line which is divided into a suitable number of equal parts, the first of which is further sub-divided into small parts. It is used to represent either two units or a unit and its fraction such as km and hm, m and dm, cm and mm etc.

- a. A plain scale consists of a line divided into suitable number of equal units. The first unit is subdivided into smaller parts.
- b. The zero should be placed at the end of the 1st main unit.

- From the zero mark, the units should be numbered to the right and the sub-divisions to the left.
- The units and the subdivisions should be labeled clearly.
- The R.F. should be mentioned below the scale.

Ex.- 1. Construct a scale of 1:40 to read metres and decimetres and long enough to measure 6 m. Mark on it a distance of 4.7 m.

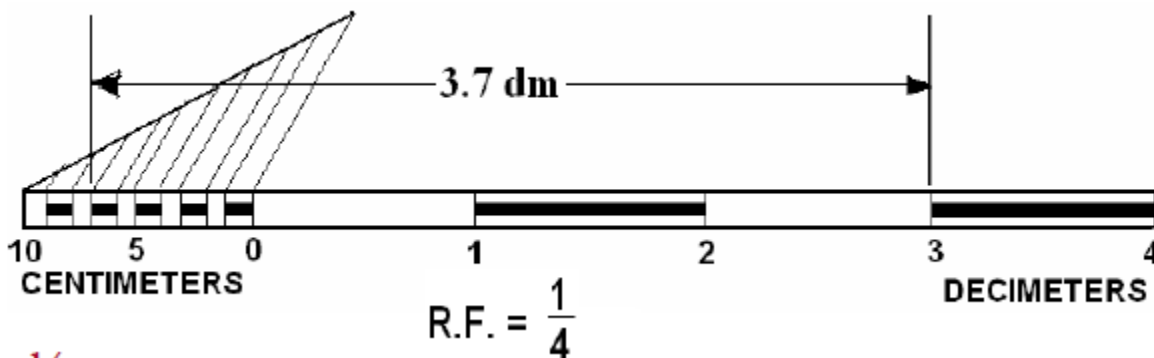


- Given (a) R.F. = 1/40, (b) Maximum = 6 m and (c) Least count = 1 dm.
- Calculate length of scale

$$L_s = \text{R.F.} \times \text{maximum length} = \frac{1}{40} \times 6 \times 100 \text{ cm} = 15 \text{ cm}$$

- Draw a rectangle having length $AB = 15 \text{ cm}$ and width $AD = 10 \text{ mm}$.
- As the length of scale represents 6 m, divide it into 6 equal parts so that each part may represent 1 metre and mark the main units as shown.
- Divide the first part OA into 10 divisions, so that each division may represent 1 dm. Mark sub-units on the scale as shown.
- Write the R.F. below the scale.
- Mark a 4.7 m length on the scale, i.e., 4 metre on the right side of the zero mark and 7 decimetre on the left side of zero mark.

Ex.-2. Construct a scale of 1:4, to show centimeters and long enough to measure up to 5 decimeters.



— 1/

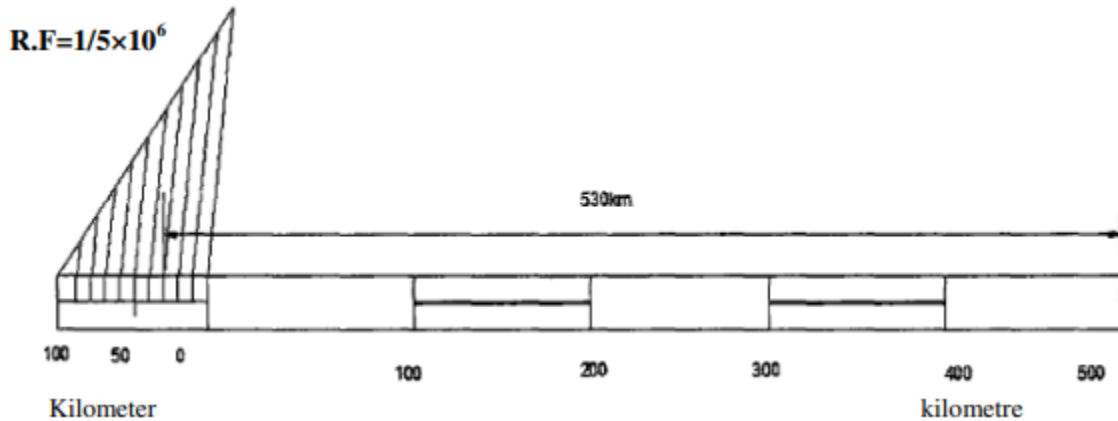
- R.F. = $\frac{1}{4}$
- Length of the scale = R.F. \times max. length = $\frac{1}{4} \times 5 \text{ dm} = 12.5 \text{ cm}$.
- Draw a line 12.5 cm long and divide it in to 5 equal divisions, each representing 1 dm.
- Mark 0 at the end of the first division and 1, 2, 3 and 4 at the end of each subsequent division to its right.
- Divide the first division into 10 equal sub-divisions, each representing 1 cm.
- Mark cm to the left of 0 as shown.

Ex.- 3. The distance between two towns is 250 km and is represented by a line of length 50mm on a map. Construct a scale to read 600 km and indicate a distance of 530 km on it.

Distance (Length) between two towns in the drawing = 50mm

Actual distance (length) = 250 km = 250*1000*1000 mm

Therefore, $R.F = 50\text{mm}/250\text{km} = 50\text{mm}/250 \times 1000 \times 1000\text{mm} = 1/5 \times 10^6$



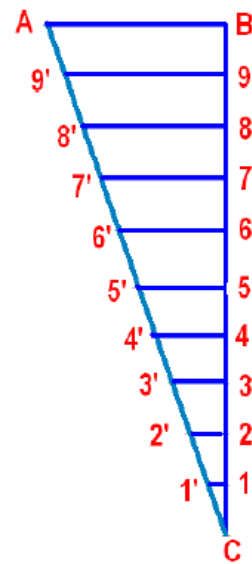
Diagonal Scales:

Diagonal scales are used to represent either three units of measurements such as metres, decimetres, centimetres or to read to the accuracy correct to two decimals (e.g. 4.35). e.g. *dm, cm & mm*, or *yard, foot & inch*.

- **Diagonal scale can measure more accurately than the plain scale.**

The principle of construction of a diagonal scale is as follows:

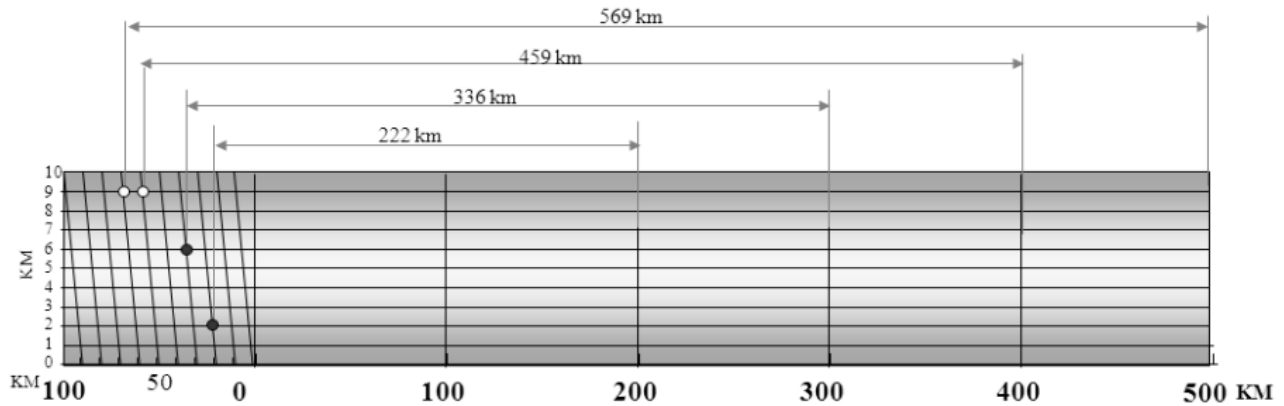
- **At end B of line AB, draw a perpendicular.**
- **Step-off ten equal divisions of any length along the perpendicular starting from B and ending at C.**
- **Number the division points 9,8,7,.....1.**
- **Through the points 1, 2, 3, etc., draw lines parallel to AB and cutting AC at 1', 2', 3', etc.**
- **Since the triangles are similar; 1'1 = 0.1 AB, 2'2 = 0.2AB, ... 9'9 = 0.9AB.**
- **Gives divisions of a given short line AB in multiples of 1/10 its length, e.g. 0.1AB, 0.2AB, 0.3AB, etc.**



Ex. 1-The distance between Delhi and Agra is 200 km. In a railway map it is represented by a line 5 cm long. Find its R.F. Draw a diagonal scale to show single km. And maximum 600 km. Indicate on it following distances. 1) 222 km 2) 336 km 3) 459 km 4) 569 km.

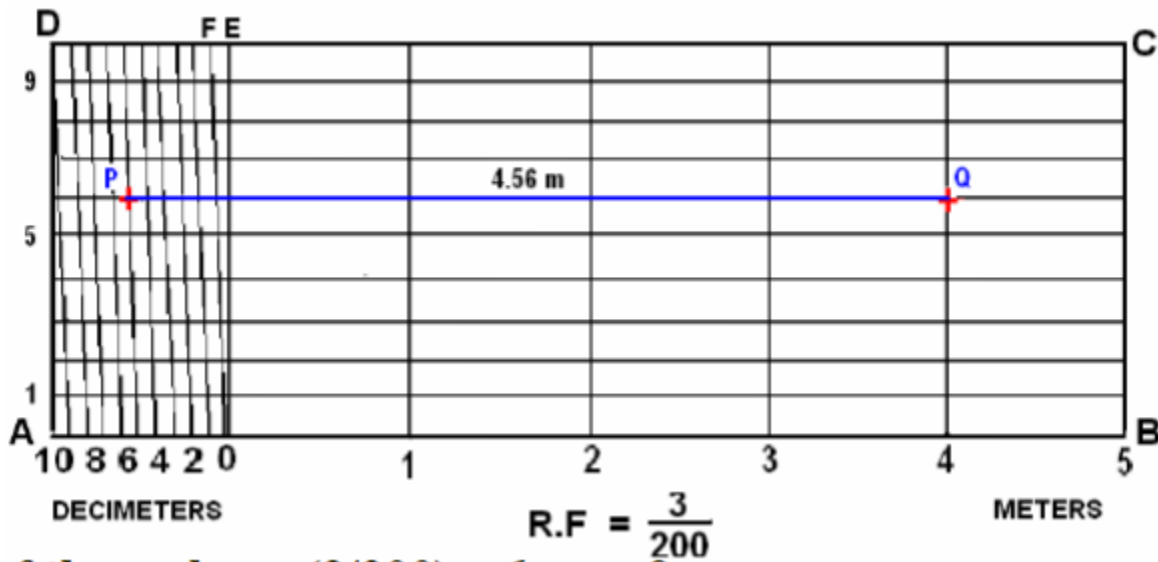
$$R.F = 5 \text{ cm} / 200 \text{ km} = 1 / 40,000,000$$

$$\text{Length of scale} = 1 / 40,000,000 \times 600 \times 10^5 = 15 \text{ cm}$$



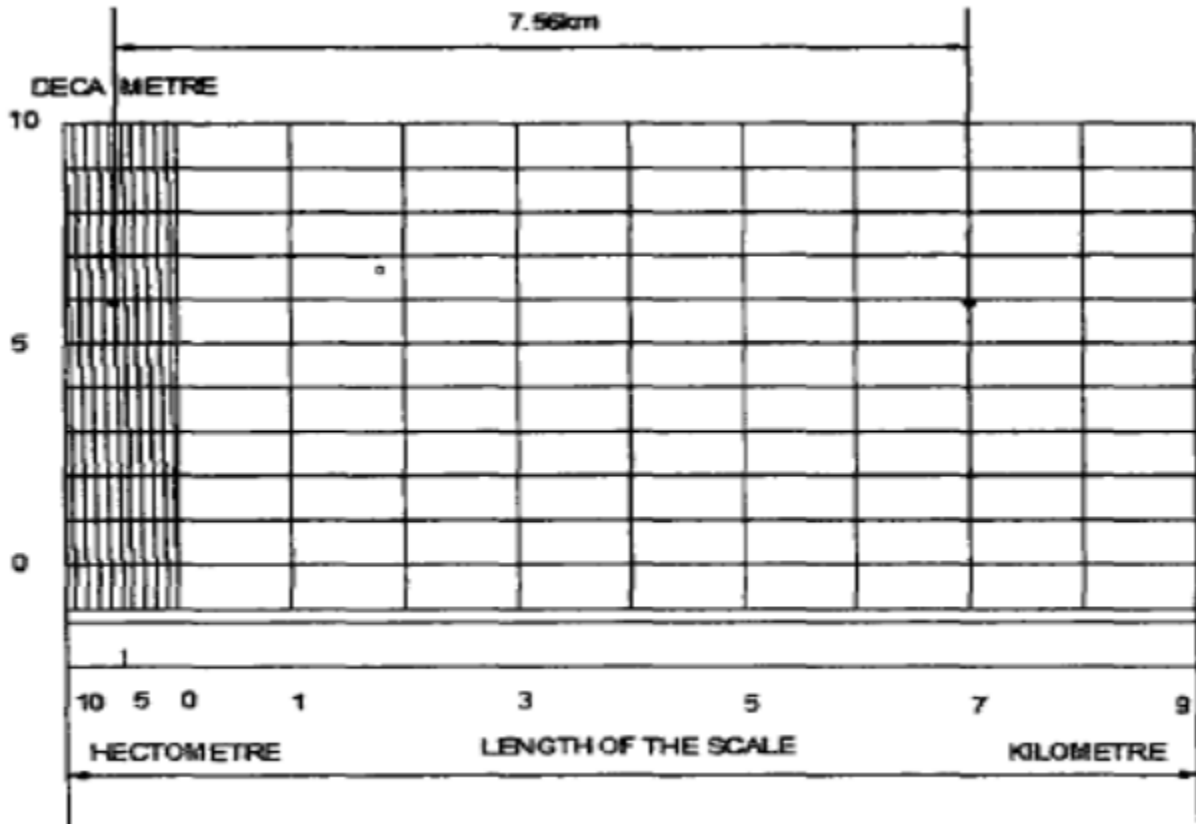
R.F. = $1 / 40,00,000$

Ex. 2-Construct a Diagonal scale of RF = $3:200$ (i.e. $1:66 \frac{2}{3}$) showing meters, decimeters and centimeters. The scale should measure up to 6 meters. Show a distance of 4.56 meters.



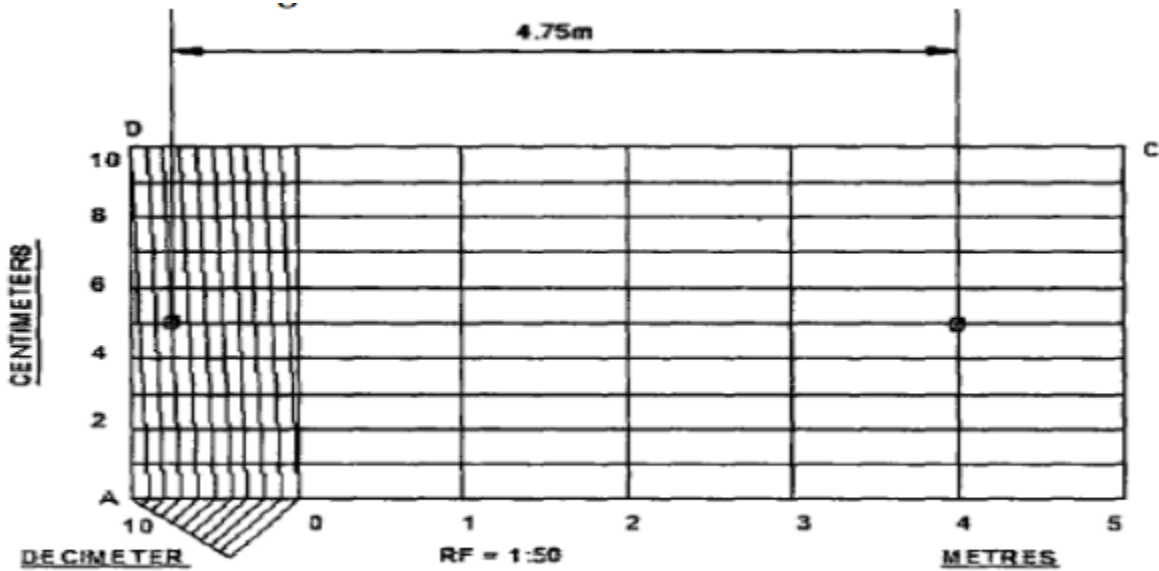
- Length of the scale = $(\frac{3}{200}) \times 6 \text{ m} = 9 \text{ cm}$
- Draw a line AB = 9 cm . Divide it in to 6 equal parts.
- Divide the first part A0 into 10 equal divisions.
- At A draw a perpendicular and step-off along it 10 equal divisions, ending at D.
- Complete the rectangle ABCD.
- Draw perpendiculars at meter-divisions i.e. 1, 2, 3, and 4.
- Draw horizontal lines through the division points on AD. Join D with the end of the first division along A0 (i.e. 9).
- Through the remaining points i.e. 8, 7, 6, ... draw lines // to D9.
- PQ = 4.56 meters

Ex. 3-An area of 144 sq cm on a map represents an area of 36 sq Km on the field. Find the RF of the scale of the map and draw a diagonal scale to show Km, hectometers and decameters and to measure up to 10 Km. Indicate on the scale a distance 7 Km, 5 hectometers and 6 decameters.



$$\mathbf{R.F. = 12/6 \times 1000 \times 100}$$

Ex. 4-Construct a diagonal scale 1/50, showing metres, decimetres and centimetres, to measure upto 5 metres. Mark a length 4.75 m on it.



$$\mathbf{R.F = 1/50}$$

Vernier Scale:

The vernier scale is a short auxiliary scale constructed along the plain or main scale, which can read up to two decimal places.(when available the space is small).

A Vernier scale consists of (i) a primary scale (main scale) and (ii) a vernier.

The primary scale is a plain scale fully divided into minor divisions.

The graduations on **the vernier** are derived from those on the primary scale.

- The smallest division on the main scale and vernier scale are 1 msd (main scale division) or 1 vsd (vernier scale division) respectively.
Generally $\{(n+ 1)$ for backward or retrograde vernier} or $\{(n-1)$ for forward or direct vernier} divisions on the main scale is divided into n (generally, $n=10$) equal parts on the vernier scale.
- When $1 \text{ vsd} < 1$ it is called **forward or direct vernier**. The vernier divisions are numbered in the same direction as those on the main scale.
- When $1 \text{ vsd} > 1$ or $(1 + 1/n)$, It is called **backward or retrograde vernier**. The vernier divisions are numbered in the opposite direction compared to those on the main scale. The least count (LC) is the smallest dimension correct to which a measurement can be made with a vernier.
- For forward vernier, $L C = (1 \text{ msd} - 1 \text{ vsd})$ when **MSD>VSD**
- For backward vernier, $L C = (1 \text{ vsd} - 1 \text{ msd})$ when **VSD>MSD**

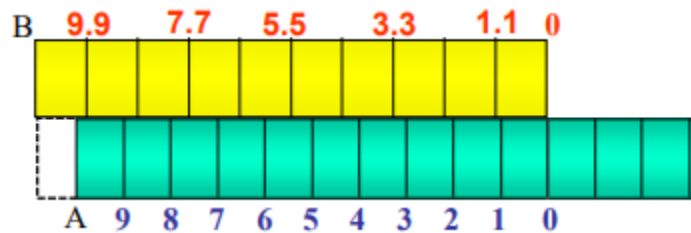


Figure to the right shows a part of a plain scale in which length A-O represents 10 cm. If we divide A-O into ten equal parts, each will be of 1 cm. Now it would not be easy to divide each of these parts into ten equal divisions to get measurements in millimeters.

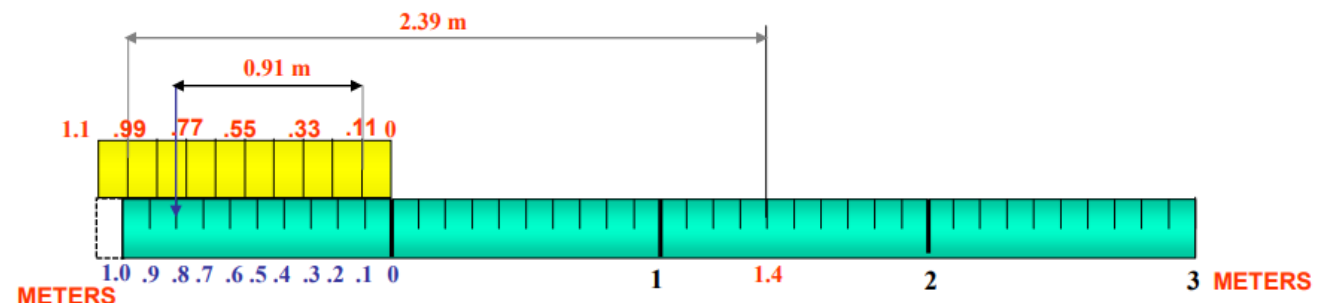
Now if we take a length BO equal to $10 + 1 = 11$ such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent $11 / 10 = 1.1$ cm.

The difference between one part of AO and one division of BO will be equal $1.1 - 1.0 = 0.1$ cm or 1 mm.

This difference is called Least Count of the scale. Minimum this distance can be measured by this scale.

The upper scale BO is the vernier. The combination of plain scale and the vernier is vernier scale.

Example 1: Draw a vernier scale of $RF = 1 / 25$ to read centimeters upto 4 meters and on it, show lengths 2.39 m and 0.91 m. (backward vernier)



SOLUTION:

Length of scale = $RF \times \text{max. Distance} = 1 / 25 \times 4 \times 100 = 16 \text{ cm}$

CONSTRUCTION: (**Main scale**)

Draw a line 16 cm long. Divide it in 4 equal parts. (Each will represent meter) Sub-divide each part in 10 equal parts. (Each will represent decimeter) Name those properly.

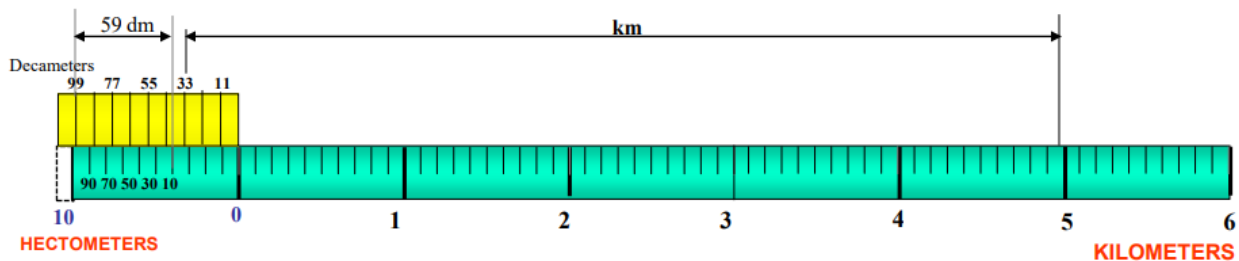
CONSTRUCTION:

(**vernier scale**) Take 11 parts of Dm length and divide it in 10 equal parts. Each will show 0.11 m or 1.1 dm or 11 cm and construct a rectangle Covering these parts of vernier.

TO MEASURE GIVEN LENGTHS:

- (1) For 2.39 m : Subtract 0.99 from 2.39 i.e. $2.39 - 0.99 = 1.4$ m The distance between 0.99 (left of Zero) and 1.4 (right of Zero) is 2.39 m
- (2) (2) For 0.91 m : Subtract 0.11 from 0.91 i.e. $0.91 - 0.11 = 0.80$ m The distance between 0.11 and 0.80 (both left side of Zero) is 0.91 m

Example 2: A map of size 500cm X 50cm wide represents an area of 6250 sq.Kms. Construct a vernier scale to measure kilometers, hectometers and decameters and long enough to measure upto 7 km. Indicate on it a) 5.33 km b) 59 decameters.



SOLUTION:

$$\begin{aligned}
 \text{RF} &= \sqrt{\frac{\text{AREA OF DRAWING}}{\text{ACTUAL AREA}}} \\
 &= \sqrt{\frac{500 \times 50 \text{ cm sq.}}{6250 \text{ km sq.}}} \\
 &= 2 / 10^5
 \end{aligned}$$

Length of scale = RF X max. Distance = $2 / 10^5 \times 7 \text{ kms} = 14 \text{ cm}$

CONSTRUCTION: (Main scale)

Draw a line 14 cm long.

Divide it in 7 equal parts. (each will represent km) Sub-divide each part in 10 equal parts. (each will represent hectometer)

CONSTRUCTION: (vernier)

Take 11 parts of hectometer part length and divide it in 10 equal parts. Each will show 1.1 hm or 11 dm and Covering in a rectangle complete scale.

TO MEASURE GIVEN LENGTHS:

a) For 5.33 km :

Subtract 0.33 from 5.33 i.e. $5.33 - 0.33 = 5.00$

The distance between 33 dm (left of Zero) and 5.00 (right of Zero) is 5.33 k m

(b) For 59 dm :

Subtract 0.99 from 0.59 i.e. $0.59 - 0.99 = - 0.4$ km (- ve sign means left of Zero) The distance between 99 dm and - .4 km is 59 dm (both left side of Zero)

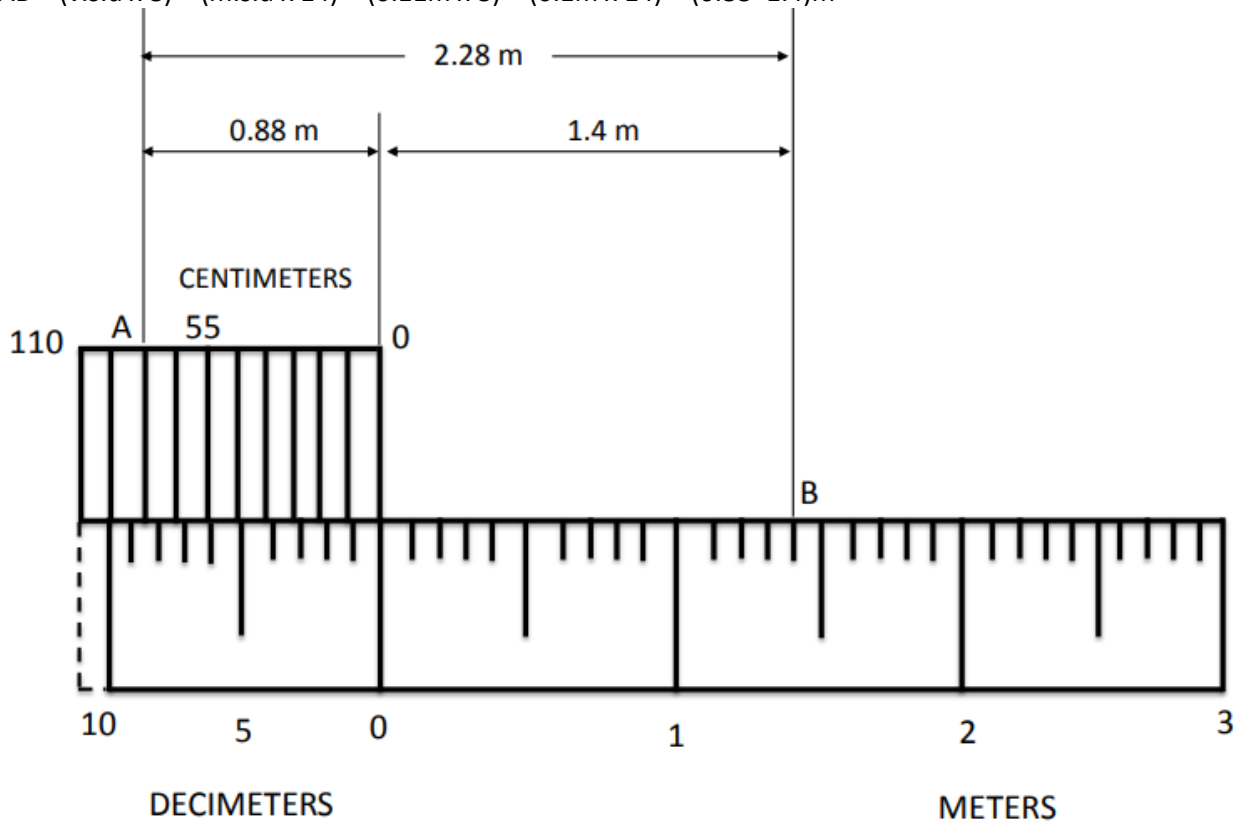
Ex. 3- Construct a Vernier scale to read meters, decimeters and centimeters and long enough to measure up to 4 m. R.F. of the scale is 1/20. Mark on your scale a distance of 2.28 m.

- Least Count = Smallest distance to be measured = 1 cm (given) = 0.01 m
- $L = \text{R.F.} \times \text{Maximum distance to be measured} = (1/20) \times 4 \text{ m} = 20 \text{ cm}$

- Main Scale: Draw a line of 20 cm length. Complete the rectangle of 20 cm x 0.5 cm.
- Divide it into 4 equal parts each representing 1 meter.
- Sub-divide each part into 10 main scale divisions. Hence 1 m.s.d. = $1\text{m}/10 = 0.1\text{ m} = 1\text{ dm}$.
- Backward Vernier: Take 11 divisions on main scale. Divide it into 10 equal parts on vernier scale.
So 1 v.s.d. = $11\text{ m.s.d.}/10 = 11 \times 1\text{ dm}/10 = 0.11\text{ m} = 1.1\text{ dm} = 11\text{ cm}$.

Mark 0, 55, 110 towards left from 0 on the vernier scale. The units of main divisions is METERS, subdivisions is DECIMETERS and vernier divisions is CENTIMETERS

- $AB = (\text{v.s.d} \times 8) + (\text{m.s.d} \times 14) = (0.11\text{m} \times 8) + (0.1\text{m} \times 14) = (0.88+1.4)\text{m}$

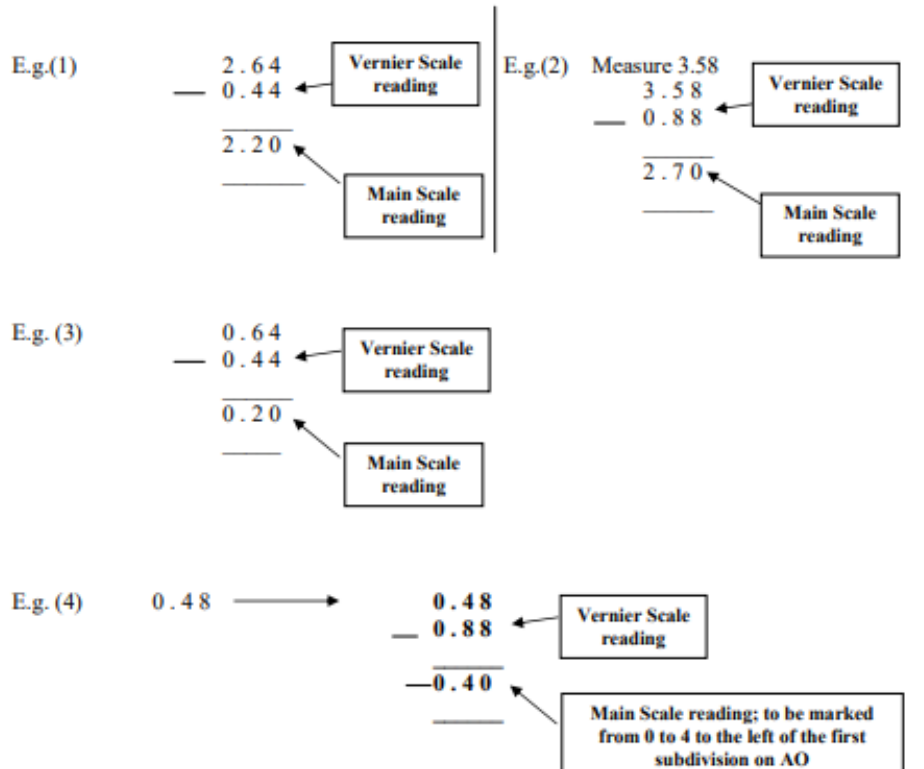


Measurements in Vernier Scales

- In Vernier scales, it is not possible to measure the sub-division directly. Only 2 types of readings can be taken. They are the main scale reading and the vernier scale reading.
- As the vernier scale reading consists of multiples of 11, subtract the multiple of 11 from the main scale reading which has the same last digit.

For E.g. 2.64 decimeters is to be measured.

Since 4 is the last digit, subtract 0.44 from it. Hence $2.64 = 0.44 + 2.20$.

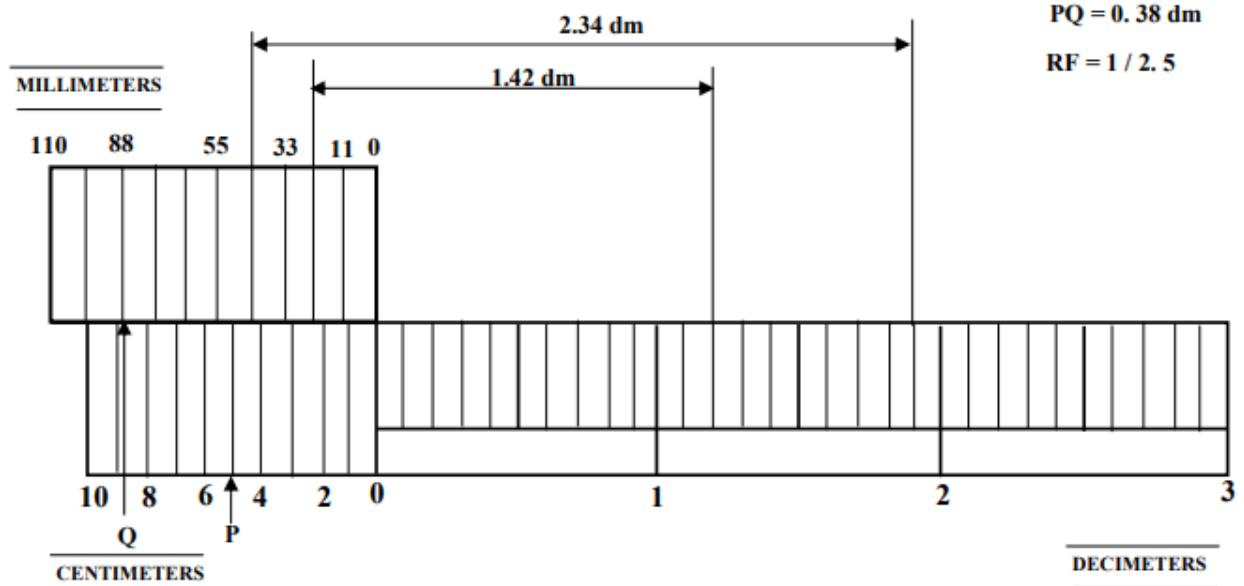


Ex. Construct a scale of R.F = 1/(2.5) to show decimeters & centimeters and by a vernier to read millimeters, to measure up to 4 decimeters. Show on it lengths 2.34 dm, 1.42 dm & 0.38 dm.

Length of scale (LOS) = RF X ML (in cm) = (1/2.5) x 4 x 10 cm (1 dm=10cm) = 16 cm.

The length of the line that is drawn on the drawing sheet is 16 cm & divided into 4 parts.

The lengths are 2.34 (0.44, 1.90), 1.42 (0.22, 1.20) & 0.38 (0.88, -0.50)
(VSD, MSD) (VSD, MSD) (VSD, MSD)



Ex. The actual length of 500m is represented by a line of 15cm on a drawing. Construct a vernier scale to read up to 400 m. Mark on the scale a length of 349m.

$$RF = 15 \text{ cm} / (500 \times 100) \text{ cm} = 3/10000.$$

Max Length (ML) = 400 m; (no. of parts of scale (n) = 4 parts, each is 100 m)

Length of scale (LOS) = RF X ML (in cm) = (3/10000) x 400 x 100 cm (1 m=100cm) = 12 cm.

The length of the line that is drawn on the drawing sheet is 12 cm & divided into 4 parts.

**The length is 349 m (99+ 250); 99 on vernier scale & 250 on main scale.
(VSD, MSD)**

