Interpolation with unevenly spaced points →

Lagrange's interpolation formula -

Let $y(x)$ be continuous and differentiable $(n+1)$ times in the interval (a,b). Given the $(n+1)$ points (x_0,y_0) (x_1,y_1) -- (x_n,y_n) where the values of x need not necessarily be equally spaced.

we find a polynomial of degree n such that,

$$L_n(x_i) = y(x_i) = y_i \quad i = 0,1,2,--n. \quad — ①$$

we consider the case of the equation of a straight line passing through two points (x_0,y_0) and (x_1,y_1). Such a polynomial, $L_1(x)$ is seen to be

$$L_1(x) = \frac{x-x_1}{x_0-x_1}y_0 + \frac{x-x_0}{x_1-x_0}y_1$$

$$L_1(x) = l_0(x)y_0 + l_1(x)y_1$$

$$= \sum_{i=0}^{l} l_i(x)y_i \quad — ②$$

where $l_0(x) = \dfrac{x-x_1}{x_0-x_1}$, $l_1(x) = \dfrac{x-x_0}{x_1-x_0}$ — ③

$l_0(x_0) = 1$, $l_0(x_1) = 0$ $l_1(x_0) = 0$ $l_1(x_1) = 1$

$$l_j(x_j) = \begin{cases} 1, & \text{if } i=j \\ 0, & \text{if } i\neq j \end{cases} \quad — ④$$

$l_i(x)$ have the property -

$$\sum_{i=0}^{l} l_i(x) = l_0(x) + l_1(x)$$

$$= \frac{x-x_1}{x_0-x_1} + \frac{x-x_0}{x_1-x_0}$$

$$\sum_{i=0}^{l} l_i(x) = 1 \quad — ⑤$$

eq. (2) is the Lagrange polynomial of degree one passing through two points (x_0, y_0) and (x_1, y_1)

the Lagrange polynomial of degree two passing through three points $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) is written as.

$$L_2(x) = \sum_{i=0}^{2} l_i(x) y_i$$

$$= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2 \quad \text{--- (6)}$$

$l_i(x)$ satisfies the formula eq. (4) and eq. (5)

$$L_n(x) = \sum_{i=0}^{n} \frac{\Pi_{n+1}(x)}{(x-x_i)\Pi'_{n+1}(x_i)} y_i \quad \text{--- (7)}$$

this is Lagrange's interpolation formula. $l_i(x)$ are called Lagrange interpolation coefficients.

changing x and y in eq. (7) we obtain the inverse inter formula-

$$L_n(y) = \sum_{i=0}^{n} \frac{\Pi_{n+1}(y)}{(y-y_i)\Pi'_{n+1}(y_i)} x_i \quad \text{--- (8)}$$

Ex (1) If $y_1 = 4$, $y_3 = 12$, $y_4 = 19$ and $y_x = 7$ find x,