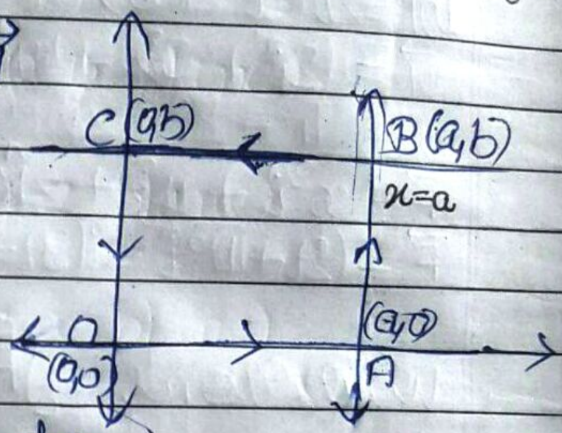
Q. Find the total work done by a force $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ in moving a point from (0,0) to (a,b) along the rectangle bounded by the line $x=0$, $x=a$, $y=0$, $y=b$.

Total Work done $= \int_C \vec{F}\cdot d\vec{r}$



$\vec{r} = x\hat{i} + y\hat{j}$

$d\vec{r} = \hat{i}\,dx + \hat{j}\,dy$

$\vec{F}\cdot d\vec{r} = (x^2+y^2)\,dx - 2xy\,dy$

Total Work Done $= \int_{OA} \vec{F}\cdot d\vec{r} + \int_{AB} \vec{F}\cdot d\vec{r}$ —— (1)

Along OA $\longrightarrow$

$y=0 \Rightarrow dy=0$

$$\int_{OA} \vec{F}\cdot d\vec{r} = \int_0^a x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^a = \frac{a^3}{3}$$

Along AB $\longrightarrow$

$x=a \Rightarrow dx=0$

$$\int_{AB} \vec{F}\cdot d\vec{r} = \int_0^b -2ay\,dy$$

$$= \left[-ay^2\right]_0^b$$

$$= -ab^2$$

from eq (1) $\longrightarrow$

Total Work Done $= \frac{a^3}{3} - ab^2$ Ans

Q. If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from (0,0) to (1,2).

Let $x = t$, then parametric eq^n of the parabola $y = 2x^2$ are — $x = t,\ y = 2t^2$

0, 1

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = t\hat{i} + 2t^2\hat{j} \implies d\vec{r} = \hat{i}\,dt + 4t\hat{j}\,dt$

$d\vec{r} = (\hat{i} + 4t\hat{j})\,dt$

$\vec{F} = 3xy\hat{i} - y^2\hat{j}$

$\vec{F} = 6t^3\hat{i} - 4t^4\hat{j}$

$\vec{F}\cdot d\vec{r} = (6t^3 - 16t^5)\,dt$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 (6t^3 - 16t^5)\,dt$$

$$= \left[\frac{\cancel{6}^{3}t^4}{\cancel{4}_{2}} - \frac{\cancel{16}^{8}t^6}{\cancel{6}_{3}}\right]_0^1$$

$$= \frac{3}{2} - \frac{8}{3}$$

$$= \frac{9-16}{6}$$

$$= \frac{-7}{6}$$ Ans