Surface Integral: Any integral which is to be evaluated over a surface is called a surface integral.
It is denoted by — $\iint_S \vec{F} \cdot \vec{ds}$ or $\iint_S \vec{F} \cdot \hat{n}\, ds$

where $ds = \dfrac{dx\,dy}{|\hat{k} \cdot \hat{n}|}$ (x-y plane)

$ds = \dfrac{dy\,dz}{|\hat{i} \cdot \hat{n}|}$ (y-z plane)

$ds = \dfrac{dz\,dx}{|\hat{j} \cdot \hat{n}|}$ (z-x plane)

Volume Integral: Any integral which is to be evaluated over a volume is called a volume integral.
It is denoted by — $\iiint_V \phi\, dV$ or $\iiint_V \vec{F}\, d\vec{V}$

Q. Evaluate → $\iint_S \vec{A} \cdot \hat{n}\, ds$ where $\vec{A} = (x+y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$

and S is the surface of the plane $2x+y+2z=6$ in the first octant.

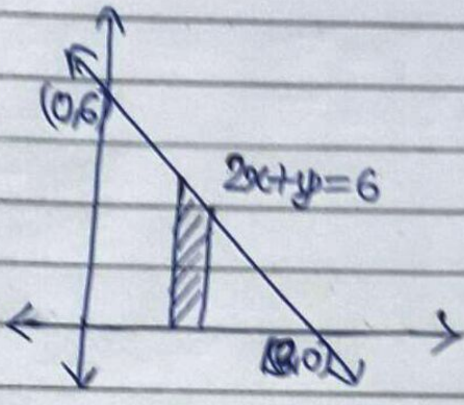
$\vec{n} = \nabla(2x+y+2z)$
$\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\hat{n} = \dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{3} = \dfrac{\vec{n}}{|\vec{n}|}$

$$ds = \frac{dx\,dy}{|\hat{k}\cdot\hat{n}|}$$

$$ds = \frac{3\,dx\,dy}{2}$$

$$\iint_S \vec{A}\cdot\hat{n}\,ds = \iint \left\{(x+y^2)\hat{i} - 2x\hat{j} + 2y\left(\frac{6-2x-y}{\cancel{2}}\right)\hat{k}\right\}\cdot\left(\frac{2\hat{i}+\hat{j}+2\hat{k}}{\cancel{3}}\right)\frac{\cancel{3}}{2}\,dy\,dx$$



$$= \int_0^3\int_0^{6-2x} \left\{2(x+y^2) - 2x + 2y(6-2x-y)\right\} dy\,dx \times \frac{1}{2}$$

$$= \frac{1}{\cancel{2}}\int_0^3\int_0^{6-2x} \cancel{2}4(3-x)y\,dy\,dx$$

$$= \cancel{2}\int_0^3 \left[\frac{y^2}{\cancel{2}}\right]_0^{6-2x} dx\,(3-x)$$

$$= 4\int_0^3 (3-x)^3\,dx$$

$$= -\cancel{4}\left[\frac{(3-x)^4}{\cancel{4}}\right]_0^3$$

$$= \cancel{-\{(-1)^4 - (3)^4\}}$$

$$= -(-81)$$

$$= 81 \quad \underline{\text{Ans}}$$