12/5/22

Q:- If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then Evaluate -

$\iiint_V \nabla \vec{F}\, dv$, where V is bounded by the planes $x=0$, $y=0$, $z=0$ & $2x+2y+z=4$

Sol^n:- $\nabla \cdot \vec{F} = \frac{\delta}{\delta x}(2x^2 - 3z) + \frac{\delta}{\delta y}(-2xy) + \frac{\delta}{\delta z}(-4x)$

$= 4x - 2x$ ~~$- 4x - 2x$~~

$= +2x.$

$$\int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} 2x \, dz\, dy\, dx$$

$$= 2\int_0^2 \int_0^{2-x} x\,[z]_0^{4-2x-2y}\, dy\, dx$$

$$= 2\int_0^2 \int_0^{2-x} x\,[4-2x-2y]\, dy\, dx$$

~~$= 2\int_0^2 \int_0^{2-x} [4-2x-2y]_0$~~

$$= 2\int_0^2 \left\{ x(4-2x)[y]_0^{2-x} - 2\left[\frac{y^2}{2}\right]_0^{2-x} \right\} dx$$

$$= 2\int_0^2 x\{(4-2x)(2-x) - (2-x)^2\}\, dx$$

~~$= 2\int_0^2 x(2-x)^3\, dx$~~

$(2-x)\,(4-2x-2+x)$
$2-x$

= ~~2~~

$x(2-x)^1$

$$= 2\int_0^2 x\left[2(2-x)^2-(2-x)^2\right]dx$$

$$= 2\int_0^2 x\,(2-x)^2\,dx$$

$$= 2\int_0^2 x\left[4+x^2-4x\right]dx$$

$$= 2\int_0^2 \left[4x+x^3-4x^2\right]dx$$

$$= 2\left[\frac{4x^2}{2}+\frac{x^4}{4}-\frac{4x^3}{3}\right]_0^2$$

$$= 2\left[8+\frac{16}{4}-\frac{32}{3}\right]$$

$$= \frac{8}{3}$$

$24-\frac{32}{3}$

$12-\frac{32}{3}$

$\frac{36}{3}$

$\frac{36}{32}$

4. $\vec{A} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$.

Evaluate

$$\iiint_V \vec{A}\cdot dV$$ where V is the region bounded by the surfaces $x=0$, $y=0$, $x=2$, $y=6$, $z=x^2$, $z=4$

Sol^n - $$\int_0^2\int_0^6\int_{x^2}^4 (2xz\hat{i} - x\hat{j} + y^2\hat{k})\,dz\,dy\,dx$$

$$I_3 = \hat{k}\int_0^2\int_0^6\int_{x^2}^4 y^2 \, dz\, dy\, dx$$

$$= \hat{k}\int_0^2\int_0^6 y^2[4-x^2]\, dy\, dx$$

$$= \hat{k}\int_0^2 [4-x^2]\left[\frac{y^3}{3}\right]_0^6 dy\, dx$$

$$= \hat{k}\int_0^2 [4-x^2]\left[\frac{\cancel{216}^{\,72}}{\cancel{3}}\right] dx$$

$\frac{36\ \cancel{6}}{24}$

$$= \hat{k}\left[288x - \frac{\cancel{72}^{\,24}x^3}{\cancel{3}_1}\right] dx$$

$$= \hat{k}\,(576 - 192)$$

$$= \hat{k}\,(384)$$

$$I = I_1 - I_2 + I_3$$

$$= 128\hat{i} - 24\hat{j} + 384\hat{k}$$