Line Integral:- Any integral which is to be evaluated along a curve is called a line integral. It is denoted by $\int_C \vec{F}\cdot d\vec{R}$ or $\int_C \vec{F}\cdot \frac{d\vec{R}}{dt}\cdot dt$.

→ If $\vec{F} = f_1\hat{i} - f_2\hat{j} + f_3\hat{k}$ and $\vec{R} = x\hat{i} - y\hat{j} + z\hat{k}$.

$d\vec{R} = \hat{i}\,dx - \hat{j}\,dy + dz\,\hat{k}$.

$$\int_C \vec{F}\cdot d\vec{R} = \int_C f_1\,dx + f_2\,dy + f_3\,dz.$$

→ If the parametric eqn of the curve C are $x = x(t)$, $y = y(t)$, $z = z(t)$ and $t = t_1$ at A, $t = t_2$ at B, then $\int_C \vec{F}\cdot\frac{d\vec{R}}{dt}\,dt = \int_{t_1}^{t_2}\left(f_1\frac{dx}{dt} + f_2\frac{dy}{dt} + f_3\frac{dz}{dt}\right)$

Total workdone by $\vec{F}$ during displacement from A to B is given by $\int_A^B \vec{F}\cdot d\vec{R}$

Q:- Suppose $\vec{F} = x^3\hat{i} + y\hat{j} + z\hat{k}$ is the force. Find the workdone by $\vec{F}$ along the line from (1, 2, 3) to (3, 5, 7)

$\vec{R} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ <u>eqn of line:</u>

$\int_C x^3\hat{i} + y\hat{j} + z\hat{k}$.

$$\Rightarrow \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x-1}{3-1} = \frac{y-2}{5-2} = \frac{z-3}{7-3}$$

$$\Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = K \text{ (say)}.$$

$x - 1 = 2K$

$y - 2 = 3K$

$z - 3 = 4K$

$$\boxed{\begin{aligned} x &= 2k+1 \\ y &= 3k+2 \\ z &= 4k+3. \end{aligned}}$$

$x = 2k+1$

$dx = 2dk$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}.$

$d\vec{r} = 2dk\hat{i} + 3dk\hat{j} + 4dk\hat{k}.$

$d\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k})dk.$

$$\text{workdone} = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_C x^3\hat{i} + y\hat{j} + z\hat{k}$$

$$= \int_C \left[(2k+1)^3\hat{i} + (3k+2)\hat{j} + (4k+3)\hat{k}\right] \cdot \left[2\hat{i} + 3\hat{j} + 4\hat{k}\right] dk.$$

$$\Rightarrow \int_C (2k+1)^3 \cdot 2 + (3k+2)(3) + (4k+3)(4)].$$

$$= \int_C [8k^3 + 1 + 6k(2k+1)] \times 2 + 9k + 6 + 16k + 12.$$

$$\Rightarrow \int_C 16k^3 + 2 + 12k(2k+1)] + 9k + 6 + 16k + 12.$$

$$\Rightarrow \int_0^1 16k^3 + 2 + 24k^2 + 12k + 9k + 18 + 16k.$$

$$\int_0^1 16k^3 + 24k^2 + 37k + 20$$

12
9
16
37

$$= \left[\frac{\cancel{16}^{4} k^4}{\cancel{4}} + \frac{\cancel{24}^{8} k^3}{\cancel{3}} + \frac{37k^2}{2} + 20k\right]_0^1$$

$$= \left[4k^4 + 8k^3 + \frac{37k^2}{2} + 20k\right]_0^1$$

$$= \left[\left(4 + 8 + \frac{37}{2} + 20\right) - 0\right]$$

$$= \left[32 + \frac{37}{2}\right] = \frac{64+37}{2} = \boxed{\frac{101}{2}}.$$