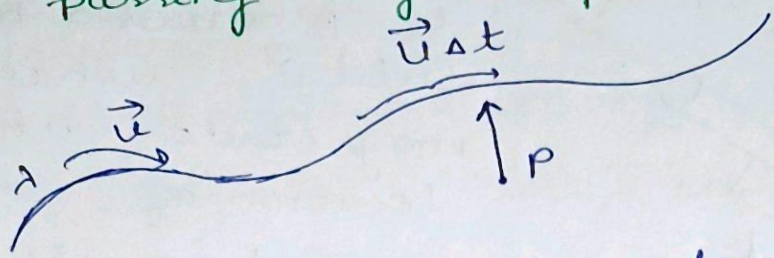
Currents →

The current in a wire is the charge per unit time passing a given point.



Current is measured in coulomb per second, or ampere (A):-

$$1\,A = 1\,C/sec. \quad -①$$

A line charge λ traveling down a wire at speed u constitutes a current.

$$I = \lambda u \quad -②$$

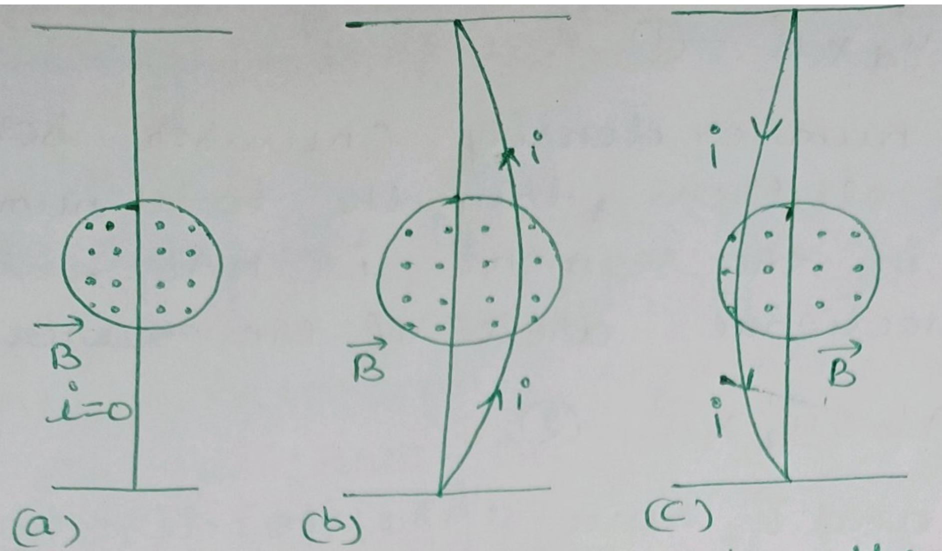
because a segment of length $u\Delta t$, carrying charge $\lambda u \Delta t$, passes point P in a time interval Δt. Current is actually a vector:-

$$\vec{I} = \lambda \vec{u} \quad -③$$

The Magnetic Force On A Current-Carrying wire →

A current is a collection of moving charges. Because a magnetic field exerts a sideways force on a moving charge, it should also exert a sideways force on a wire carrying a current.

A sideways force is exerted on the conduction electrons in the wire, but since the electrons cannot escape sideways, the force must be transmitted to the wire itself.



A flexible wire passes b/w the poles of a magnet
(a) there is no current in the wire.
(b) A current is established in the wire.
(c) the current is reversed.

We consider the individual charges flowing in a wire. We use the free e^- model for current in a wire, assuming the electrons to move with a constant velocity, the drift velocity $\vec{v}_d$.
→ the actual direction of motion of the electrons is of course opposite to the direction we take for the current i in the wire.
the wire passes through a region in which a uniform field $\vec{B}$ exists.
the sideways force on each electron due to the magnetic field is $-e\vec{v}_d \times \vec{B}$.
Let us consider the total sideways force on a segment of the wire of length L.
the same force acts on each electron in the segment, and the total force F_B on the segment is therefore equal to the number N of electrons times the force on each electron:-

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$$\vec{F}_B = -Ne\vec{v}_d \times \vec{B} \quad -(1)$$

If n is the number density (number per unit volume) of electrons, then the total number N of electrons in the segment is nAL, where A is the cross-sectional area of the wire,

$$\vec{F}_B = -nALe\vec{v}_d \times \vec{B} \quad -(2)$$

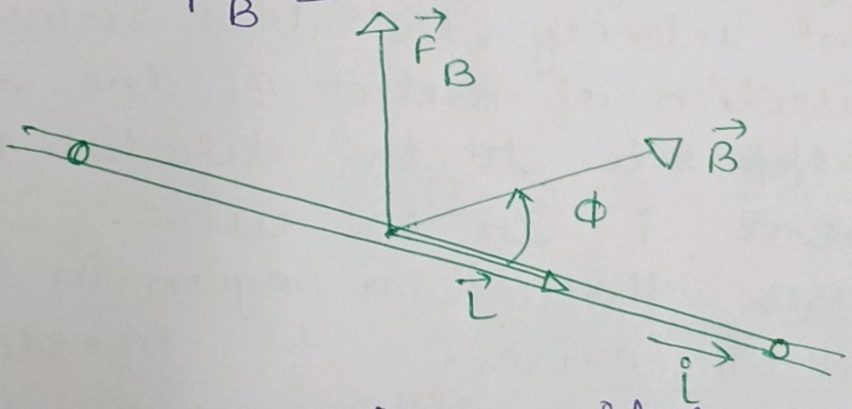
the vectors $\vec{v}_d$ and $\vec{L}$ have opposite directions and can be written as in form-

$nALev_d = iL$ using vectors as,

$$-nALe\vec{v}_d = i\vec{L} \quad -(3)$$

So an expression for the force on the segment

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad -(4)$$



If the field is uniform over the length of the wire segment and the direction of the current makes an angle ϕ with the field, then the magnitude of the force is.

$$F_B = iLB\sin\phi \quad -(5)$$

If $\vec{L}$ is parallel to $\vec{B}$, then the force is zero. If the segment is $\perp$ to the

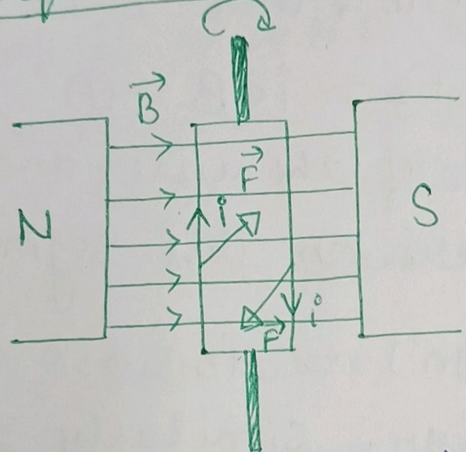
the magnitude of the force is

$$F_B = iLB \quad —(6)$$

If the wire is not straight or field is not uniform, we can imagine the wire to be broken into small segments of length dL. So the force on each segment can then be written,

$$\boxed{d\vec{F}_B = i\,d\vec{L} \times \vec{B}} \quad —(7)$$

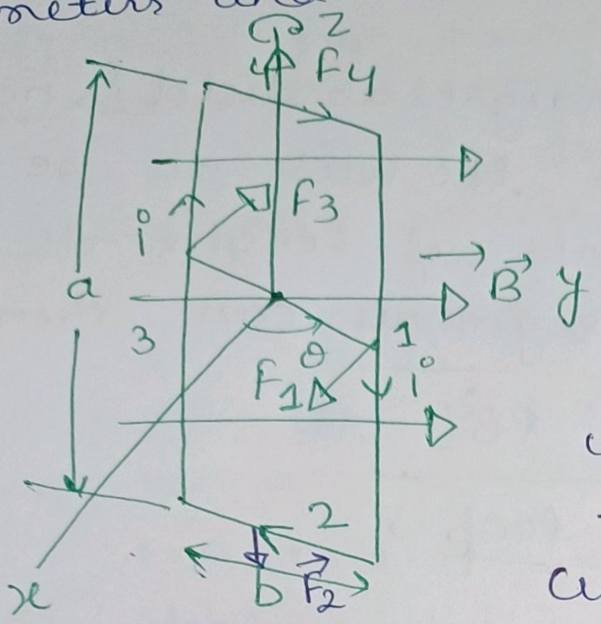
the torque on a current loop →



An electric motor. Loop carries an electric current. the magnetic forces on the loop sides of the loop produces a torque that tends to rotate the loop clockwise.

When the loop is oriented so that the field is in the plane of the loop. the magnetic forces on the short ends are zero because $\vec{B}$ and $\vec{L}$ are parallel. on the long ends of the rectangular loop the forces are equal but point in opposite directions, so net force on the loop is zero.

However, there is a net torque that tends to rotate the loop about its axis in a clockwise direction. this shows how the combination of an electric current and a magnetic field can produce the rotary motion of the electric motor. this principle is also responsible for the action of analog volt-meters and ammeters.



we consider a rectangular loop of length a and width b carrying a current i. the plane of the loop makes an angle θ with the x-axis.

we define the vector $\vec{L}$ as being in the direction of the current, then $\vec{L}$ is $\perp$ to $\vec{B}$.

for those sides $F_1 = F_3 = iaB$ — (1)

the angle b/w side 2 of the wire and $\vec{B}$ is $\theta + 90°$ we find the force on this segment to be.

$$F_2 = ibB \sin(\theta + 90°) = ibB\cos\theta \quad (2)$$

in the (-ve) z-direction, similarly the force on the side 4 is

$$F_4 = ibB\sin(90° - \theta) = ibB\cos\theta \quad (3)$$

in the positive z-direction.

$\vec{F_2}$ and $\vec{F_4}$ are equal in magnitude and opposite in direction they sum to zero. Same for $\vec{F_1}$ and $\vec{F_3}$. the net force on the loop is zero, so its centre of mass does not accelerate under the action of the magnetic force.

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Even though the net force is zero, the net torque is nonzero. Relative to the z-axis, the forces F_1 and F_2 each have moment arms of $\left(\frac{b}{2}\right)\sin\theta$, and so the magnitude of the total torque is.

$$\tau = 2(iab)\left(\frac{b}{2}\right)\sin\theta \quad -(4)$$

factor 2 enters because both forces contribute equally to the torque.

The torque has its maximum magnitude when the loop is oriented so that the magnetic field lies in the plane of the loop ($\theta = 90°$). The torque is zero when the magnetic field is $\perp$ to the plane of the loop ($\theta = 0°$).

If the loop were constructed as a coil of N turns of wire, then the total torque on the coil would be $\tau = NiABsin\theta$

$$\quad -(5)$$

$$\boxed{\vec{\tau} = NiA\hat{n} \times \vec{B}}$$

Here $\hat{n}$ is the unit vector which is perpendicular to the plane of the loop.