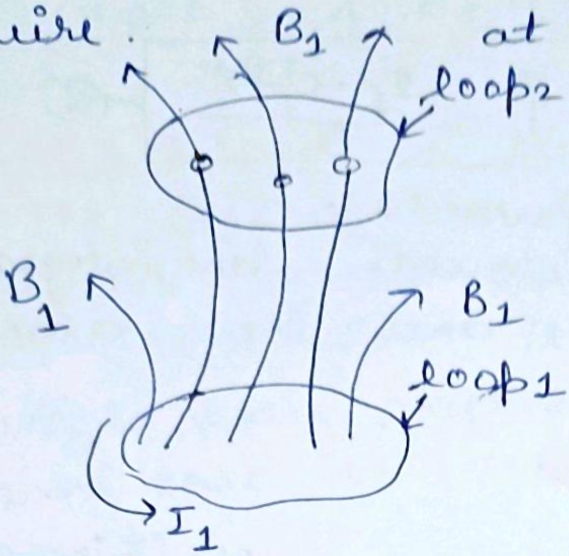
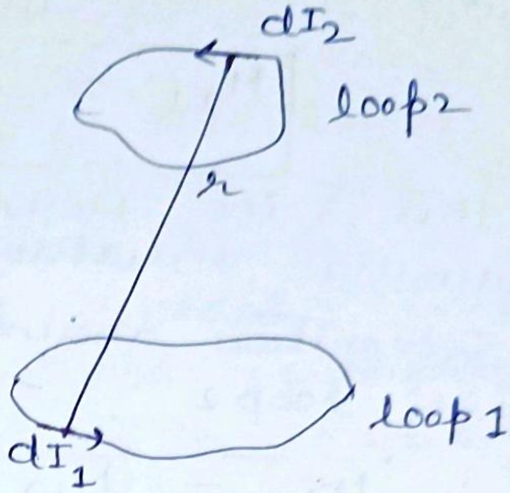
Inductance

Suppose we have two loops of wire at rest.





If we run a steady current I_1 around loop 1, it produces a magnetic field B_1. Some of the field lines pass through loop 2, let Φ_2 be the flux of $\vec{B_1}$ through 2.

Now we calculate $\vec{B_1}$ by Biot - savart law

$$\vec{B_1} = \frac{\mu_0}{4\pi} I_1 \oint \frac{dl_1 \times \hat{r}}{r^2} \quad (1)$$

$$B_1 \propto I_1$$

the flux through loop 2 :-

$$\Phi_2 = \int B_1 \cdot da_2 \quad (1)$$

$$\Phi_2 = M_{21} I_1$$

$M_{21} \rightarrow$ proportionality constant known as the mutual inductance of the two loops.

we can replace magnetic field in the terms of magnetic vector potential.

$$\Phi_2 = \int (\nabla \times A_1) \cdot da_2 = \oint A_1 \cdot dl_2 \quad (3)$$

As we know that $\vec{A}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r')}{r} d\tau' \quad (4)$

$$\Phi_2 = \oint \left(\frac{\mu_0}{4\pi} \int \frac{\vec{J}(r')}{r} d\tau' \right) \cdot dl_2$$

Continue

Hence, $\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{r} \right) \cdot d\mathbf{l}_2$

$$\boxed{M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r}} \quad \text{—(5)}$$

this is the Neumann formula.
this formula involves a double line integral one integration around loop 1 and the other around loop 2

$$\boxed{M_{21} = M_{12}}$$

M_{21} is a purely geometrical quantity, having to do with the sizes, shapes and relative positions of the two loops.

Now we vary the current in loop 1. the flux through loop 2 will vary accordingly then

$$\mathcal{E}_2 = \frac{-d\Phi_2}{dt} = -M \frac{dI_1}{dt} \quad \text{—(6)}$$

& induced emf

according to the faraday's law | A changing current not only induces an emf in any nearby loops.

It also induces an emf in the source loop itself

$$\Phi = LI \quad \text{—(7)}$$

Here, the proportionality constant L is called the self-inductance of the loop.

$$\mathcal{E} = -L \frac{dI}{dt} \quad \text{—(8)}$$

If the current changes then the emf induced in the loop is – eq. (8)

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Unit of inductance is Henries (H). which is $\dfrac{\text{volt} \times \text{second}}{\text{Ampere}}$.

Lenz's law → Accarding to this law, "The flux of the magnetic field due to the induced current opposes the change in flux that causes the induced current."

lenz's law refers to induced currents, which means that it applies only to closed conducting circuits.

If the change in flux in an increase, then Lenz's law requires that the direction of the induced current appose the increase, the flux from the magnetic field of the induced current would pass through the loop in a direction opposite to that of the original flux that is increasing.

If the change in flux is a decrease, the flux from the magnetic field of the induced current opposes the decrease, that is it tends to add to the original flux to keep it from decreasing.

Calculation of Inductance – Suppose the current in the inductor sets up a magnetic field $\vec{B}$, which can be calculated from the size and shape of the inductor, and from the distribution of current. the magnetic flux ϕ_B through each turn of the coil is calculated. the flux has the same value for each of the N turns of the coil, so the total flux is $N\phi_B$. this quantity is known as flux linkage of the inductor.

Now the emf can be found from faraday's law -

$$\mathcal{E} = -\frac{d(N\phi_B)}{dt} \quad -①$$

$$\mathcal{E} = -L\frac{di}{dt} \quad -②$$

So by equating these two equations.

$$\int \cancel{-}\frac{d(N\phi_B)}{dt} = \cancel{-}\int L\frac{di}{dt}$$

$$N\phi_B = Li$$

$$\boxed{L = \frac{N\phi_B}{i}} \quad -③$$

By this formula, we can calculate the inductance directly from the flux linkages.

the inductance of a solenoid -

Now apply eq.(3) formula to calculate L for a section of length l of a long solenoid of cross-sectional area A

the magnetic field inside the solenoid

$$B = \mu_0 n i \quad -①$$

$n \rightarrow$ no. of turns per unit length

the no. of flux linkages in the length l is $N\phi_B = (nl)\,BA$ -②

substituting the value of B from eq.① to eq.②

$$N\phi_B = (nl)\,\mu_0 n i A$$

$$N\phi_B = \mu_0 n^2 l i A \quad -③$$