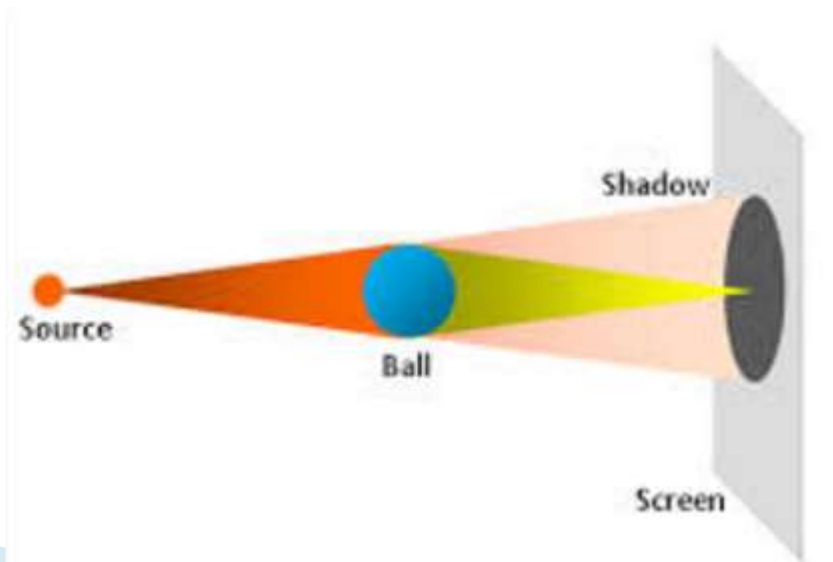
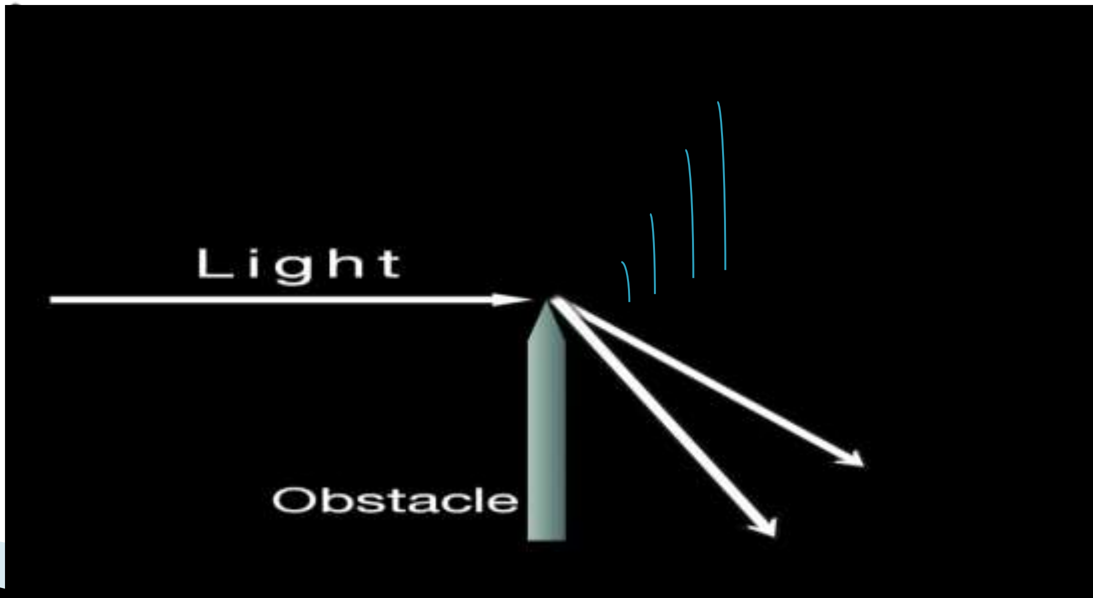
Diffraction of light

The phenomena of bending of light around the corners and edges of an obstacle object and their spreading into geometrical shadow is called diffraction



Reason:Diffraction can be explained d on the basis of Hygen's wave theory. When light is incident on a sharp edge of an object (obstacle), the point on edge become secondary sources of wave emitting spherical wavefronts which travels in all directions. As according to Fresnel, Diffraction phenomena is due to mutual interference of these secondary wavefronts .



Classes of Diffraction

Diffraction is classified into two catagories :

(1) **Fraunhofer Diffraction**: Fraunhofer studied the diffraction of light by keeping the source and the screen at infinite distance from the slit which is known as Fraunhofer Diffraction.

(2) **Fresnel Diffraction**: Fresnel studied the diffraction of light by keeping the source and the screen at finite distance from the slit which is known as Fresnel Diffraction .

Assignment Questions

- Give Five differences between Interference and diffraction.
- Give Five differences between Fraunhofer and Fresnel diffraction

Resultant of n-harmonic waves

$y1 = a \sin \omega t$

$y2 = a \sin (\omega t + \phi)$

$y3 = a \sin (\omega t + 2\phi)$

.. . . .

$yn = a \sin (\omega t + (n-1)\phi)$

Now resultant:

$Y = y1 + y2 + y3 + \ldots\ldots\ldots\ldots + yn$

$Y = a \sin \omega t + a \sin (\omega t + \phi) + a \sin (\omega t + 2\phi) + \ldots\ldots\ldots + a \sin (\omega t + (n-1)\phi)$

$Y = [a \sin(n\phi/2)/\sin(\phi/2)] \sin (\omega t + \delta)$

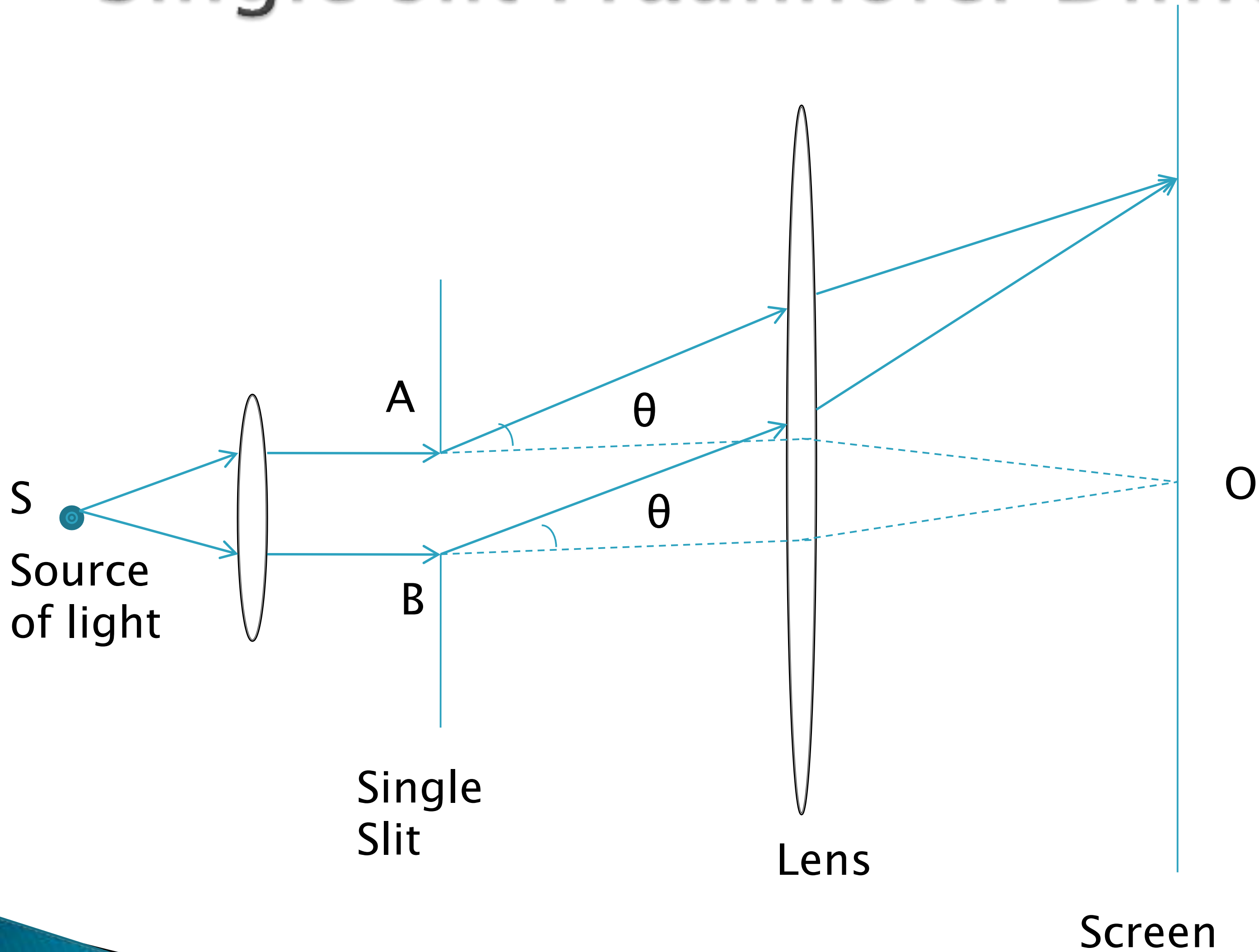
Where:

$[a \sin(n\phi/2)/\sin(\phi/2)]$ = resultant amplitude=A

and δ = resultant phase

$A = [a \sin(n\phi/2)/\sin(\phi/2)]$-------(1)

Single slit Fraunhofer Diffraction:



Slit width =e
Angle of Diffraction = θ

From Figure:

The path difference corresponding to n numbers of rays = BN

The path difference between two consecutive rays = BN/n

Corresponding phase difference $\phi = 2\pi (BN)/\lambda n$

$\phi = 2\pi e \sin\theta/\lambda n$

$\phi = 2\alpha/n$ ------------(1)

Where $\alpha = \pi e \sin\theta/\lambda$ ------(2)

We know,

$A = [a \sin(n\phi/2)/\sin(\phi/2)]$-------(3)

Using eq(1) and (3), we have

$$A = a \sin\alpha / \sin(\alpha/n) \quad \text{---------(4)}$$

For small (α/n) :

$$\sin(\alpha/n) \sim (\alpha/n)$$

- Now, $A = a \sin \alpha / (\alpha / n)$
- $A = (an \sin \alpha) / \alpha$

$A = (R \sin \alpha) / \alpha$ ------(5)

Where $R = an$

and $A^2 = (R^2 \sin^2 \alpha) / \alpha^2$

Intensity,

$$I = A^2 = (R^2 \sin^2 \alpha) / \alpha^2 \text{ -------(6)}$$

Case1: Principle Maxima

When $\alpha \longrightarrow 0$

$\lim_{\alpha \longrightarrow 0} (\sin \alpha) / \alpha = 1$

Therefore, $I_{max} = R^2$

Case 2: minima

When $\sin \alpha = 0$,

Or $\alpha = m\pi$

By eq. 2 ,

$$\pi e \sin\theta/\lambda = m\pi$$

$$e \sin\theta = m\lambda$$

Now , $I_{min} = 0$

Case3: Secondary Maxima,

$$dI/d\alpha = 0$$

By eq.(6) we have,

$$d\,[(R^2 \sin^2 \alpha)/\alpha^2]/d\alpha = 0$$

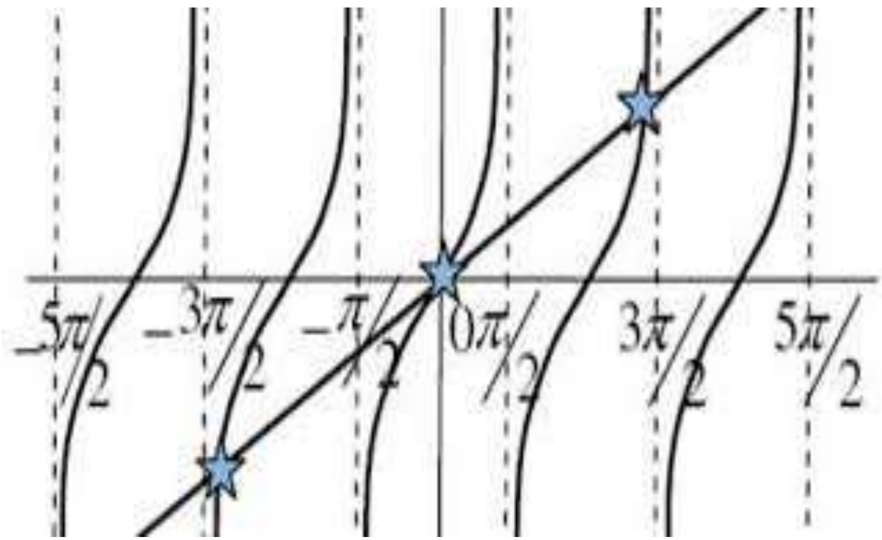
On solving, we have,

$$\alpha \cos\alpha - \sin\alpha = 0$$

$$\tan\alpha = \alpha \quad \text{------}(7)$$

y = α represent a straight line
and y = tanα represents a discontinuous curves having number of branches.

At α =0 ,we have principle maxima
At α = 3 π/2, 5π/2 , 7 π/2,…. We have secondary maximas



Now for $\alpha = 3\pi/2, 5\pi/2, 7\pi/2, \ldots$ We have secondary maxima as given below:

(i) First secondary Maxima:

$$I_1 = R^2 \sin^2(3\pi/2)/(3\pi/2)^2$$

$$I_1 = 4R^2 / 9\pi^2$$

(i) Second Secondary Maxima:

$$I_2 = R^2 \sin^2(5\pi/2)/(5\pi/2)^2$$

$$I_2 = 4R^2 / 25\pi^2$$

(i) Third Secondary Maxima:

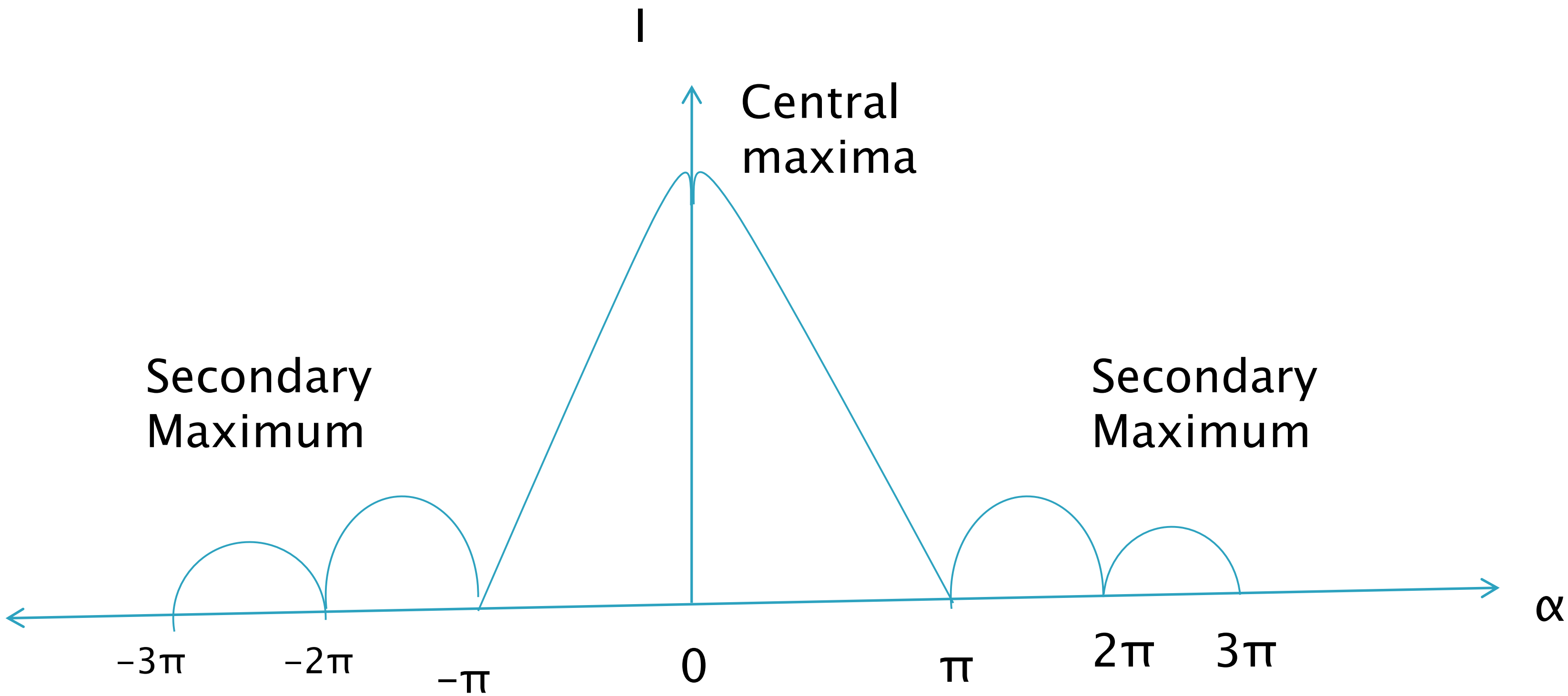
$$I_3 = R^2 \sin^2(7\pi/2)/(7\pi/2)^2$$

$$I_3 = 4R^2 / 49\pi^2$$

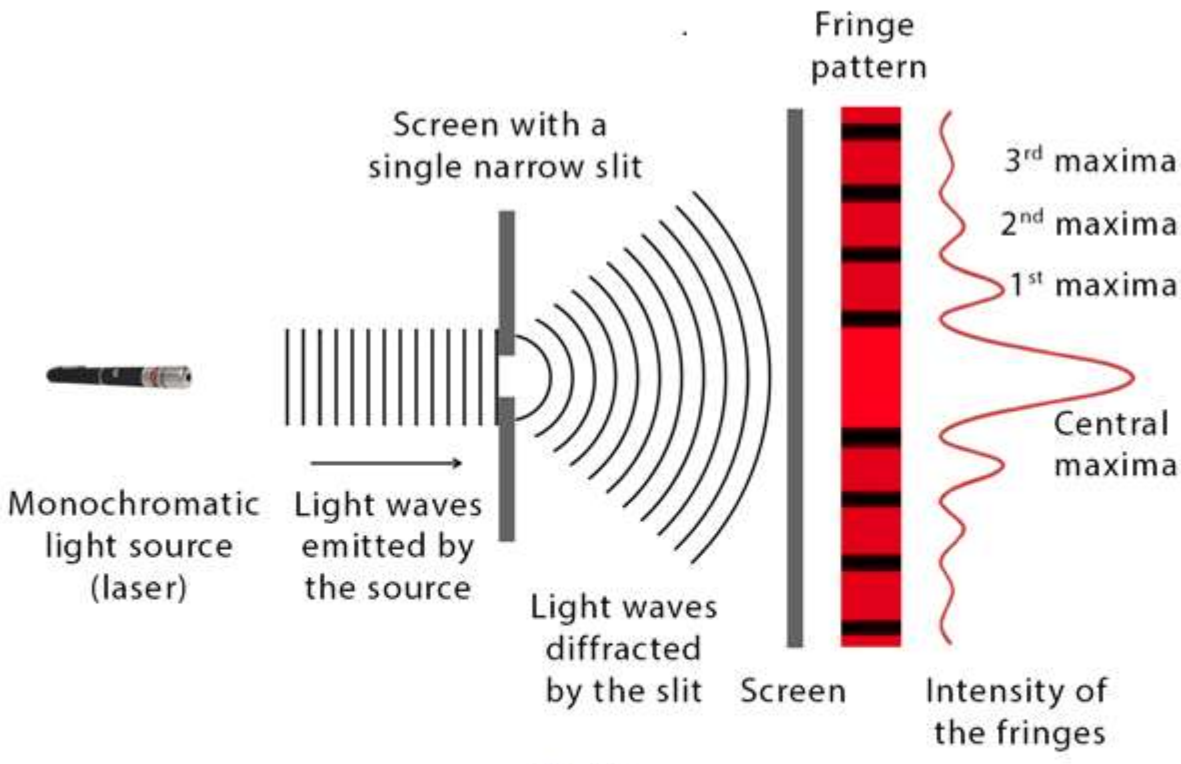
Similarly I_4, I_5, I_6,…………so on

Therefore Ratio of Secondary maxima:

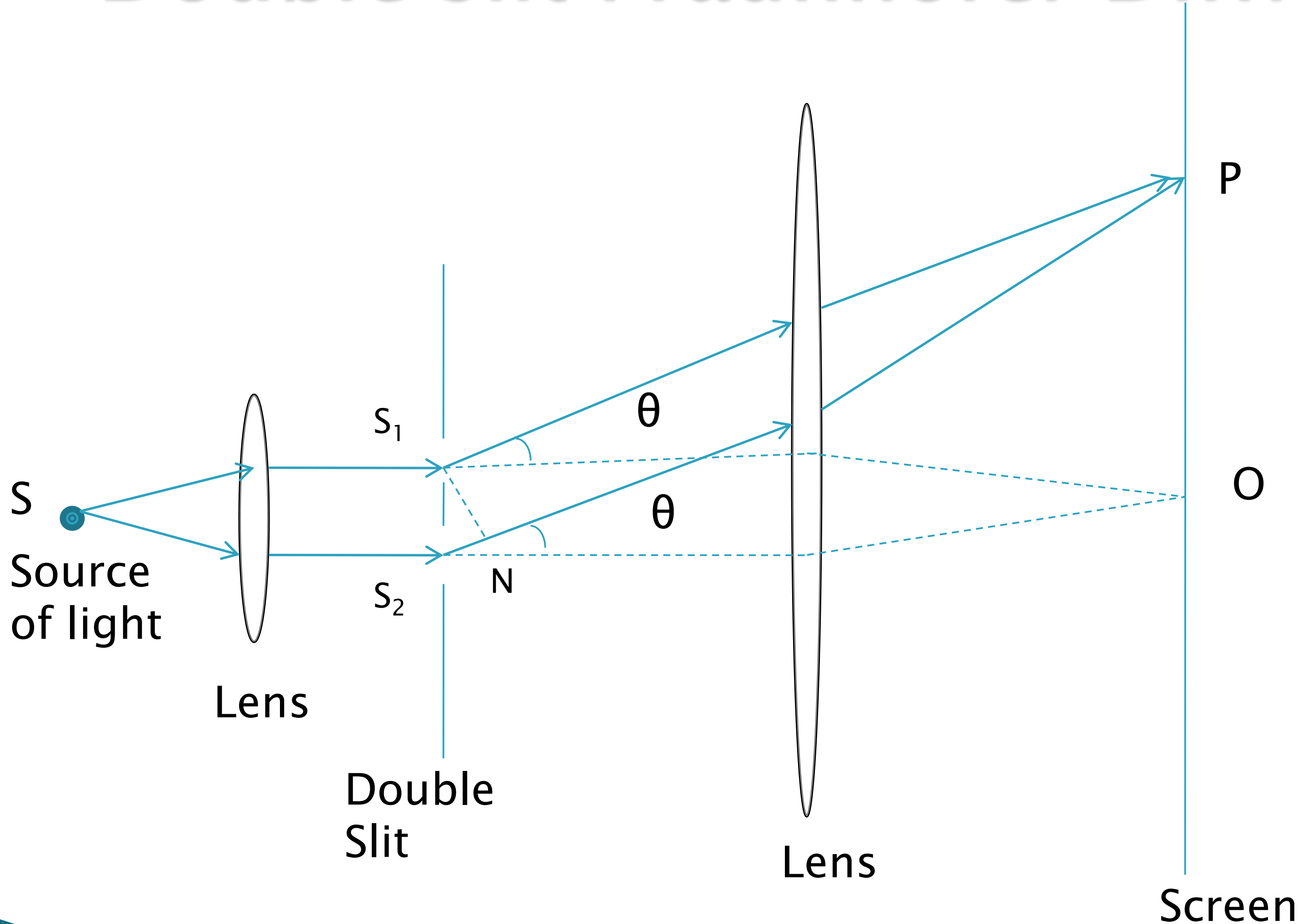
$I_1 : I_2 : I_3 : \ldots\ldots = 4/9\pi^2 : 4/25\pi^2 : 4/49\pi^2 : \ldots\ldots\ldots$



Single-Slit Diffraction



Double slit Fraunhofer Diffraction:



$S_1 S_2 = e/2+a+e/2 = e+a$

Here, Slit width $= e$
Separation between two slits $= a$
Angle of Diffraction $= \theta$

From Figure:

The path difference S_1P and $S_2P = S_2N = (e+a)\sin\theta$

Corresponding phase difference $\phi = 2\pi (S_2N)/\lambda$

$\phi = 2\pi (e+a) \sin \theta/\lambda$

$\phi = 2\beta$ -----------(1)

Where $\beta = \pi (e+a) \sin \theta/\lambda$ ------(2)

We know, for single slit

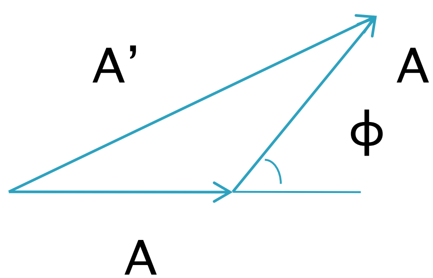
$A = R^2 \sin^2\alpha / \alpha^2$

Resultant amplitude for double slits:

$A'^2 = A^2 + A^2 + 2A A \cos \phi$

$= 2A^2 + 2A^2 \cos 2\beta$

$= 2A^2 (1 + \cos 2\beta)$



$A'^2 = 4A^2 \cos^2\beta$ ---------(3)

- Now,

$$A'^2 = 4(R^2 \sin^2 \alpha) \cos^2\beta / \alpha^2$$

and Intensity,

$$I = A'^2 = 4(R^2 \sin^2 \alpha) \cos^2\beta / \alpha^2 \text{ -------(4)}$$

<u>Here</u>

- $(R^2 \sin^2 \alpha)/\alpha^2$ term is due to single slit diffraction , so we have principle Maximum, minimum, secondary maxima
- And $4 \cos^2\beta$ is due to double slit diffraction.So we have:

<u>Case1: Maximum</u>

When $\cos^2\beta = 1$

$$\beta = m\pi \quad \text{where } m= 1, 2, 3.....$$

By eq2 we have , $\pi (e+a) \sin \theta/\lambda = m\pi$

$$(e+a) \sin \theta = m \lambda \text{ -------(5)}$$

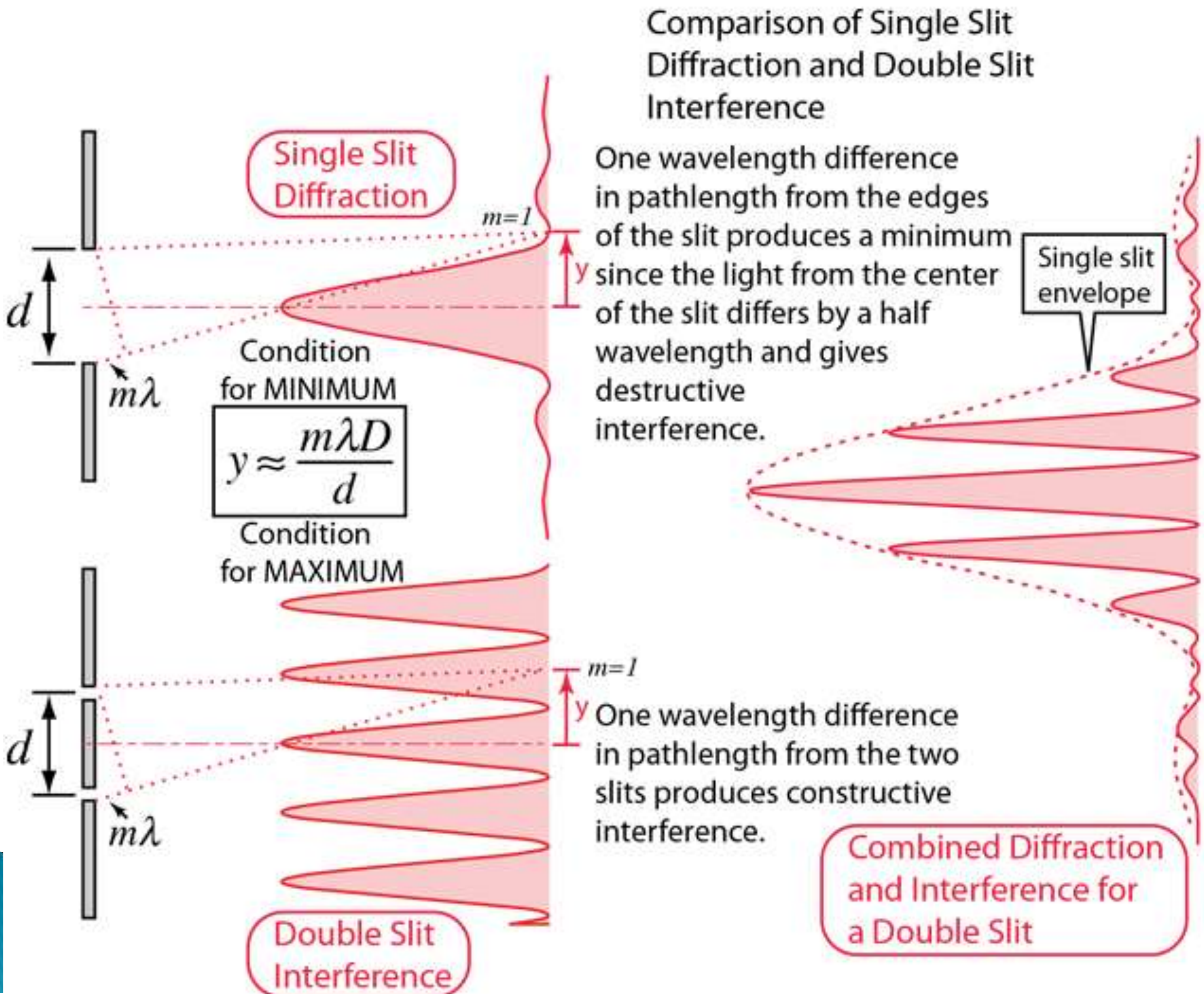
<u>Case2: Minimum</u>

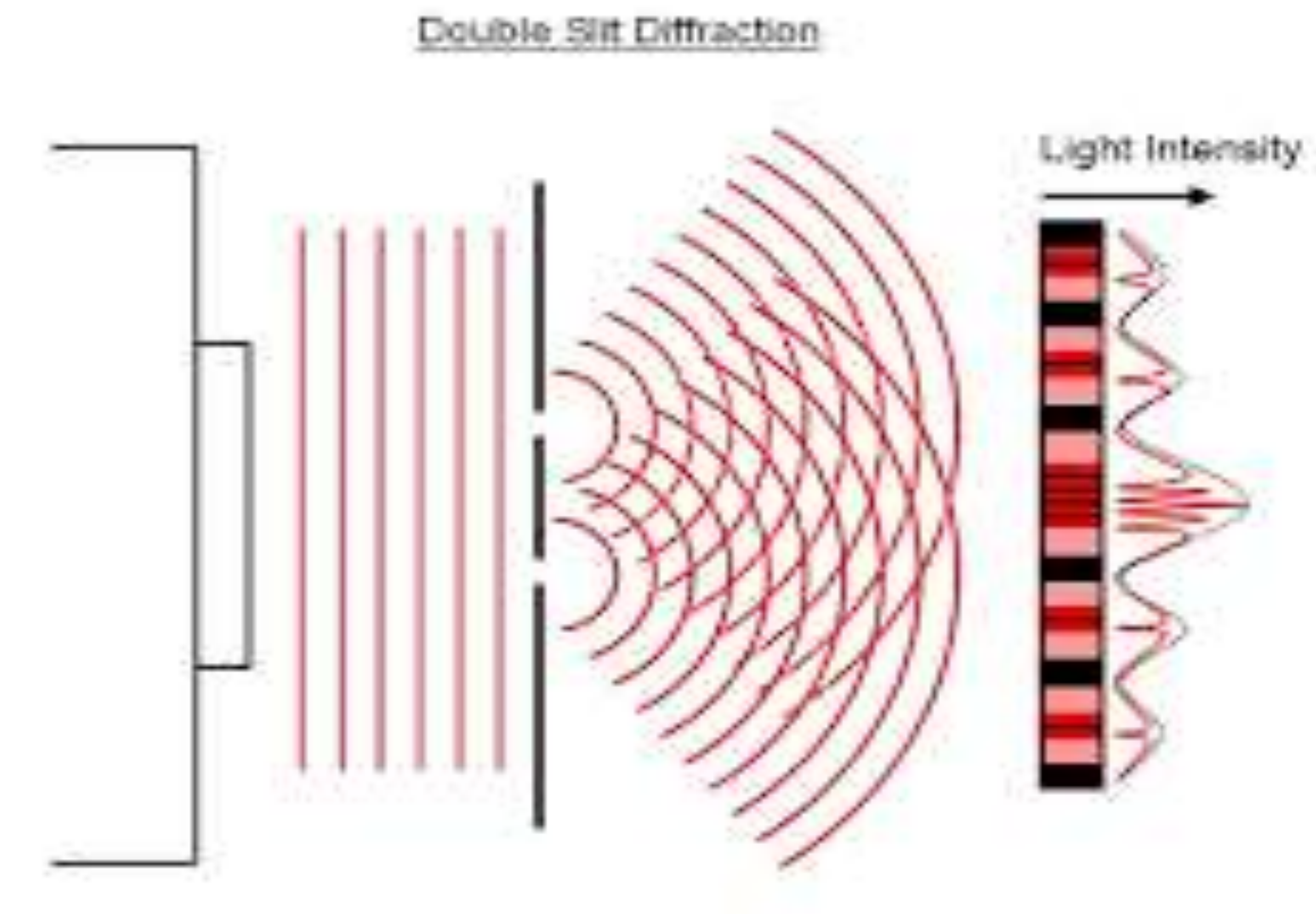
When $\cos^2\beta = 0$

$$\beta = (2m+1)\pi/2 \quad \text{where } m=0,1,2,3,......$$

By eq2 we have , $\pi (e+a) \sin \theta/\lambda = (2m+1)\pi/2$

$$(e+a) \sin \theta = (2m+1) \lambda/2 \text{ -------(6)}$$


Comparison of Single Slit Diffraction and Double Slit Interference
Single Slit Diffraction
m=1
d
mλ
y
One wavelength difference in pathlength from the edges of the slit produces a minimum since the light from the center of the slit differs by a half wavelength and gives destructive interference.
Single slit envelope
Condition for MINIMUM
$y \approx \frac{m\lambda D}{d}$
Condition for MAXIMUM
m=1
d
mλ
y
One wavelength difference in pathlength from the two slits produces constructive interference.
Combined Diffraction and Interference for a Double Slit
Double Slit Interference



Double Slit Diffraction
Light Intensity

Missing order : when condition for maximum intensity in interference (double slit) and the condition for minimum intensity in diffraction (single slit) are simultaneously satisfied then those orders are missing from the spectrum known as missing orders.

condition for maximum intensity in interference (double slit)

$$(e+a) \sin \theta = m \lambda \text{ -----(1)}$$

condition for minimum intensity in diffraction (single slit)

$$e \sin \theta = n \lambda \text{ -----(2)}$$

Eq(1)/eq(2): $(e+a) \sin \theta / e \sin \theta = m \lambda / n \lambda$

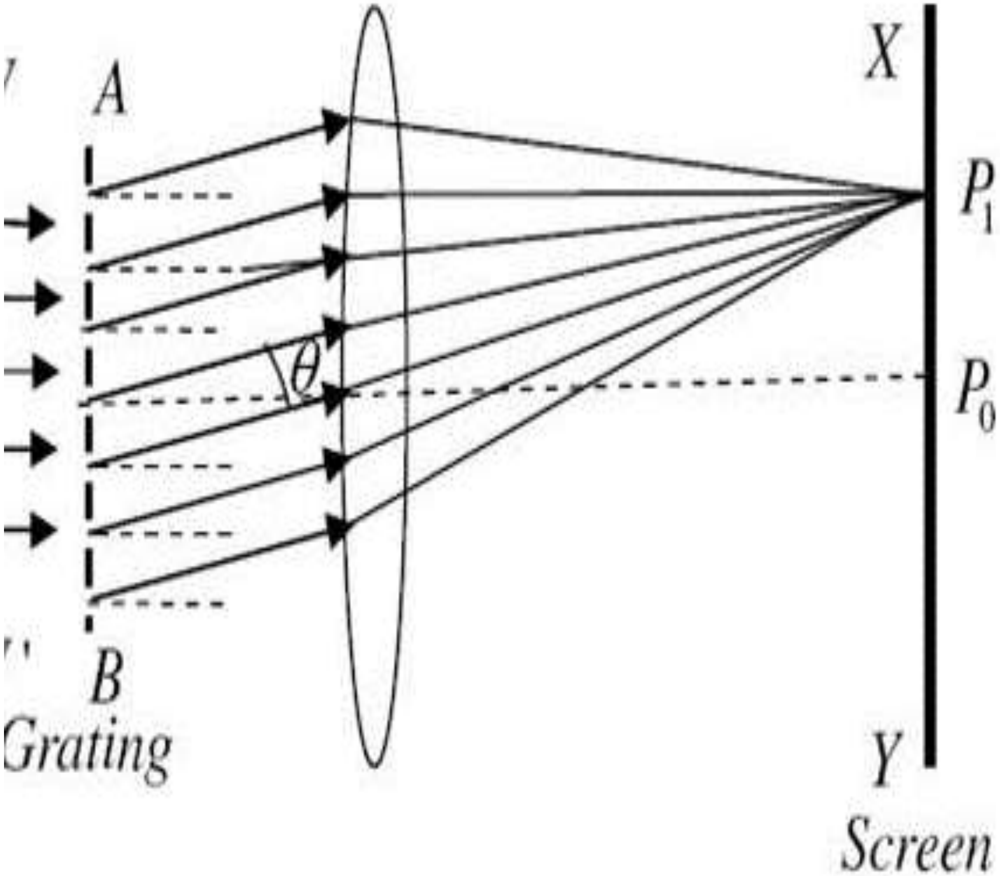
$$(e+a)/ e = m / n \text{ -----(3)}$$

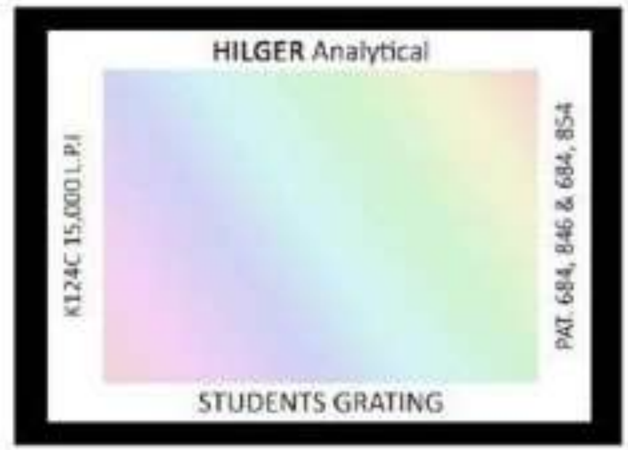
Grating or N-slit

- It consist of large numbers of parallel slits of equal width separated by opaque spaces.
- All opaque spaces have equal width.
- For lab purpose: It is constructed by drawing lines on a flat glass plate using a fine diamond point. The lines becomes opaque and the region between two lines, which is transparent acts as the slit.
- width of each Slit =e

  Separation between two slits = a

  N = Total no. of lines on grating





N-Slits system or Grating **Grating using in Lab**

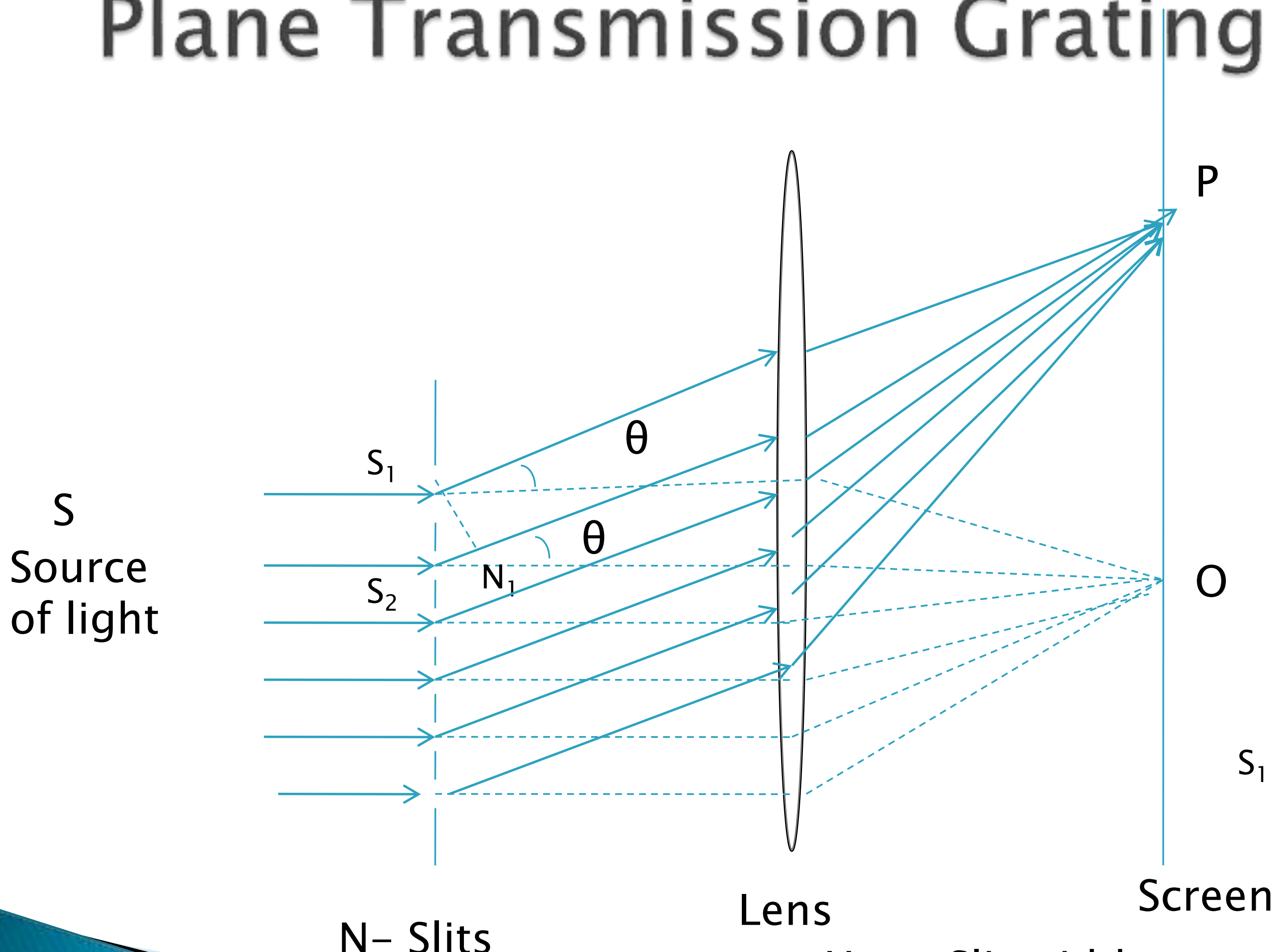
- Let ‘e’ be the width of each slit and‘a’ the width of each opaque space. Then (e+a) is known as grating element.

  Grating element = e+a = 1/ no. of lines per cm

  = 2.54/ no. of lines per inch

- The number of lines per inch of grating are written over it by the manufacturer.
- In figure XY is the screen.
- Suppose a parallel beam of monochromatic light of wavelength 'λ ' be incident normally on the grating. By Huygen’s principle, each of the slit sends secondary wavelets in all directions. Now, the secondary wavelets travelling in the direction of incident light will focus at a point P_o on the screen. This point P_o will be a central maximum.

N- slit Fraunhofer Diffraction or Plane Transmission Grating:



$S_1 S_2 = e/2+a+e/2 = e+a$

Here, Slit width $= e$
Separation between two slits $= a$
Angle of Diffraction $= \theta$

From Figure:

The path difference between two consecutive diffracted beams of light = S_2N_1 = (e+a)Sinθ

Corresponding phase difference $\phi = 2\pi (S_2N_1)/\lambda$

$\phi = 2\pi (e+a) \sin \theta/\lambda$

$\phi = 2\beta$ ------------(1)

Where $\beta = \pi (e+a) \sin \theta/\lambda$ ------(2)

Resultant amplitude for N- slits:

$A' = A \sin N\beta/\sin \beta$ -------3

We know, for single slit

$A^2 = R^2 \sin^2\alpha \,/\, \alpha^2$

Therefore, $A' = R \sin\alpha \sin N\beta/ \alpha \sin \beta$

and $A'^2 = R^2 \sin^2\alpha \sin^2 N\beta/ \alpha^2 \sin^2\beta$ ----4 and Resultant Intensity,

$$I = A'^2 = R^2 \sin^2\alpha \sin^2 N\beta / \alpha^2 \sin^2\beta \text{ -------(5)}$$

Here

- $(R^2 \sin^2 \alpha)/\alpha^2$ term is due to single slit diffraction , so we have principle Maximum, minimum, secondary maxima
- And $\sin^2 N\beta/\sin^2\beta$ is due to N– slit diffraction. So we have:

Case1: Principle Maximum

When $\sin \beta = 0$

$\beta = m\pi$ where m= 0,1, 2, 3…..

By eq2 we have , $\pi (e+a) \sin \theta/\lambda = m\pi$

$(e+a) \sin \theta = m \lambda$ -------(6)

For this case $(\sin N\beta/\sin\beta)$

$= \lim_{\beta \to m\pi} (d \sin N\beta / d \beta)/(d \sin\beta / d \beta)$

$= N$

and $I_p = A = R^2 \sin^2\alpha \, N^2/ \alpha^2$ -------(7)

Case2: Minimum

When $\sin N\beta = 0$

Where $\sin \beta \neq 0$

$$N\beta = m\pi \quad \text{where } m = 0, 1, 2, 3\ldots..$$

By eq2 we have ,

$$N \pi (e+a) \sin \theta/\lambda = m\pi$$

$$(e+a) \sin \theta = m \lambda/N \quad \text{-------}(8)$$

So Intensity ,

$$I_{min} = 0$$

Case3: Secondary Maxima,

$$dI/d\beta = 0$$

By eq.(6) we have,

$$[(R^2 \sin^2 \alpha)/\alpha^2]\{d[\sin^2 N\beta/\sin^2\beta]/d\beta\} = 0$$

On solving, we have,

$$N \cos N\beta \sin \beta - \cos\beta \sin N\beta = 0$$

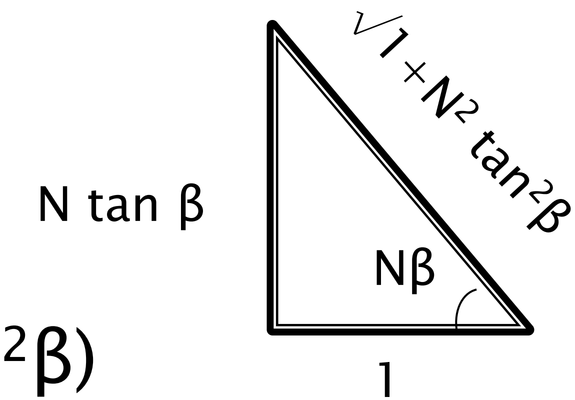
or $$\cos\beta \sin N\beta = N \cos N\beta \sin \beta$$

$$\tan N\beta = N \tan \beta$$

or $$\tan N\beta = N \tan \beta/1$$

From Figure:

$$\sin N\beta = N \tan \beta/\sqrt{(1+N^2 \tan^2\beta)}$$



$$\sin N\beta = N \tan\beta / \sqrt{(1+N^2 \tan^2\beta)}$$

$$\sin^2 N\beta = (N^2 \tan^2 \beta)/(1+N^2 \tan^2\beta)$$

$$\sin^2 N\beta/\sin^2\beta = (N^2 \tan^2 \beta)/ \sin^2\beta(1+N^2 \tan^2\beta)$$

$$\sin^2 N\beta/\sin^2\beta = N^2/ \sin^2\beta(\cot^2\beta + N^2)$$

$$\sin^2 N\beta/\sin^2\beta = N^2/ (\cos^2\beta + N^2 \sin^2\beta)$$

$$\sin^2 N\beta/\sin^2\beta = N^2/ (1-\sin^2\beta + N^2 \sin^2\beta)$$

$$\sin^2 N\beta/\sin^2\beta = N^2/ \{1 + (N^2 - 1)\sin^2\beta\}$$

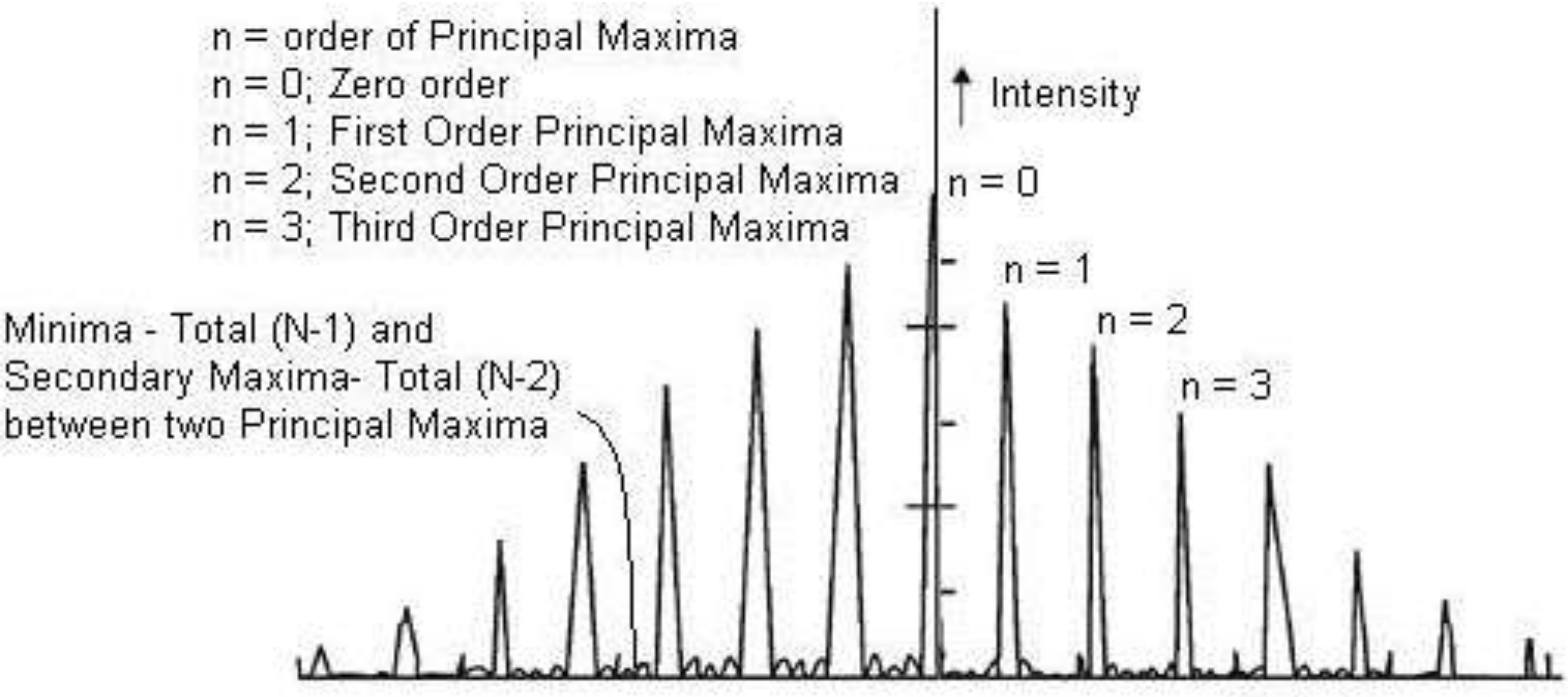
Now put this in Equation (5), we have

Intensity expression for secondary maxima:

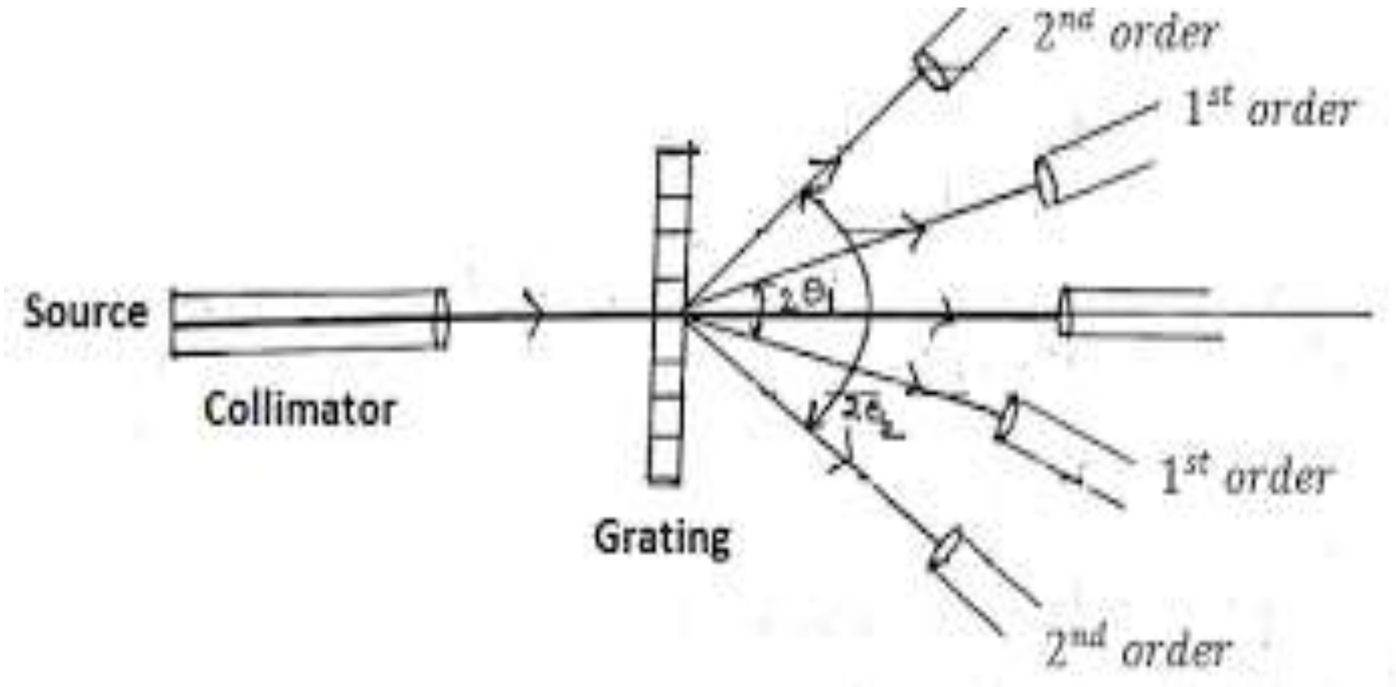
$$I_s = R^2 \sin^2\alpha \, N^2/ \alpha^2 \{1 + (N^2 - 1)\sin^2\beta\} \text{ -------(9)}$$

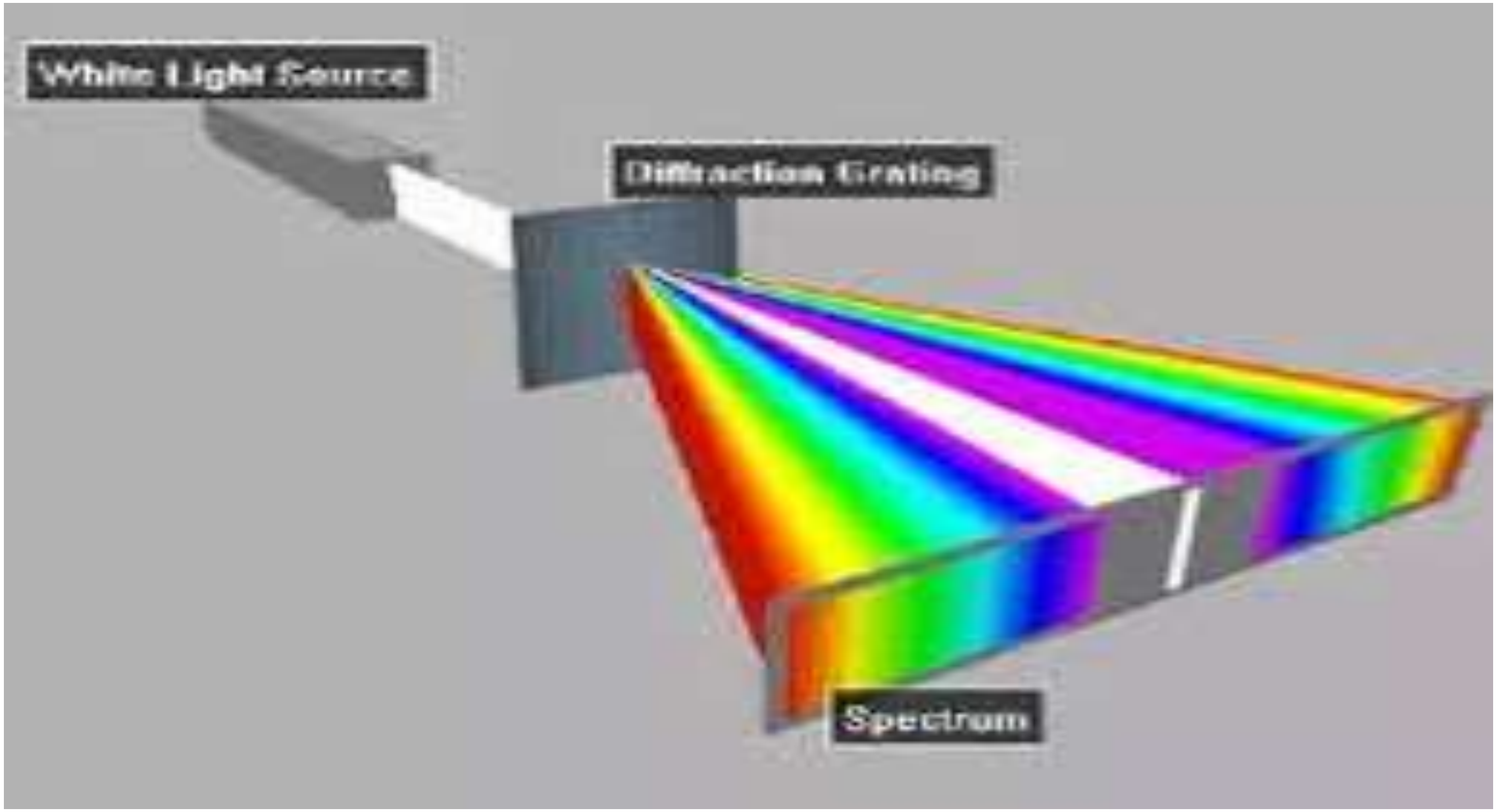
or, $I_s = I_p/\{1 + (N^2 - 1)\sin^2\beta\}$

N-slits diffraction pattern or Grating Spectra



Grating experiment in Lab




White Light Source
Diffraction Grating
Spectrum

Dispersive Power of Grating

- Rate of change in angle of diffraction with wavelength called its dispersive power.
- D.P. $= d\theta/d\lambda$ --------(1)

Derivation:

we know equation of grating is:

$$(e+a)\sin\theta = n\lambda \quad \text{-------(2)}$$

On differentiating w.r.t. λ ,

$$(e+a)\cos\theta \, d\theta/d\lambda = n$$

Or, $d\theta/d\lambda = n/(e+a)\cos\theta$

$$\text{D.P.} = d\theta/d\lambda = n/(e+a)\cos\theta$$