Microscopic Theory of Dielectric Polarizability

If the electric field E induces an average dipole moment per unit volume P, and k is the dielectric constant, is given by.

$$\boxed{k - 1 = \frac{P}{\epsilon_0 E}} \quad \text{—(1)} \qquad [\epsilon_r = k]$$

Here we are considering the case of the polarization of the gases. there are two types of molecules

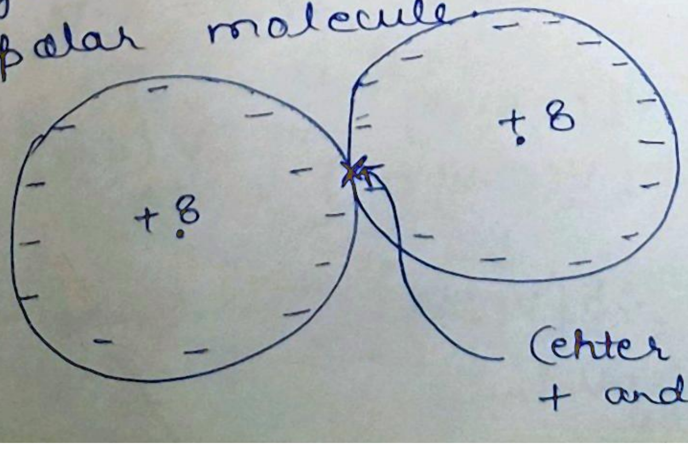
(1) Gas has a symmetric pair of atoms and molecule of there is no inherent dipole moment.

(2) Other molecules like water which has a permanent electric dipole moment.

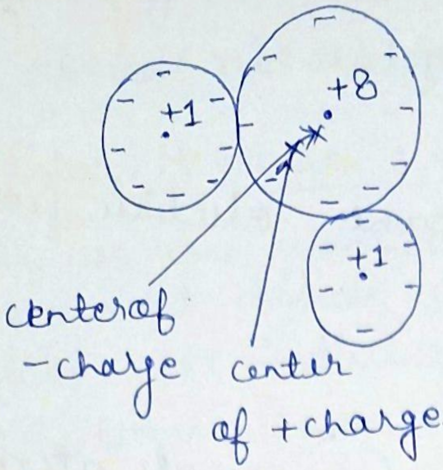
[water has a nonsymmetric arrangement of hydrogen and oxygen atoms]

Since the centre of gravity of the negative charge and the centre of gravity of the positive charge do not coincide, the total charge distribution of the molecule has a dipole moment. Such a molecule is called a polar molecule.

In oxygen, because of the symmetry of the molecule, the centers of gravity of the positive and negative charges are the same, so it is the nonpolar molecule.



@ oxygen molecule



$\swarrow p_0$

the water (H_2O) molecule has a permanent dipole moment p_0.

Electronic Polarization - In the case of a monatomic gas (as helium). When an atom of such a gas is in an electric field, the electrons are pulled one way by the field while the nucleus is pulled the other way. there is a slight net displacement of the centres of charge and a dipole moment is induced.

For the small fields, the amount of displacement of the centres of charge, and a dipole moment is induced.

and so also the dipole moment is proportional to the electric field. the displacement of the e^- distribution which provides this kind of induced dipole moment is called electronic polarization.

when an atom is placed in an oscillating electric field the centre of charge of the electron obeys the equation.

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = q_e E \quad (1)$$

↓ Restoring force

↓ force from outside electric field.

Solution of eq. ① is.

$$x = \frac{q_e E}{m(\omega_0^2 - \omega^2)} \quad -②$$

In the case of constant electric field

$\omega = 0$

$$x = \frac{q_e E}{m\omega_0^2} \quad -③$$

the dipole moment p of a single atomis.

$$p = q_e x = \frac{q_e^2 E}{m\omega_0^2} \quad -④$$

the dipole moment $\vec{p}$ is proportional to $\vec{E}$.

$$\vec{p} = \alpha E_0 E \quad -⑤$$

α = polarizability
has dimension of L^3.

$$\alpha = \frac{q_e^2}{\epsilon_0 m\omega_0^2} = \frac{4\pi e^2}{m\omega_0^2} \quad -⑥$$

If there are N atoms in a unit volume the polarization P - the dipole moment per unit volume.

$$\vec{P} = N\vec{p} = N\alpha \epsilon_0 E \quad -⑦$$

$$K - 1 = \frac{P}{\epsilon_0 E} = N\alpha.$$

$$K \cancel{\epsilon_A} - 1 = \frac{P}{\epsilon_0 E} = \frac{4\pi N e^2}{m\omega_0^2}$$

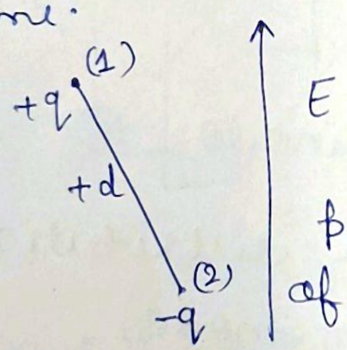
the dielectric constant K of different gases should depend on the density of the gas and on the frequency ω_0 of the optical absorption.

Polar Molecules : orientation polarization -

If we are considering a molecule which carries a permanent dipole moment p_0. with no electric field, the individual dipoles point in random directions, so the net moment per unit volume is zero. But if we have applied an electric field, then two conditions arise-

① there is an extra dipole moment induced because of the forces on the electrons. which gives the same kind of electronic polarizability as in case of nonpolar molecule.

② the electric field tends to line up the individual dipoles to produce a net moment per unit volume.



consider a dipole of moment p_0 in an electric field. the energy of the positive charge is $q\phi(1)$ and the negative charge energy is $-q\phi(2)$.

So the energy of the dipole is.

$$U = q\phi(1) - q\phi(2) = q\vec{d}\cdot\vec{\nabla}\phi$$

$$U = -\vec{p_0}\cdot\vec{E} = -p_0 E\cos\theta \quad (1)$$

the energy is lower when the dipoles are lined up with the field.

In the thermal equilibrium the relative number of molecules with the potential energy U is proportional to $e^{U/kT}$ (2)

If $n(\theta)$ be the number of molecules per unit solid angle at θ, we have-

$$n(\theta) = n_0 e^{+p_0 E \cos\theta / kT} \quad -(3)$$

$$n(\theta) = n_0 \left(1 + \frac{p_0 E \cos\theta}{kT}\right) \quad (4)$$

the average value of $\cos\theta$ over all angles is zero, so the integral is just n_0 times the total solid angle 4π,

$$n_0 = \frac{N}{4\pi} \quad -(5)$$

to calculate polarization P (net dipole moment per unit volume)

we take the vector sum of all the molecular moments in a unit volume.

$$P = \sum_{\substack{\text{unit} \\ \text{volume}}} p_0 \cos\theta_i$$

$$P = \int_0^{\pi} n(\theta)\, p_0 \cos\theta \; 2\pi \sin\theta \, d\theta \quad -(6)$$

(integrating over angular distribution, the solid angle at θ is $2\pi \sin\theta\, d\theta$)

$$P = \int_0^{\pi} n(\theta)\, p_0 \cos\theta \; 2\pi \sin\theta \, d\theta.$$

$$P = \int_0^{\pi} \left(n_0 + \frac{n_0 p_0 E \cos\theta}{kT}\right) p_0 \cos\theta \; 2\pi \sin\theta \, d\theta$$

$$P = \frac{N p_0^2 E}{3kT} \quad -(7)$$

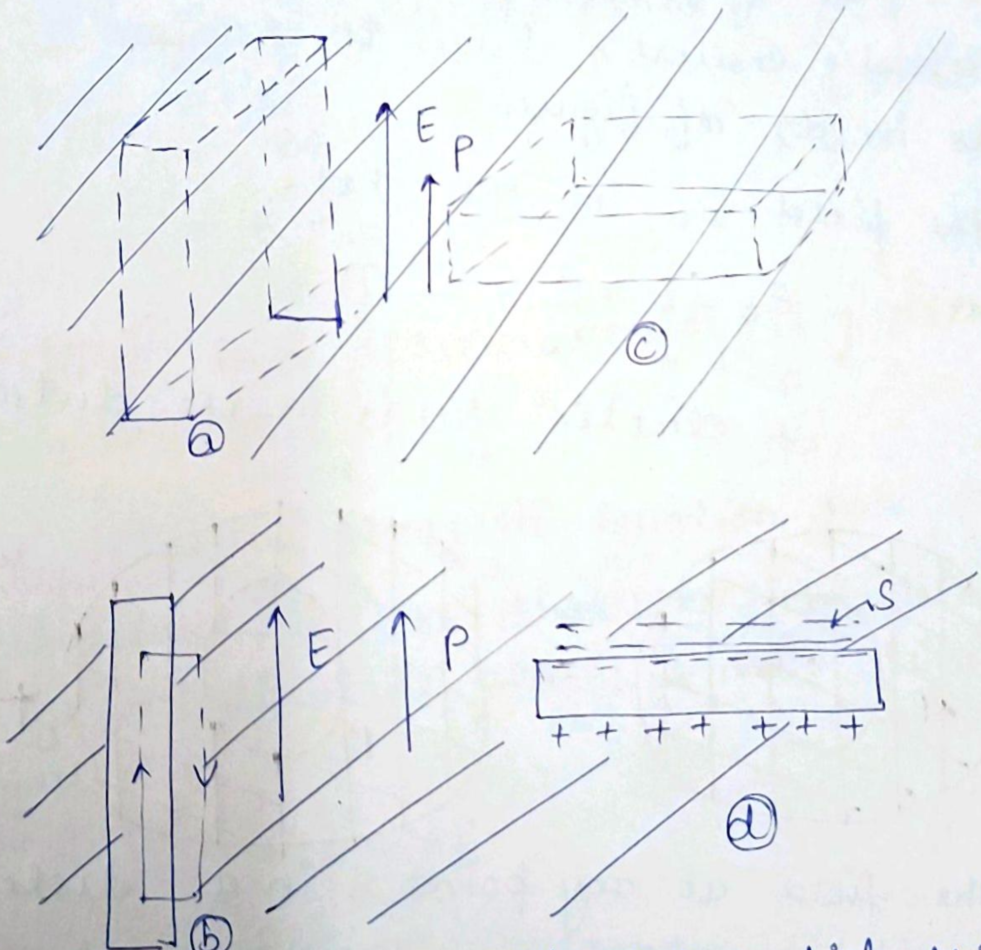
$$P \propto E \; , \; P \propto \frac{1}{T}$$

$$k - 1 = \frac{P}{\epsilon_0 E} = \frac{N p_0^2}{3 \epsilon_0 k T} \quad -(8)$$

the permanent moment p_0 appears squared. for these reasons.

① In the given electric field, the aligning force depends upon p_0 and the mean moment which is produced by the liningup is again proportional to p_0. the average induced moment is proportional to p_0^2.

the behaviour of the electric field in the cavities of a dielectric →



the field in a slot cut in a dielectric depends on the shape and orientation of the slot.

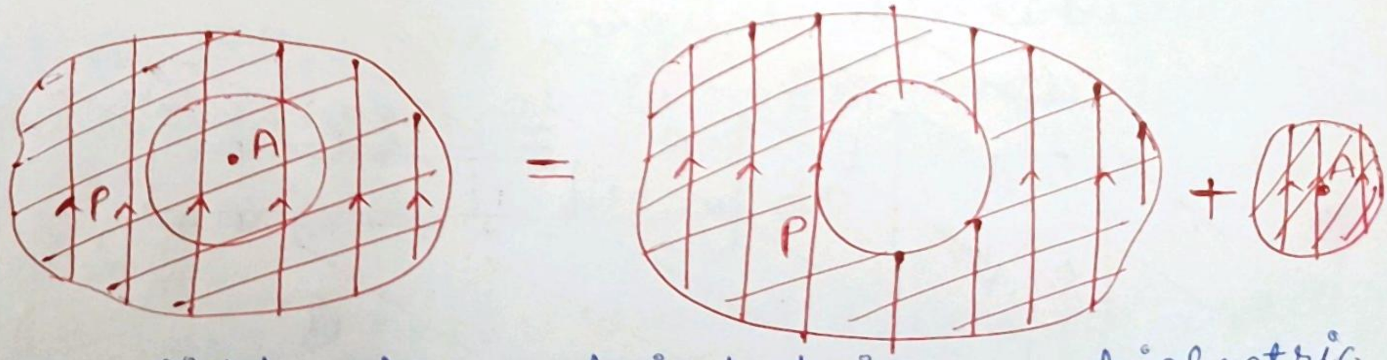
Suppose if we cut a slot in a polarized dielectric with the slot oriented parallel to the field (a), Since $\nabla \times \vec{E} = 0$ the line integral of $\vec{E}$ around the ~~cavity~~ curve Γ. the field inside the slot must give a contribution which just cancels the part from the field outside. therefore the field E_0 actually found in the centre of a long thin slot is equal to E, the average electric field found in dielectric.

in (C) another slot whose large sides are $\perp$ to E, in this case, the field E_0 in the slot is not the same as E because polarization charges appear on the surfaces. If we apply Gauss's law to surface S drawn as in (d) of figure.

the field E_0 in the slot-

$$E_0 = E + \frac{P}{\varepsilon_0} \quad — (1)$$

E electric field in the dielectric.



the field at any point A in a dielectric can be considered as the sum of the field in a spherical hole plus the field due to a spherical plug.

If we imagine carving out a spherical hole in a uniformly polarized material, we remove a sphere of polarized material. the fields inside the dielectric, before the sphere was removed, is the sum of the fields from all charges outside the spherical volume plus the fields from the charges within the polarized sphere.

In the uniform dielectric,

$$E = E_{hole} + E_{plug} \quad -② $$

is the field inside a sphere which is uniformly polarized.

the electric field inside the sphere is uniform

$$E_{plug} = \frac{-P}{3\epsilon_0} \quad -③$$

$$E_{hole} = E + \frac{P}{3\epsilon_0} \quad -④$$

the field in a spherical cavity is greater than the average field by the amount $\frac{P}{3\epsilon_0}$.

the Clausius-Mossotti Equation →

In a liquid we expect that the field which will polarize an individual atom is more like E_{hole} than just E.

$$p = \alpha \epsilon_0 E$$

$$P = Np = N\alpha \epsilon_0 E$$

$$\vec{P} = N\alpha \epsilon_0 \left(E + \frac{P}{3\epsilon_0} \right) \quad -⑤$$

$$P = \frac{N\alpha}{1-(N\alpha/3)} \epsilon_0 E \quad -⑥$$

Remembering that $k-1$ is just $\frac{P}{\epsilon_0 E}$, we have

$$\boxed{k-1 = \frac{N\alpha}{1-\left(\frac{N\alpha}{3}\right)}} \quad -(7)$$

this gives us the dielectric constant of a liquid in terms of α, the atomic polarizability. this is called the Clausius-Mossotti equation.

If $N\alpha$ is very small, as it is for a gas (because the density N is small) then the term $\frac{N\alpha}{3}$ can be neglected compared with 1. and we get

$$\boxed{k-1 = N\alpha} \quad -(8)$$