The Electric Field -

If we have several point charges $q_1, q_2, \cdots q_n$ at distances $r_1, r_2, \cdots r_n$ from Q, the total force on Q is evidently

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \cdots\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1 \hat{r}_1}{r_1^2} + \frac{q_2 \hat{r}_2}{r_2^2} + \frac{q_3 \hat{r}_3}{r_3^2}\right)$$

$$\boxed{\vec{F} = Q\vec{E}}$$

$$\boxed{\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$

$\vec{E}$ is called the electric field of the source charges. Here Q is the test charge / source charge.

$\vec{E}(r)$ is the function of position($\vec{r}$) because the separation vector $\vec{r}_i$ depend on the location of the field Point P.

that the electric field is a vector quantity that varies from point to point and is determined by the configuration of source charges.

Continuous Charge Distributions -

If the charge is distributed continuously over some region, the sum becomes an integral.

$$\boxed{\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r^2}\hat{r}\, dq}$$

$dq \Rightarrow \lambda\, dl' \sim \sigma\, da' \sim \rho\, d\tau'$

charge per unit length, charge per unit area, charge per unit volume the electric field ...

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int_P \frac{\lambda(r')}{r^2} \hat{r} \, dl'$$

for a surface charge

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma(r')}{r^2} \hat{r} \, da'$$

and for a volume charge

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(r')}{r^2} \hat{r} \, d\tau'$$

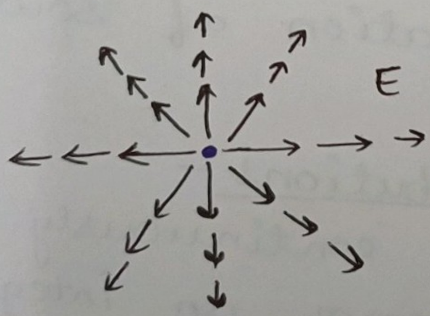
Divergence and Curl of Electrostatic Fields-

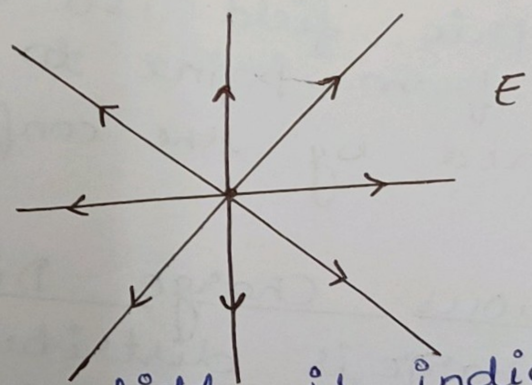
Field Lines, Flux and Gauss's law-

A single point charge q, situated at the origin.

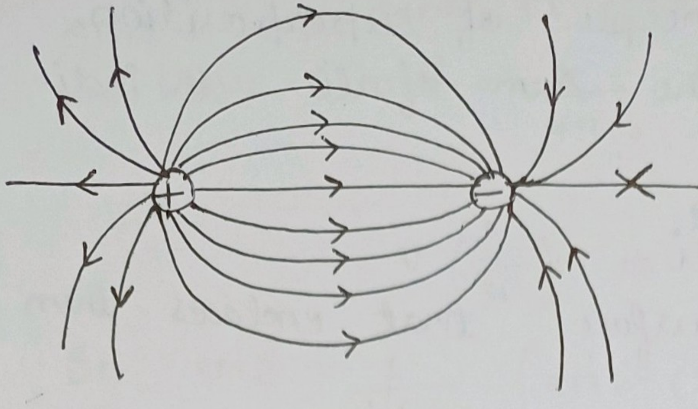
$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

the field falls off like $\frac{1}{r^2}$ the vector get shorter as we go farther away from the origin, they always point radially outward.

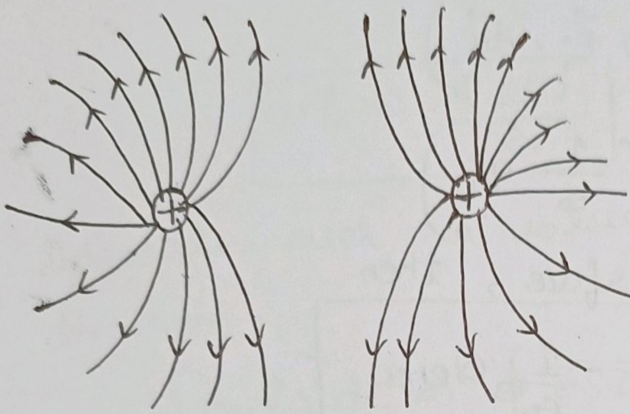




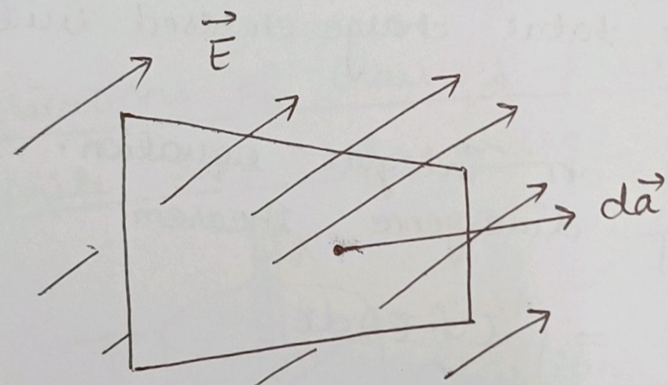
the magnitude of the field is indicated by the density of the (arrows) & field lines, it's strong near the center while the field lines are close together, and weak farther out, where are relatively far apart.



(Equal and opposite charges)



(Equal charges)



the flux of $\vec{E}$ through a surface S

$$\boxed{\Phi_E = \int_S \vec{E} \cdot d\vec{a}}$$

the flux is proportional to the no. of lines drawn, because the field strength, is proportional to the density of field lines. and hence $\vec{E} \cdot d\vec{a}$ is proportional to the number of lines passing through the infinitesimal area da.

the flux through any surface enclosing the charge is q/ϵ_0.

According to the principle of superposition, the total field is the sum of all the individual fields:

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i$$

the flux through a surface that encloses them all, then is,

$$\oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^{n} \left(\oint \vec{E}_i \cdot d\vec{a} \right)$$

$$= \sum_{i=1}^{n} \left(\frac{1}{\epsilon_0} q_i \right)$$

for any closed surface, then.

$$\boxed{\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc.}}$$

where $Q_{enc.}$ is the total charge enclosed within the surface.

Gauss's law is an integral equation.

By applying divergence theorem.

$$\oint_S \vec{E} \cdot d\vec{a} = \int_v (\nabla \cdot E) \, d\tau$$

$Q_{enc.}$ in terms of the charge density ρ.

$$Q_{enc} = \int_v \rho \, d\tau$$

$$\int_v (\nabla \cdot E) \, d\tau = \int_v \left(\frac{\rho}{\epsilon_0} \right) d\tau$$

the integrands must be equal.

$$\boxed{\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho}$$

It is the Gauss's law is the diffential

The Divergence of $\vec{E}$ $\rightarrow$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int_{\text{all space}} \frac{\hat{r}}{r^2} \rho(r') \, d\tau'$$