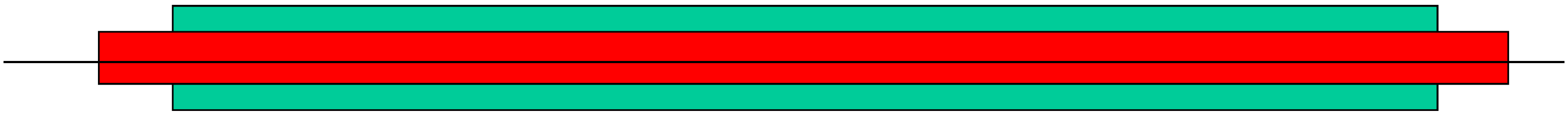
Temperature Profiles in Heat Exchangers

Thermodynamics & Heat Transfer Analysis of Infinitesimal Heat Exchanger



$$dT_h = -\frac{\delta \dot{Q}_{loss}}{\dot{m}_h c_{p,h}} \qquad dT_c = \pm \frac{\delta \dot{Q}_{gain}}{\dot{m}_c c_{p,c}}$$

$$d\left(\Delta T_{comm}\right) = d\left(T_h - T_c\right) = dT_h - dT_c$$

$$d\left(\Delta T_{comm}\right) = \left[\frac{-\delta \dot{Q}_{loss}}{\dot{m}_h c_{p,h}} - \frac{\pm \delta \dot{Q}_{gain}}{\dot{m}_c c_{p,c}}\right]$$

Heat Transfer in an infinitesimal HX

$$\delta \dot{Q}_{loss} = \delta \dot{Q}_{gain} = U\left(T_h - T_c\right) dA$$

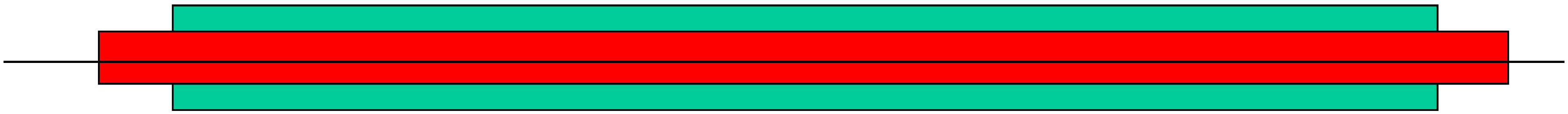
Synergism between HT & TD:

$$d\left(\Delta T_{comm}\right) = -U\left(T_h - T_c\right)dA\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right]$$

$$d\left(\Delta T_{comm}\right) = -U\left(\Delta T_{comm}\right)dA\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right]$$

$$\frac{d\left(\Delta T_{comm}\right)}{\left(\Delta T_{comm}\right)} = -UdA\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right]$$

For A finite HX:



$$\int_{\Delta T_{comm,1}}^{\Delta T_{comm,2}} \frac{d\left(\Delta T_{comm}\right)}{\left(\Delta T_{comm}\right)} = -U\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right]\int_0^A dA$$

$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -U\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right]A$$

For A finite HX:

$$\dot{m}_{cold} c_{p,c} \left(T_{c,out} - T_{c,in}\right) = \dot{Q} = \dot{m}_{hot} c_{p,h} \left(T_{h,in} - T_{h,out}\right)$$

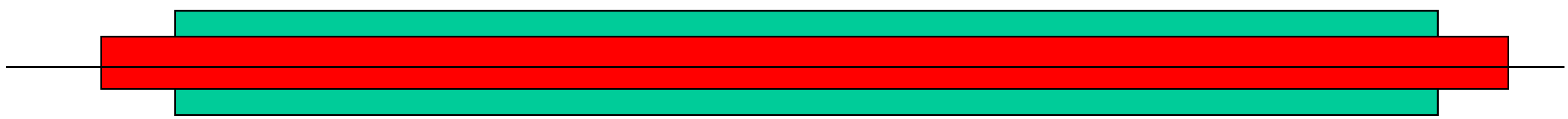
$$\frac{1}{\dot{m}_{cold} c_{p,c}} = \frac{\left(T_{c,out} - T_{c,in}\right)}{\dot{Q}} \quad \& \quad \frac{1}{\dot{m}_{hot} c_{p,h}} = \frac{\left(T_{h,in} - T_{h,out}\right)}{\dot{Q}}$$

$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -U\left[\frac{1}{\dot{m}_h c_{p,h}} + \frac{\pm 1}{\dot{m}_c c_{p,c}}\right] A$$

$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -\frac{U}{\dot{Q}}\left[\left(T_{h,in} - T_{h,out}\right) + \left\{\pm\left(T_{c,out} - T_{c,in}\right)\right\}\right] A$$

$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -\frac{U}{\dot{Q}}\left[\left(T_{h,in} - T_{h,out}\right) + \left\{\pm\left(T_{c,out} - T_{c,in}\right)\right\}\right]A$$

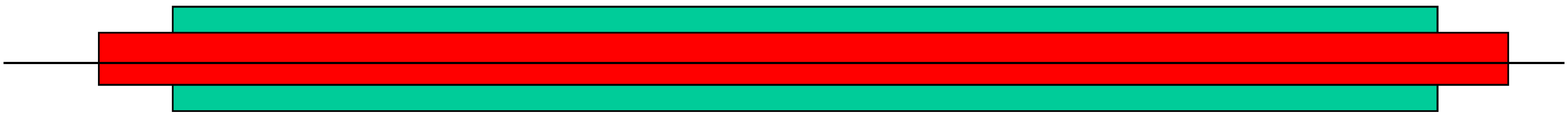
Parallel flow configuration



$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -\frac{U}{\dot{Q}}\left[\left(\Delta T_{comm,1} - \Delta T_{comm,2}\right)\right]A$$

$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -\frac{U}{\dot{Q}}\left[\left(T_{h,in} - T_{h,out}\right) + \left\{\pm\left(T_{c,out} - T_{c,in}\right)\right\}\right]A$$

Counter flow configuration



$$\ln\left[\frac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right] = -\frac{U}{\dot{Q}}\left[\left(\Delta T_{comm,1} - \Delta T_{comm,2}\right)\right]A$$

Capacity of A Finite Simple Heat Exchanger

$$\frac{\dot{Q}}{UA} = \frac{\left[\left(\Delta T_{comm,2} - \Delta T_{comm,1}\right)\right]}{\ln\left[\dfrac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right]}$$

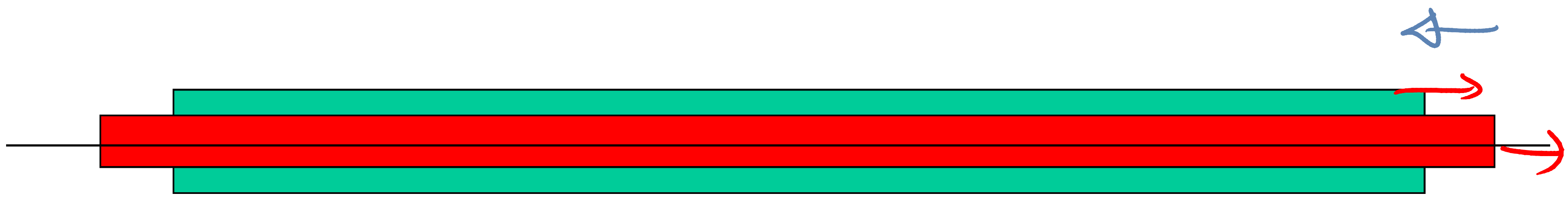
A representative temperature difference for heat communication:

$$\Delta T_{LM} = \frac{\left(\Delta T_{comm,2} - \Delta T_{comm,1}\right)}{\ln\left[\dfrac{\Delta T_{comm,2}}{\Delta T_{comm,1}}\right]}$$

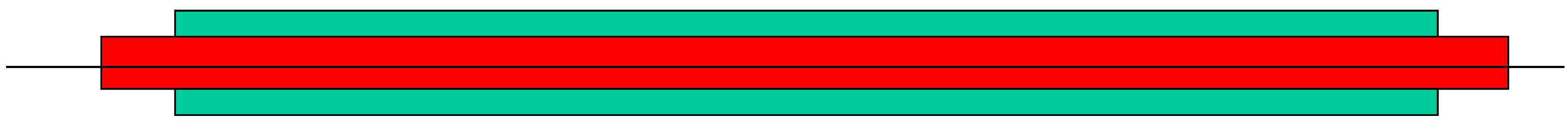
Discussion on LMTD

- This LMTD is valid of simple tube in tube HXs only.
- LMTD can be easily calculated, when the fluid inlet temperatures are know and the outlet temperatures are specified.
- Lower the value of LMTD, higher the value of overall value of UA.
- For given end conditions, counter flow gives higher value of LMTD when compared to co flow.
- Counter flow generates more temperature driving force with same entropy generation.
- This nearly equal to mean of many local values of ΔT.

Counter flow generates more temperature driving force with same entropy generation.

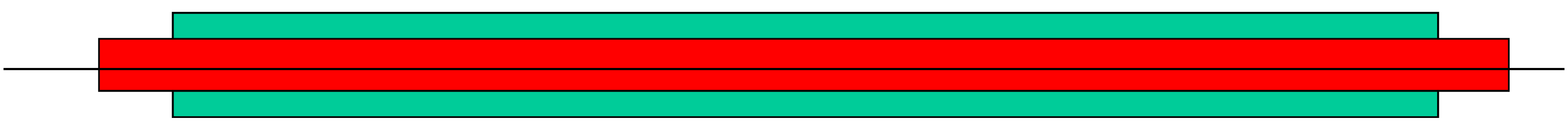


LMTD is nearly equal to mean of many local values of ΔT



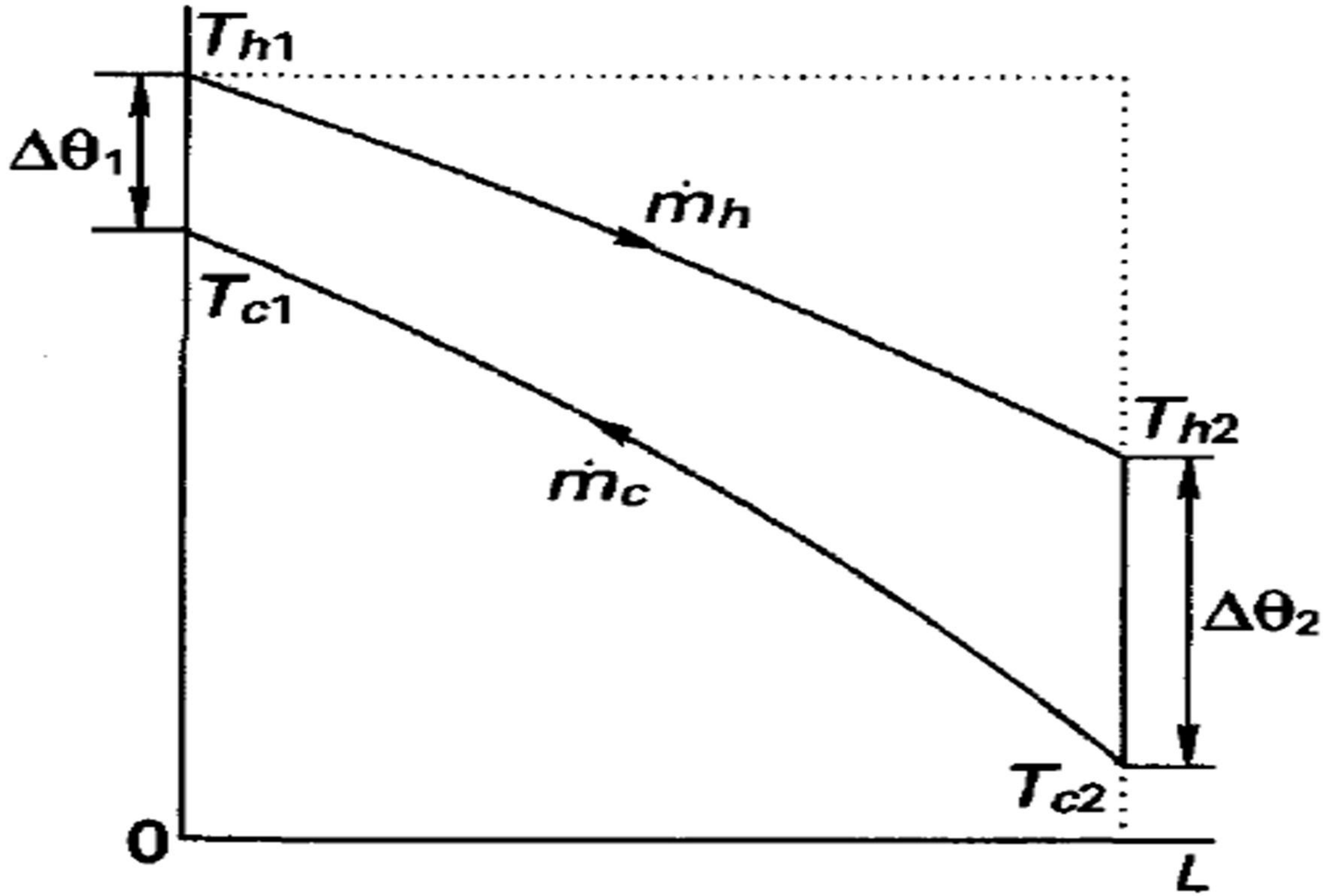
The Problem of Sizing of A Heat Exchanger

Contraflow	*Parallel flow*
Given: $\dot{Q}$ (*duty*) $\dot{m}_h$, C_h, T_{h1} $\dot{m}_c$, C_c, T_{c2}	Given: $\dot{Q}$ (*duty*) $\dot{m}_h$, C_h, T_{h1} $\dot{m}_c$, C_c, T_{c1}
Find: *geometry*	Find: *geometry*



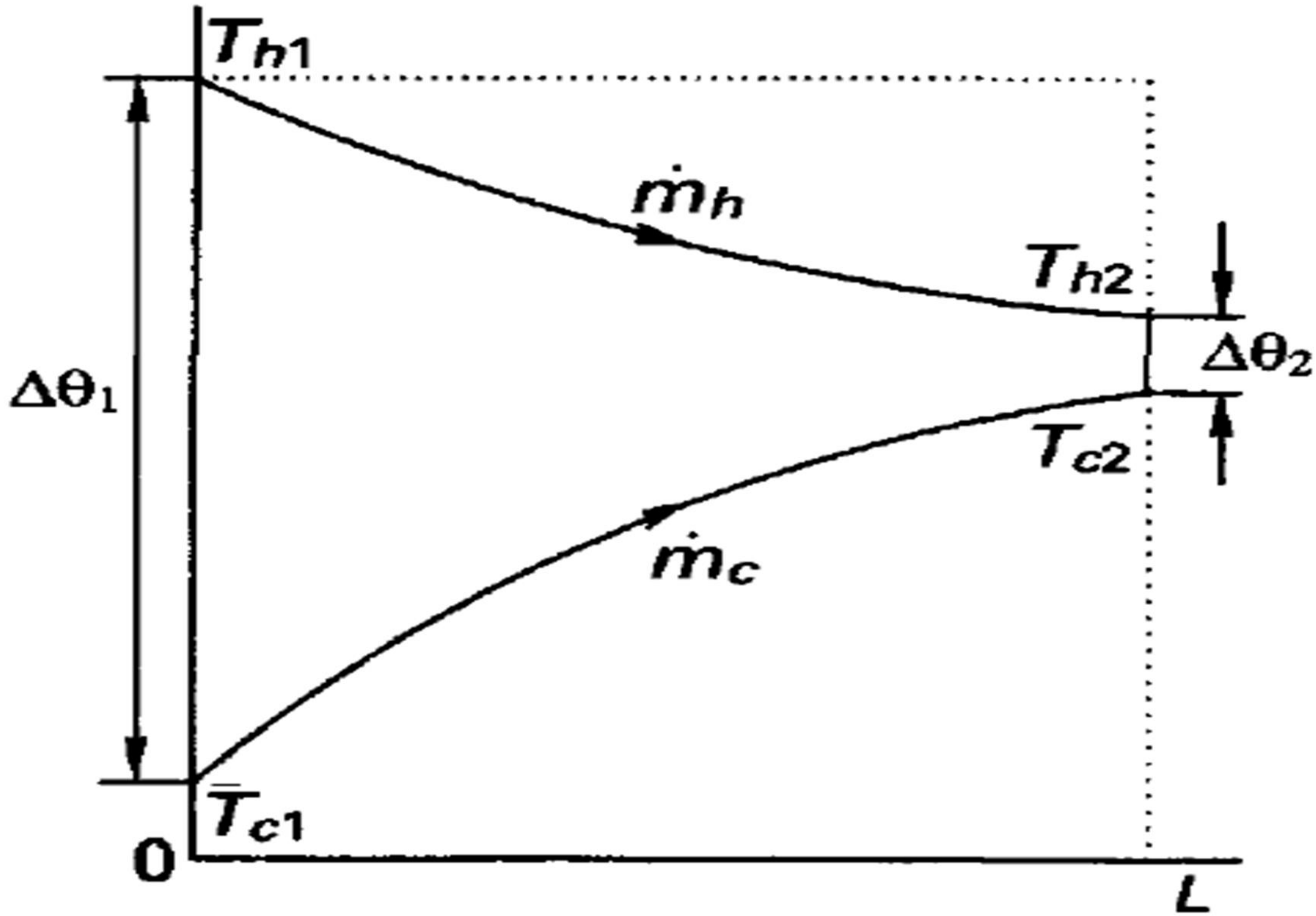
The Problem of Finding Capacity

Contraflow	*Parallel flow*
Given: *geometry* $\dot{m}_h$, C_h, T_{h1} $\dot{m}_c$, C_c, T_{c2}	Given: *geometry* $\dot{m}_h$, C_h, T_{h1} $\dot{m}_c$, C_c, T_{c1}
Find: $\dot{Q}$ *(duty)*	Find: $\dot{Q}$ *(duty)*



Contraflow profiles

$$\delta\dot{Q} = -\dot{m}_{cold}c_{p,c}dT_c = -\dot{m}_{hot}c_{p,h}dT_h = UdA(T_h - T_c)$$



Parallel flow profiles

$$\delta\dot{Q} = \dot{m}_{cold} c_{p,c} dT_c = -\dot{m}_{hot} c_{p,h} dT_h = UdA\left(T_h - T_c\right)$$