collinear array is usually mounted vertically in order to increase overall gain and directivity in the horizontal direction. Stacking of dipole antennas in the fashion of doubling their number with proper phasing produces a 3 dB increase in directive gain.

Parasitic Arrays-In some way it is similar to broad side array, but only one element is feddirectly from source, other element arc electromagnetically coupled because of its proximity to the feed element. Feed element is called driven element while other elements are called parasitic elements. A parasitic element lengthened by $5 \%$ to driven element act as reflector and another element shorted by $5 \%$ acts as director. Reflector makes the radiation maximum in perpendicular direction toward driven element and direction helps in making maximum radiation perpendicular to next parasitic element. The simplest parasitic array has three elements: reflector, driven element and director, and is used, for example in Yagi-Uda array antenna. The phase and amplitude of the current induced in a parasitic element depends upon its tuning and the spacing between elements and driven element to which it is coupled. Variation in spacing between driven element and parasitic elements changes the relative phases and this proves to be very convenient. It helps in making the radiation pattern unidirectional. A distance of $\lambda / 4$ and phase difference of $\pi / 2$ radian provides a unidirectional pattern. A properly designed parasitic array with spacing $0.1 \lambda$ to $0.15 \lambda$ provides a frequency bandwidth of the order of $2 \%$, gain of the order of 8 dB and FBR of about 20 dB . It is of great practical importance, especially at higher frequencies between 150 and 100 MHz , for Yagi array used for TV reception.

The simplest array configuration is array of two point sources of same polarization and separated by a finite distance. The concept of this array can also be extended to more number of elements and finally an array of isotropic point sources can be formed.

Based on amplitude and phase conditions of isotropic point sources, there are three types of arrays:
(a) Array with equal amplitude and phases
(b) Array with equal amplitude and opposite phases
(c) Array with unequal amplitude and opposite phases

## Two Point Sources with Currents Equal in Magnitude and Phase



Fig. 5 Two element array
Consider two point sources $A_{1}$ and $A_{2}$, separated by distance $d$ as shown in the Fig. 5. Consider that both the point sources are supplied with currents equal in magnitude and phase. Consider point $P$ far away from the array. Let the distance between point $P$ and point sources $A_{1}$ and $A_{2}$ be $r_{1}$ and $r_{2}$ respectively. As these radial distances are extremely large as compared with the distance of separation between two point sources i.e. d, we can assume,
$r_{1}=r_{2}=r$
The radiation from the point source $A_{2}$ will reach earlier at point $P$ than that from point source $A_{1}$ because of the path difference. The extra distance is travelled by the radiated wave from point source $A_{1}$ than that by the wave radiated from point source $A_{2}$.

Hence path difference is given
by, Path difference $=d \cos \not \subset$
The path difference can be expressed in terms of wavelength as, Path difference $=(\mathrm{d} \cos$ © $/ / \lambda \ldots$...(2)
Hence the phase angle $v$ is given by,
Phase angle $w=2 \pi$ (Path difference)

$$
\begin{array}{ll}
\therefore & \left.\psi=2 \pi\left(\frac{\mathrm{~d} \cos \phi}{\lambda}\right) \right\rvert\, \\
\therefore & \psi=\frac{2 \pi}{\lambda} \mathrm{~d} \cos \phi \mathrm{rad} \tag{3}
\end{array}
$$

But phase shift $\beta=2 \pi / \lambda$, thus equation (3) becomes,
$\therefore \quad \psi=\beta \mathrm{d} \cos \phi \mathrm{rad}$
Let $E_{1}$ be the far field at a distant point $P$ due to point source $A_{1}$. Similarly let $E_{2}$ be the far field at point $P$ due to point source $A_{2}$. Then the total field at point $P$ be the addition of the two field components due to the point sources $A_{1}$ and $A_{2}$. If the phase angle between the two fields is $v=\beta \mathrm{d} \cos v$ then the far field component at point $P$ due to point source $A_{1}$ is given by,

$$
\begin{equation*}
E_{1}=E_{0} \cdot e^{-i \frac{\psi}{2}} \tag{5}
\end{equation*}
$$

Similarly the far field component at point $P$ due to the point source $A_{2}$ is given by,

$$
\begin{equation*}
E_{2}=E_{0} \cdot e^{j \frac{V}{2}} \tag{6}
\end{equation*}
$$

Note that the amplitude of both the field components is $\mathrm{E}_{0}$ as currents are same and the point sources are identical.
The total field at point $P$ is given by,

$$
\begin{aligned}
& E_{T}=E_{1}+E_{2}=E_{0} \cdot e^{-j \frac{\psi}{2}}+E_{0} \cdot e^{i \frac{\psi}{2}} \\
& \therefore \quad E_{T}=E_{0}\left(e^{-j \frac{\psi}{2}}+e^{j \frac{\psi}{2}}\right)
\end{aligned}
$$

Rearranging the terms on R.H.S., we get,

$$
\begin{equation*}
\therefore \quad \mathrm{E}_{\mathrm{T}}=2 \mathrm{E}_{0}\left(\frac{\mathrm{e}^{\mathrm{j} \frac{\psi}{2}}+\mathrm{e}^{-\mathrm{j} \frac{\psi}{2}}}{2}\right) \tag{7}
\end{equation*}
$$

By trigonometric identity,
$\frac{\mathrm{e}^{\mathrm{j} \theta}+\mathrm{e}^{-\boldsymbol{\theta} \theta}}{2}=\cos \theta$.
Hence equation (7) can be written as,

$$
\begin{equation*}
\mathrm{E}_{\mathbf{T}}=2 \mathrm{E}_{0} \cos \left(\frac{\psi}{2}\right) \tag{8}
\end{equation*}
$$

Substituting value of $\Psi$ from equation (4), we get,.

$$
\begin{equation*}
\therefore \quad \mathrm{E}_{\mathrm{T}}=2 \mathrm{E}_{0} \cos \left(\frac{\beta \mathrm{~d} \cos \phi}{2}\right) \tag{9}
\end{equation*}
$$

Above equation represents total field in intensity at point $P$. due to two point sources having currents of same amplitude and phase. The total amplitude of the field at point $P$ is $2 E_{0}$ while the phase shift is $\beta$ dcosv/2

The array factor is the ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$
\therefore \quad \text { A.F. } \left.=\frac{\left|\mathrm{E}_{\mathrm{T}}\right|}{\left|\mathrm{E}_{\max }\right|} \right\rvert\,
$$

But maximum field is Ernax $=2 \mathrm{E}_{0}$

$$
\therefore \quad \text { A.F. }=\frac{\left|\mathrm{E}_{T}\right|}{\left|2 \mathrm{E}_{0}\right|}=\cos \left(\pi \frac{\mathrm{d}}{\lambda} \cos \phi\right)
$$

The array factor represents the relative value of the field as a function of $v$ defines the radiation pattern in a plane containing the line of the array.

## Maxima direction

From equation (9), the total field is maximum when ${ }^{\cos \left(\frac{\beta d \cos \phi}{2}\right)_{\text {is maximum. As we know, }} \text {. }{ }^{\text {m }} \text {. }}$ the variation of cosine of a angle is $\pm 1$. Hence the condition for maxima is given by,

$$
\cos \left(\frac{\beta \mathrm{d} \cos \phi}{2}\right)= \pm 1
$$

Let spacing between the two point sources be $\lambda / 2$. Then we can write,

$$
\begin{equation*}
\cos \left[\frac{\beta(\lambda / 2) \cos \phi}{2}\right]= \pm 1 \tag{10}
\end{equation*}
$$

i.e. $\cos \left[\frac{\frac{2 \pi}{\lambda} \cdot \frac{\lambda}{2} \cos \phi}{2}\right]= \pm 1$.
i.e. $\cos \left(\frac{\pi}{2} \cos \phi\right)= \pm 1$
i.e. $\frac{\pi}{2} \cos \phi_{\max }=\cos ^{-1}( \pm 1)= \pm n \pi$, where $n=0,1,2, \ldots \ldots .$.

If $\mathrm{n}=0$, then

$$
\frac{\pi}{2} \cos \phi_{\max }=0
$$

i.e. $\quad \cos \phi_{\max }=0$
i.e.

$$
\begin{equation*}
\phi_{\text {max }}=90^{\circ} \text { or } 270^{\circ} \tag{11}
\end{equation*}
$$

## Minima direction

Again from equation (9), total field strength is minimum when $\cos \left(\frac{\beta \mathrm{d} \cos \phi}{2}\right)$ is minimum i.e.
0 as cosine of angle has minimum value 0 . Hence the condition for minima is given by,

$$
\begin{equation*}
\therefore \quad \cos \left(\frac{\beta \mathrm{d} \cos \phi}{2}\right)=0 \tag{12}
\end{equation*}
$$

Again assuming $d=\lambda / 2$ and $\beta=2 \pi / \lambda$, we can write

$$
\begin{aligned}
& \cos \left(\frac{\pi}{2} \cos \phi_{\text {min }}\right)=0 \\
& \therefore \quad \frac{\pi}{2} \cos \phi_{\min }=\cos ^{-1} 0= \pm(2 n+1) \frac{\pi}{2}, \text { where } n=0,1,2, \ldots \ldots \ldots .
\end{aligned}
$$

$$
\text { If } \mathrm{n}=0 \text {, then, }
$$

$$
\frac{\pi}{2} \cos \phi_{\min }= \pm \frac{\pi}{2}
$$

i.e. $\cos \phi_{\min }= \pm 1$
i.e.

$$
\begin{equation*}
\phi_{\min }=0^{\circ} \text { or } 180^{\circ} \tag{13}
\end{equation*}
$$

## Half power point direction:

When the power is half, the voltage or current is $1 / \sqrt{ } 2$ times the maximum value. Hence the condition for half power point is given by,

$$
\begin{equation*}
\cos \left(\frac{\beta \mathrm{d} \cos \phi}{2}\right)= \pm \frac{1}{\sqrt{2}} \tag{14}
\end{equation*}
$$

Let $d=\lambda / 2$ and $\beta=2 \pi / \lambda$, then we can write,
$\cos \left(\frac{\pi}{2} \cos \phi\right)= \pm \frac{1}{\sqrt{2}}$
i.e. $\frac{\pi}{2} \cos \phi=\cos ^{-1}\left( \pm \frac{1}{\sqrt{2}}\right)= \pm(2 n+1) \frac{\pi}{4}$, where $n=0,1,2, \ldots$

If $\mathrm{n}=0$, then

$$
\begin{align*}
\frac{\pi}{2} \cos \phi_{\mathrm{HPPD}} & = \pm \frac{\pi}{4} \\
\text { i.e. } \cos \phi_{\mathrm{HPPD}} & = \pm \frac{1}{2} \\
\text { i.e. } \phi_{\mathrm{HPPD}} & =\cos ^{-1}\left( \pm \frac{1}{2}\right) \\
\therefore \quad \phi_{\mathrm{HPPD}} & =60^{\circ} \text { or } 120^{\circ} \tag{15}
\end{align*}
$$

The field pattern drawn with $\mathrm{E}_{\top}$ against $v$ for $\mathrm{d}=\lambda / 2$, then the pattern is bidirectional as shown in Fig 6. The field pattern obtained is bidirectional and it is a figure of eight.
If this pattern is rotated by $360^{\circ}$ about axis, it will represent three dimensional doughnut shaped space pattern. This is the simplest type of broadside array of two point sources and it is called Broadside couplet as two radiations of point sources are in phase.


Fig. 6 Field pattern for two point source with spacing $d=\lambda / 2$ and fed with currents equal in magnitude andphase.

