Two Point Sources with Currents Equal in Magnitudes but Opposite in Phase

Consider two point sources separated by distance d and supplied with currents equal in magnitude but opposite in phase. Consider Fig. 5 all the conditions are exactly same except the phase of the currents is opposite i.e. 180°. With this condition, the total field at far point P is given by,

$$E_T = (-E_1) + (E_2) \quad \ldots(1)$$

Assuming equal magnitudes of currents, the fields at point P due to the point sources A_1 and A_2 can be written as,

$$E_1 = E_0 e^{-j\frac{\psi}{2}} \quad \ldots(2)$$

and

$$E_2 = E_0 e^{j\frac{\psi}{2}} \quad \ldots(3)$$

Substituting values of E_1 and E_2 in equation (1), we get

$$E_T = -E_0 \cdot e^{-j\frac{\psi}{2}} + E_0 \cdot e^{j\frac{\psi}{2}}$$

$$\therefore \quad E_T = E_0\left(-e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right)$$

Rearranging the terms in above equation, we get,

$$\therefore \quad E_T = (j2)\, E_0\left(\frac{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}}{j2}\right) \quad \ldots(4)$$

By trigonometry identity,

$$\frac{e^{j\theta} - e^{-j\theta}}{2} = \sin\frac{\theta}{2}.$$

Equation (4) can be written as,

$$E_T = j2E_0 \sin\left(\frac{\psi}{2}\right) \quad \ldots(5)$$

Now as the condition for two point sources with currents in phase and out of phase is exactly same, the phase angle can be written as previous case.

Phase angle = $\beta d \cos\upsilon$ ...(6) Substituting value of phase angle in equation (5), we get,

$$\boxed{E_T = j(2E_0)\sin\left(\frac{\beta d \cos\phi}{2}\right)} \quad \ldots(7)$$

Maxima direction

From equation (7), the total field is maximum when $\sin\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum i.e. ±1 as the maximum value of sine of angle is ±1. Hence condition for maxima is given by,

$$\boxed{\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1} \qquad \ldots(8)$$

Let the spacing between two isotropic point sources be equal to d=λ/2
Substituting d=λ/2 and β=2π/λ, in equation (8), we get,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\text{i.e.} \quad \frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, \ldots\ldots$$

If n = 0. then

$$\frac{\pi}{2}\cos\phi_{max} = \pm\frac{\pi}{2}$$

$$\text{i.e. } \cos\phi_{max} = \pm 1$$

$$\text{i.e.} \quad \boxed{\phi_{max} = 0° \text{ and } 180°} \qquad \ldots(9)$$

Minima direction

Again from equation (7), total field strength is minimum when $\sin\left(\frac{\beta d \cos\phi}{2}\right)$ is minimum i.e. 0.

Hence the condition for minima is given by,

$$\boxed{\sin\left(\frac{\beta d \cos\phi}{2}\right) = 0} \qquad \ldots(10)$$

Assuming d=λ/2 and β=2π/λ in equation (10), we get,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\text{i.e.} \quad \frac{\pi}{2}\cos\phi = \pm n\pi, \text{ where } n = 0, 1, 2, \ldots\ldots$$

If n = 0, then

$$\frac{\pi}{2}\cos\phi_{min} = 0$$

$$\text{i.e. } \cos\phi_{min} = 0$$

$$\text{i.e.} \quad \boxed{\phi_{min} = +90° \text{ or } -90°} \qquad \ldots(11)$$

Half Power Point Direction (HPPD)

When the power is half of maximum value, the voltage or current equals to $1/\sqrt{2}$ times the respective maximum value. Hence the condition for the half power point can be obtained from equation (7) as,

$$\boxed{\sin\left(\frac{\beta d\cos\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}} \qquad \ldots(12)$$

Let $d=\lambda/2$ and $\beta=2\pi/\lambda$, we can write,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\text{i.e. } \frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{4}, \quad \text{where } n = 0, 1, 2.$$

If $n = 0$, we can write,

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\text{i.e. } \cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\therefore \quad \boxed{\phi_{HPPD} = 60° \text{ or } 120°} \qquad \ldots(13)$$

Thus from the conditions of maxima, minima and half power points, the field pattern can be drawn as shown in the Fig. 7.

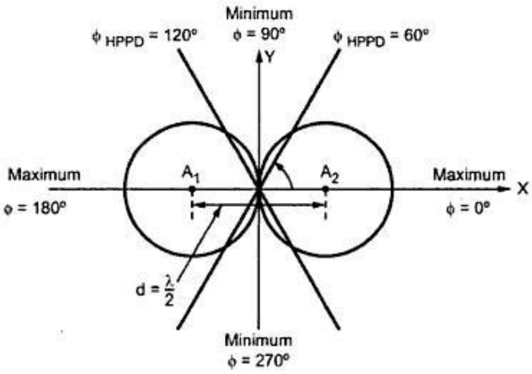

Fig. 7 **Field pattern for two point sources with spacing d = d=λ/2 and fed with currents equal in magnitude but out of phase by 180^0.**

As compared with the field pattern for two point sources with inphase currents, the maxima have shifted by 90° along X-axis in case of out-phase currents in two point source array. Thus the maxima are along the axis of the array or along the line joining two point sources. In first case, we have obtained vertical figure of eight. Now in above case, we have obtained horizontal figure of eight. As the maximum field is along the line joining the two point sources, this is the simple type of the end fire array.

Two point sources with currents unequal in magnitude and with any phase

Let us consider Fig. 5. Assume that the two point sources are separated by distance d and supplied with currents which are different in magnitudes and with any phase difference say α. Consider that source 1 is assumed to be reference for phase and amplitude of the fields E_1 and E_2, which are due to source 1 and source 2 respectively at the distant point P. Let us assume that E_1 is greater than E_2 in magnitude as shown in the vector diagram in Fig. 8.

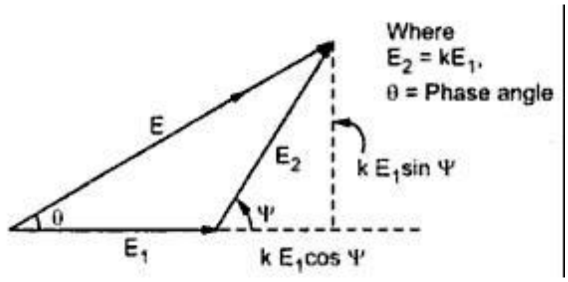

Fig. 8 Vector diagram of fields E_l and E_2

Now the total phase difference between the radiations by the two point sources at any far point P is given by,

$$\boxed{\Psi = \frac{2\pi}{\lambda}\cos\phi + \alpha} \qquad \dots (1)$$

where α is the phase angle with which current I_2 leads current I_l. Now if α = 0, then the condition is similar to the two point sources with currents equal in magnitude and phase. Similarly if α = 180", then the condition is similar to the two point source with currents equal in magnitude but opposite in phase. Assume value of phase difference as $0 < \alpha < 180^0$. Then the resultant field at point P is given by,

$$E_T = E_1 e^{j0} + E_2 e^{j\psi}$$ ...(source 1 is assumed to be reference hence phase angle is 0)

$$\therefore \quad E_T = E_1 + E_2 e^{j\psi}$$

$$\therefore \quad E_T = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

Let $$\frac{E_2}{E_1} = k \qquad \dots(2)$$

Note that $E_1 > E_2$, the value of k is less than unity. Moreover the value of k is given by, $0 \leq k \leq 1$

$$\therefore \quad \boxed{E_T = E_1 [1 + k(\cos\psi + j\sin\psi)]} \qquad \dots (3)$$