The magnitude of the resultant field at point P is given by,

$$|E_T| = |E_1 [1 + k\cos\psi + j\,k\sin\psi]|$$

$$\therefore \quad \boxed{|E_T| = E_1 \sqrt{(1+k\cos\psi)^2 + (k\sin\psi)^2}} \quad \ldots (4)$$

The phase angle between two fields at the far point P is given by,

$$\therefore \quad \boxed{\theta = \tan^{-1} \frac{k\sin\psi}{1+k\cos\psi}} \quad \ldots (5)$$

n Element Uniform Linear Arrays

At higher frequencies, for point to point communications it is necessary to have a pattern with single beam radiation. Such highly directive single beam pattern can be obtained by increasing the point sources in the arrow from 2 to n say. An array of n elements is said to be linear array if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line. Consider a general n element linear and uniform array with all the individual elements spaced equally at distance d from each other and all elements are fed with currents equal in magnitude and uniform progressive phase shift along line as shown in the Fig. 9.

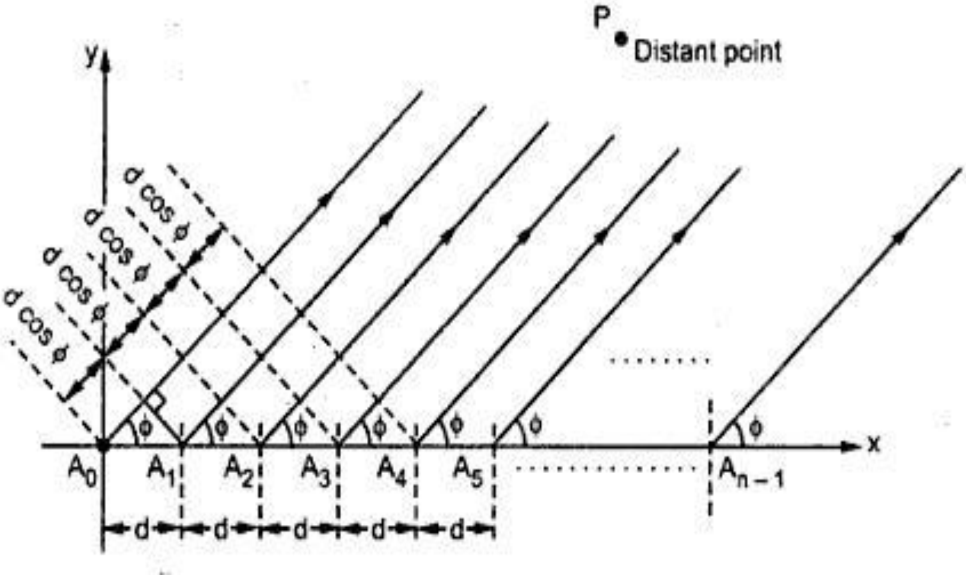


Fig. 9 Uniform, linear array of n elements

The total resultant field at the distant point P is obtained by adding the fields due to n individual sources vectorically. Hence we can write,

$$E_T = E_0 \cdot e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \ldots + E_0 e^{j(n-1)\psi}$$

$$\therefore \quad \boxed{E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}]} \quad \ldots (1)$$

Note that $\psi = (\beta d \cos\psi + \alpha)$ indicates the total phase difference of the fields from adjacent sources calculated at point P. Similarly α is the progressive phase shift between two adjacent point sources. The value of α may lie between 0^0 and 180^0. If $\alpha = 0^0$ we get n element uniform linear broadside array. If $\alpha = 180^0$ we get n element uniform linear endfire array.

Multiplying equation (1) by $e^{j\psi}$, we get,

$$E_T e^{j\psi} = E_0 [e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{jn\psi}] \quad \ldots (2)$$

Subtracting equation (2) from (1), we get,

$$E_T - E_T e^{j\psi} = E_0 \{[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}] - [e^{j\psi} + e^{j2\psi} + \ldots + e^{jn\psi}]\}$$

$$E_T (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right]} \quad \ldots (3)$$

Simply mathematically, we get

$$E_T = E_0 \left[\frac{e^{j\frac{n\psi}{2}}\left(e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

According to trigonometric identity,

$$e^{-j\theta} - e^{j\theta} = -2j \sin\theta,$$

The resultant field is given by,

$$E_T = E_0 \left[\frac{\left(-j2\sin\frac{n\psi}{2}\right)e^{j\frac{n\psi}{2}}}{\left(-j2\sin\frac{\psi}{2}\right)e^{j\frac{\psi}{2}}}\right]$$

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] e^{j\left(\frac{n-1}{2}\right)\psi}} \quad \ldots (4)$$

This equation (4) indicates the resultant field due to n element array at distant point P. The magnitude of the resultant field is given by, The phase angle θ of the resultant field at point P is given by,

$$\therefore \quad \boxed{\theta = \frac{(n-1)}{2}\psi = \frac{(n-1)}{2}\ \beta\, d \cos\phi + \alpha} \qquad \ldots (6)$$

Array of n elements with Equal Spacing and Currents Equal in Magnitude and Phase • Broadside Array

Consider 'n' number of identical radiators carries currents which are equal in magnitude and in phase. The identical radiators are equispaced. Hence the maximum radiation occurs in the directions normal to the line of array. Hence such an array is known as Uniform broadside array. Consider a broadside array with n identical radiators as shown in the Fig. 10.

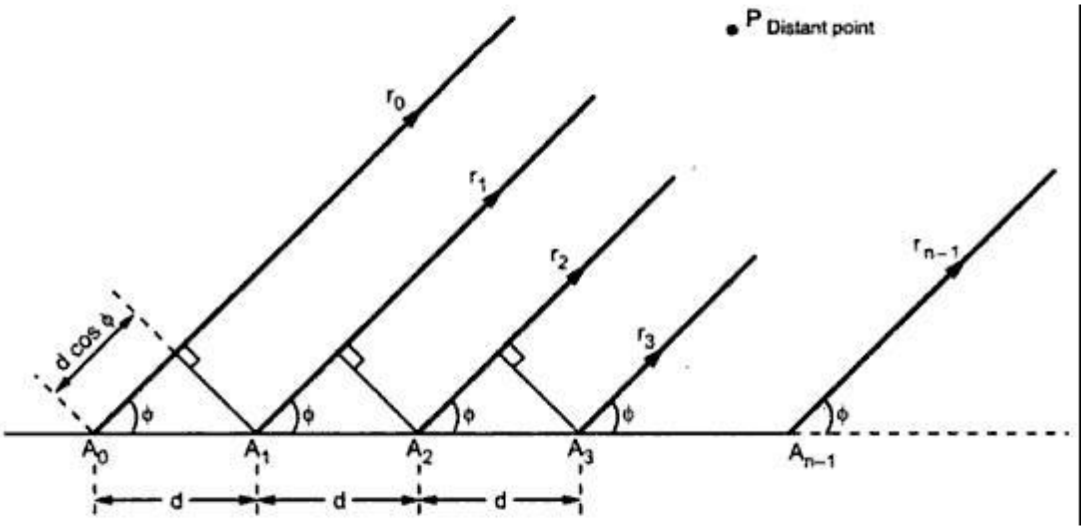


Fig 10 Array of n elements with Equal Spacing

The electric field produced at point P due to an element A_0 is given by,

$$E_0 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \qquad \ldots (1)$$

As the distance of separation d between any two array elements is very small as compared to the radial distances of point P from A_0, A_1, ...A_{n-1}, we can assume r_0, r_1, ...r_{n-1} are approximately same.

Now the electric field produced at point P due to an element A_1 will differ in phase as r_0 and r_1 are not actually same. Hence the electric field due to A_1 is given by,

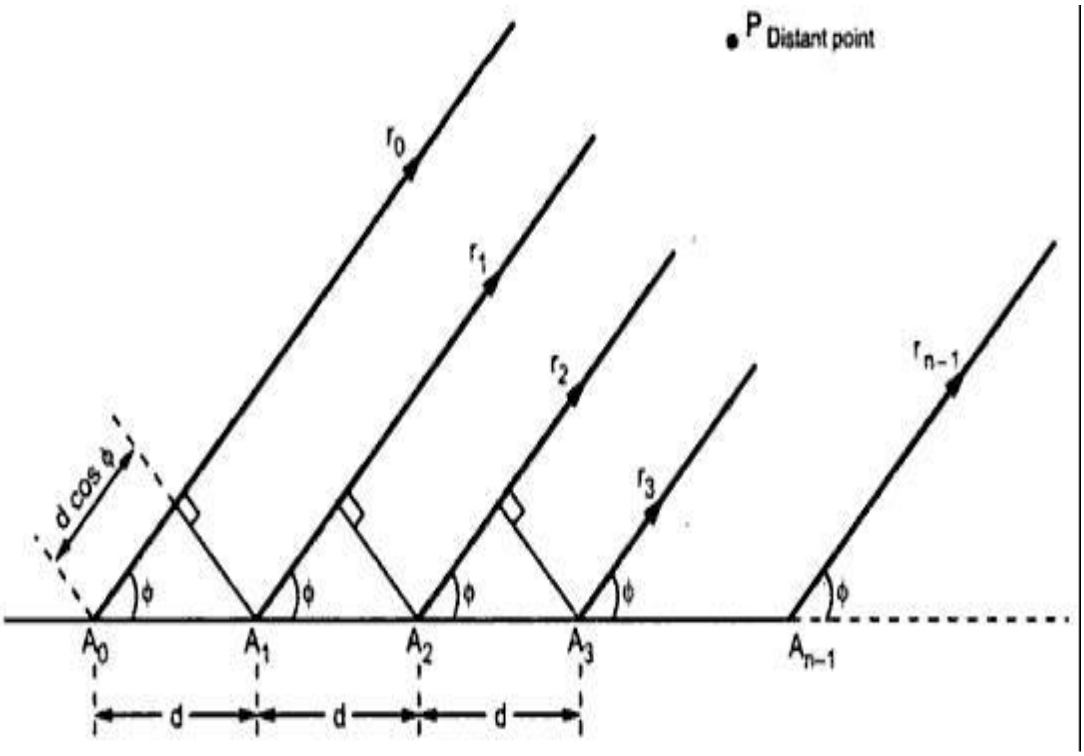
$$\therefore \quad \boxed{E_T = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} \right]} \qquad \dots (5)$$

Exactly on the similar lines we can write the electric field produced at point P due to an element A_2 as,

$$E_2 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_2}\right] e^{-j\beta r_2}$$

$$\therefore \quad E_2 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta(r_1 - d\cos\phi)} \qquad \dots r_2 = r_1 - d\cos\phi$$

$$\therefore \quad E_2 = \left\{\frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta r_1}\right\} e^{j\beta d\cos\phi}$$



But the term inside the bracket represent E_1

$$\therefore \quad E_2 = E_1 e^{j\beta d\cos\phi}$$

From equation (2), substituting the value of E_1, we get,

$$E_2 = \left[E_0 e^{j\beta d\cos\phi}\right] e^{j\beta d\cos\phi}$$

$$\therefore \quad E_2 = E_0 \cdot e^{j2\beta d\cos\phi} \qquad \text{... (3)}$$

Similarly, the electric field produced at point P due to element A_{n-1} is given by,

$$E_{n-1} = E_0 \cdot e^{j(n-1)\beta d\cos\phi} \qquad \text{... (4)}$$

The total electric field at point P is given by,

$$E_T = E_0 + E_1 + E_2 + \ldots + E_{n-1}$$

$$\therefore \quad E_T = E_0 + E_0\, e^{j\beta d\cos\phi} + E_0\, e^{j2\beta d\cos\phi} + \ldots + E_0\, e^{j(n-1)\beta d\cos\phi}$$

Let $\beta d\cos\upsilon = \upsilon$, then rewriting above equation,

$$E_T = E_0 + E_0\, e^{j\psi} + E_0\, e^{j2\psi} + \ldots + E_0\, e^{j(n-1)\psi}$$

$$\therefore \quad E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}\right] \qquad \text{... (5)}$$

Consider a series given by $s = 1 + r + r^2 + \ldots + r^{n-1}$ where $r = e^{j\upsilon}$ ... (i)

Multiplying both the sides of the equation (i) by r, $s \cdot r = r + r^2 + \ldots + r^n$ ... (ii)

Subtracting equation (ii) from (i), we get. $s(1-r) = 1-r^n$

$$\therefore \quad s = \frac{1-r^n}{1-r} \qquad \text{... (iii)}$$

Using equation (iii), equation (5) can be modified as,

$$E_T = E_0\left[\frac{1-e^{jn\psi}}{1-e^{j\psi}}\right]$$

$$\therefore \qquad \frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right]}{e^{j\frac{\psi}{2}}\left[e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right]} \qquad \text{... (6)}$$

From the trigonometric identities,

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\text{and} \quad e^{-j\theta} - e^{j\theta} = -j\,2\sin\theta$$

Equation (6) can be written as,

$$\frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{n\psi}{2}\right)\right]}{e^{j\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{\psi}{2}\right)\right]}$$

$$\therefore \qquad \frac{E_T}{E_0} = e^{j\frac{(n-1)\psi}{2}}\left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \qquad \text{... (7)}$$

The exponential term in equation (7) represents the phase shift. Now considering magnitudes of the electric fields, we can write,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \qquad \text{... (8)}$$

Properties of Broadside Array

1. **Major lobe**

   In case of broadside array, the field is maximum in the direction normal to the axis of the array. Thus the condition for the maximum field at point P is given by,

   $$\psi = 0 \quad \text{i.e.} \quad \beta d\cos\phi = 0 \qquad \text{...(9)}$$

   $$\text{i.e.} \quad \cos\phi = 0$$

   $$\text{i.e.} \quad \boxed{\phi = 90^\circ \text{ or } 270^\circ} \qquad \text{...(10)}$$

   Thus $\upsilon = 90^0$ and 270^0 are called directions of principle maxima.

2. **Magnitude of major lobe**

The maximum radiation occurs when υ=0. Hence we can write,

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\}$$

$$= \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)} \right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \qquad \ldots(11)$$

where, n is the number of elements in the array.

Thus from equation (10) and (11) it is clear that, all the field components add up together to give total field which is 'n' times the individual field when $\upsilon = 90^0$ and 270^0.

3. **Nulls**

The ratio of total electric field to an individual electric field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

Equating ratio of magnitudes of the fields to zero,

$$\therefore \quad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

The condition of minima is given by,

$$\therefore \quad \boxed{\sin n\frac{\psi}{2} = 0; \quad \text{but } \sin\frac{\psi}{2} \neq 0} \qquad \ldots(12)$$

Hence we can write,

$$\sin n\frac{\psi}{2} = 0$$

$$\text{i.e. } n\frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

$$\text{Now } \psi = \beta d\cos\phi = \frac{2\pi}{\lambda}(d)\cos\phi$$

$$\therefore \quad \frac{n}{2}\left(\frac{2\pi}{\lambda}d\right)\cos\phi_{min} = \pm m\pi$$

$$\text{i.e.} \quad \frac{nd}{\lambda}\cos\phi_{min} = \pm m$$

$$\therefore \quad \boxed{\phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)} \quad \ldots(13)$$

where, n= number of elements in the array d=

spacing between elements in meter

λ= wavelength in meter

m= constant= 1, 2 , 3....

Thus equation (13) gives direction of nulls

4. Side Lobes Maxima

The directions of the subsidary maxima or side lobes maxima can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \quad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots} \quad \ldots(14)$$

Hence sin(nυ/2), is not considered. Because if nυ/2=π/2 then sin nυ/2 =1 which is the direction of principle maxima.

Hence we can skip sin nυ/2 = ±π/2 value Thus, we get

$$\psi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \ldots\ldots\ldots\ldots$$

$$\text{Now} \quad \psi = \beta d\cos\phi = \left(\frac{2\pi}{\lambda}\right)d\cos\phi$$

Now equation for υ can be written as,

$$\frac{2\pi}{\lambda}d\cos\phi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \ldots\ldots\ldots\ldots$$

$$\therefore \quad \cos\phi = \frac{\lambda}{2\pi d}\left[\pm\frac{(2m+1)}{n}\pi\right] \quad \text{where } m = 1, 2, 3, \ldots\ldots\ldots$$

$$\therefore \quad \boxed{\phi = \cos^{-1}\left[\pm\frac{\lambda(2m+1)}{2nd}\right]} \quad \ldots(15)$$

The equation (15) represents directions of subsidary maxima or side lobes maxima.