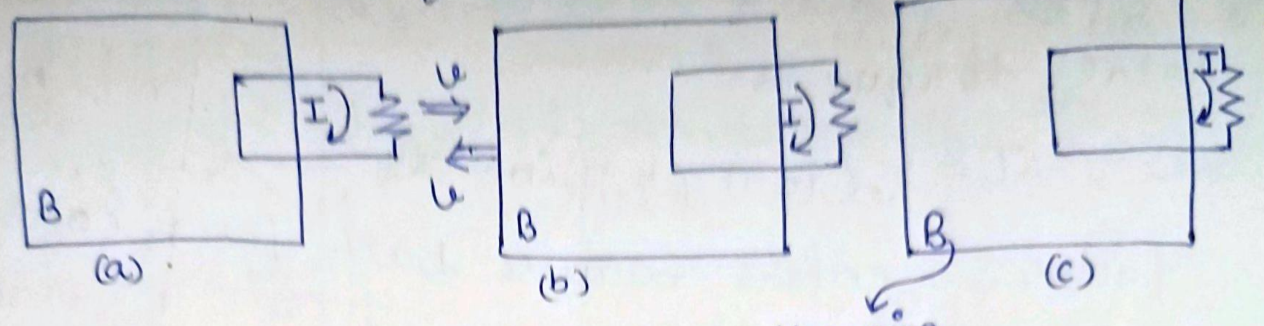
Faraday's law →

In 1831 Michael Faraday reported on a series of experiments, including three that can be characterized as follows:-



Experiment (1) Pulled a loop of wire to the right through a magnetic field (a). A current flowed in the loop.

Experiment (2) Moved the magnet to the left holding the loop still, Again, a current flowed in the loop.

Experiment (3) With both the loop and the magnet at rest, changed the strength of the field. Once again, current flowed in the loop.

An motional electromotive force (emf)

$$\mathcal{E} = -\frac{d\phi}{dt}$$

A changing magnetic field induces an electric field.

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

then $\vec{E}$ is related to the change in $\vec{B}$ by the equation,

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t} \cdot da}$$

this is faraday's law in integral form

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

this is faraday's law in differential form

the induced electric field →

there are two distinct kinds of electric field: those attributable directly to electric charges and those associated with changing magnetic fields. the former can be calculated using coulomb's law, and latter can be found by faraday's law. $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ and

Ampere's law. $\nabla \times \vec{B} = \mu_0 \vec{J}$.

By Gauss's law $\nabla \cdot \vec{E} = 0$ if $\rho = 0$.

for magnetic fields $\nabla \cdot \vec{B} = 0$ always.

the faraday-induced electric fields are determined by $\left(-\frac{\partial \vec{B}}{\partial t}\right)$ in exactly the same way as magnetostatic fields are determined by $\mu_0 \vec{J}$.

In integral form $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$.

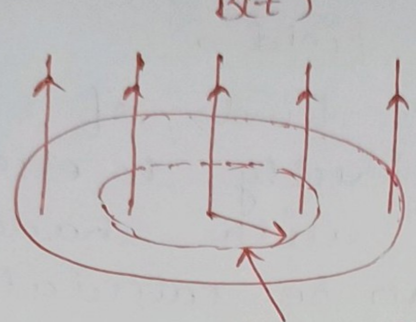
faraday's law in integral form.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

the rate of change of magnetic flux through the amperian loop plays the role formerly assigned to $\mu_0 I$.

Q. A uniform magnetic field $\vec{B}(t)$, pointing straight up, fills the shaded circular region. If $\vec{B}$ is changing with time, what is the induced electric field?

Sol.



Amperian loop of radius r.

$$\oint \vec{E}\cdot d\vec{l} = E \times 2\pi r = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\left(\pi r^2 B(t)\right)$$

$$= -\pi r^2 \frac{dB(t)}{dt}$$

$d\phi = B \cdot da$

$$\oint \vec{E}\cdot d\vec{l} = -\pi r^2 \frac{dB(t)}{dt}.$$

So $E \times 2\cancel{\pi} r = -\cancel{\pi} r^2 \dfrac{dB(t)}{dt}$

$$\boxed{\vec{E} = \frac{-r}{2}\frac{dB}{dt}\hat{\phi}}$$

If B is increasing, E runs clockwise.