Field Strength of Space Wave →

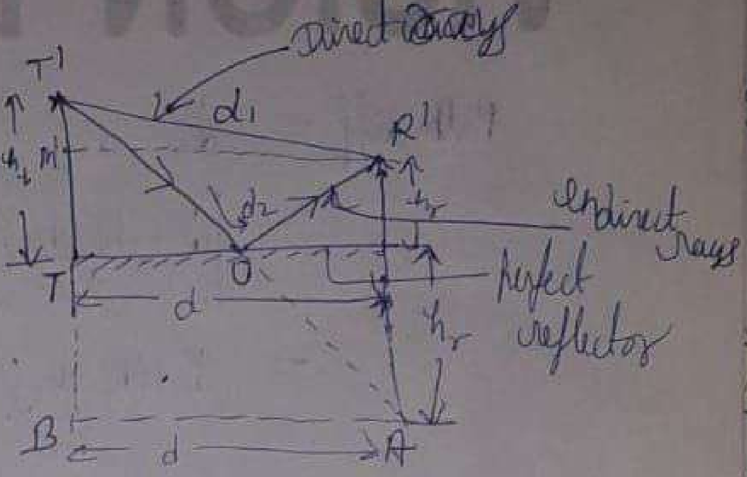
Let us consider the earth is flat (i.e curvature of earth is neglected.

Let h_t → height of transmitting antenna

h_r → ,, ,, receiving antenna

d → distance b/w T_x & R_x Antenna

d_1 → direct rays path, d_2 → Indirect rays path



In $\Delta T'R'M'$ →

$$T'R' = T'M'^2 + M'R'^2$$

$$d_1^2 = d^2 + (h_t - h_r)^2 \rightarrow (1)$$

In $\Delta T'BA$ →

$$d_2^2 = (h_t + h_r)^2 + d^2 \rightarrow (2)$$

from eqn (1) →

$$d_1 = \left[d^2 + (h_t - h_r)^2\right]^{1/2}$$

$$d_1 = d\left\{1 + \left(\frac{h_t - h_r}{d}\right)^2\right\}^{1/2}$$

$$d_1 = d\left\{1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right\}$$

$$\boxed{d_1 = d + \frac{1}{2d}(h_t - h_r)^2} \rightarrow (3)$$

from (2) →

$$d_2 = \left\{d\left(1 + \left(\frac{h_t + h_r}{d}\right)^2\right)\right\}^{1/2}$$

$$\boxed{d_2 = d + \frac{1}{2d}(h_t + h_r)^2} \rightarrow (4)$$

Let E_0 → field strength due to direct signal (rays)

Let KE_0 is field strength due to indirect rays.

Where K → ground reflection coefficient

Path diff b/w direct & indirect rays →

$$E_R = E_0 \left[1 + k(\cos\theta - j\sin\theta)\right] = E_0\left\{(1+k\cos\theta) - kj\sin\theta\right\}$$

$$|E_R| = E_0\sqrt{(1+k\cos\theta)^2 + k^2\sin^2\theta}$$

$$|E_R| = E_0\sqrt{1+k^2\cos^2\theta + 2k\cos\theta + k^2\sin^2\theta}$$

$$|E_R| = E_0\sqrt{1+2k\cos\theta + k^2}$$

assume $k=1$ (becoz earth to be perfect) $\beta = 180°$

$$|E_R| = E_0\sqrt{1+2\cos\theta+1}$$

$$= E_0\sqrt{2+2\left(2\cos^2\frac{\theta}{2}-1\right)}$$

$$= E_0\sqrt{\cancel{2}+4\cos^2\frac{\theta}{2}-\cancel{2}}$$

$$|E_R| = E_0 \times 2\cos\frac{\theta}{2}$$

$$= 2E_0\cos\left(\frac{\alpha+\pi}{2}\right)$$

$$|E_R| = 2E_0\sin\frac{\alpha}{2}$$

$$|E_R| = 2E_0\sin\left(\frac{4\pi h_t h_r}{2\lambda d}\right) \rightarrow (6)$$

as $d >> h_t, h_r$

so $\sin\left(\frac{4\pi h_t h_r}{2\lambda d}\right) \simeq \frac{4\pi h_t h_r}{2\lambda d}$

by eqn (6) $|E_R| = \cancel{2}E_0 \times \frac{4\pi h_t h_r}{\cancel{2}\lambda d}$

$$\boxed{|E_R| = \frac{E_0\, 4\pi h_t h_r}{\lambda d}} \quad \text{or} \quad \boxed{E_R = \frac{4\pi E_0\, h_t h_r}{\lambda d}}$$

Path diff $= d_2 - d_1$

$$= \left[d + \frac{(h_t+h_r)^2}{2d}\right] - \left[d + \frac{(h_t-h_r)^2}{2d}\right]$$

$$= \cancel{d} + \frac{(h_t+h_r)^2}{2d} - \cancel{d} - \frac{(h_t-h_r)^2}{2d}$$

$$= \frac{1}{2d}\left\{\cancel{h_t^2} + \cancel{h_r^2} + 2h_th_r - \cancel{h_t^2} - \cancel{h_r^2} + 2h_th_r\right\}$$

$$= \frac{4h_th_r}{2d}$$

$$\boxed{\text{Path diff} = \frac{2h_th_r}{d}}$$

Phase diff $= \frac{2\pi}{\lambda}$ (Path diff)

$$\text{Phase diff} = \frac{2\pi}{\lambda} \times \frac{2h_th_r}{d}$$

$$\boxed{\text{Phase diff } \alpha = \frac{4\pi h_th_r}{\lambda d}} \rightarrow 5$$

Phase diff due to reflection from ground $\beta = 180^0$

Total phase diff is $\theta = \alpha + \beta$

Resultant field strength at receiver point $\rightarrow E = E_0 + KE_0$

$$E_R = E_0 (1 + Ke^{-j\theta})$$